HADRONIC VACUUM POLARIZATION AND $(g-2)_\mu$

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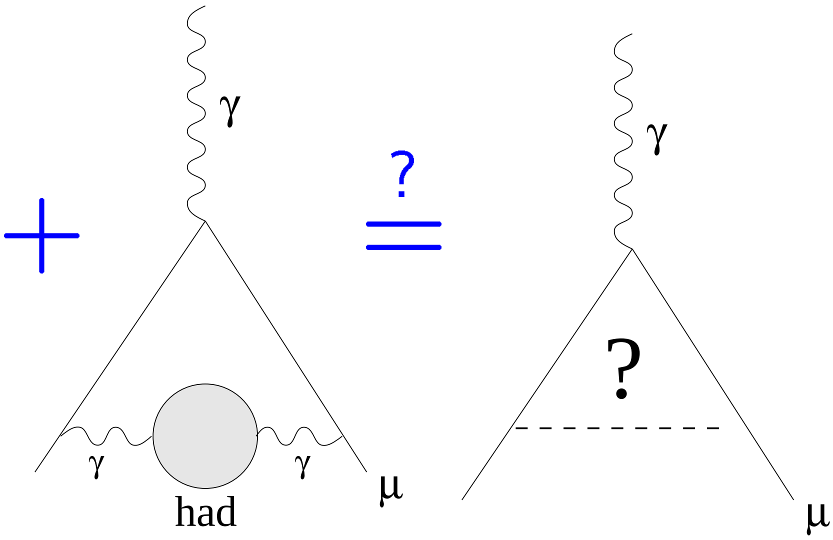
Hommage to BNL $(g-2)_\mu$ Collaboration



Outline

Introduction

- Muon Magnetic Anomaly and
- Hadronic Vacuum Polarization

The BNL Discrepancy and Who is Wrong ?

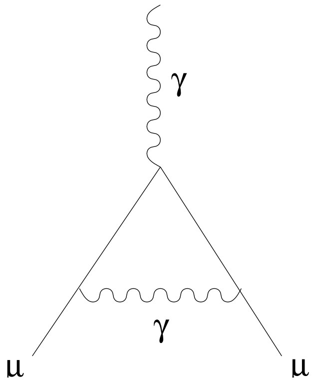
Contribution from Hadronic Vacuum Polarization

(and all what we have learned about QCD from hadronic τ decays)

- τ Data, e^+e^- Data and CVC
- Perturbative QCD
- Theoretical Constraints using Cauchy Integrals

Discussion of Critical Issues and Outlook

MAGNETIC ANOMALY



QED prediction: $\Gamma_\mu = e\gamma_\mu + a\frac{i\,e}{2m}\sigma_{\mu\nu}q_\nu$

Schwinger 1948 (Nobel 1965):

$$a = \alpha/2\pi = 0.001161\ldots$$

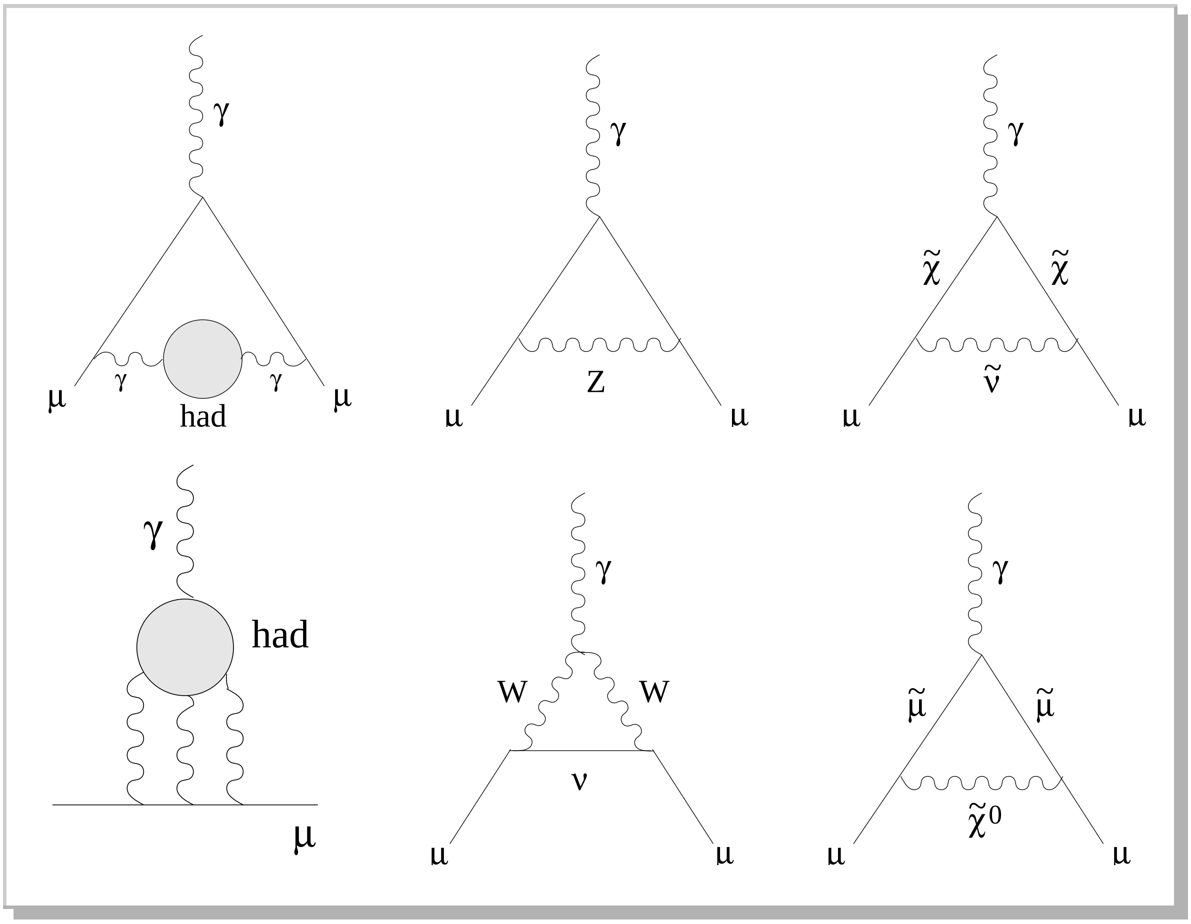
Computed up to $4^{\rm th}$ order [Kinoshita *et al.*]

($5^{\rm th}$ order estimated [Mohr, Taylor]):

$$a_\mu^{\rm QED} \simeq \sum_{n=1} C_n \left(\frac{\alpha}{\pi}\right)^n$$

$$\simeq (11614098.1 + 41321.8 + 3014.2 + 36.7 + 0.6) \times 10^{-10}$$

Other sources:



THE MUONIC $(g-2)_\mu$

Contributions to the Standard Model Prediction:

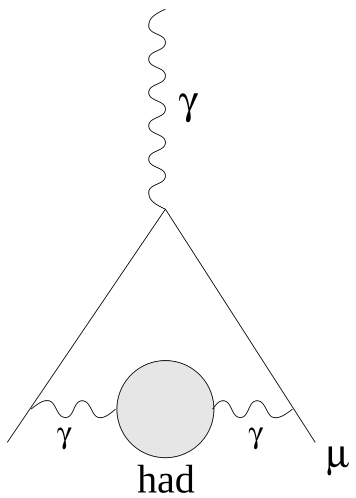
$$a_\mu(\mathrm{SM}) \equiv \left(\frac{g-2}{2}\right)_\mu = a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{had}} + a_\mu^{\mathrm{weak}} ,$$

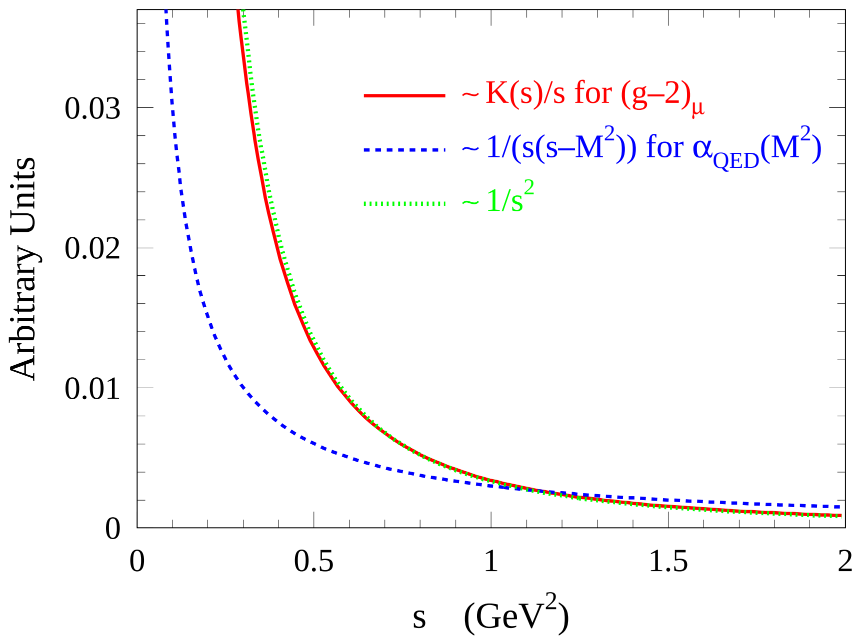
Situation 1995:

Source	$\sigma(a_\mu)$	References
QED	$\simeq 3 \times 10^{-11}$	[Schwinger, +others]
Z, W exchange	$\simeq 4 \times 10^{-11}$	[Czarnecki *et al*'95, +others]
Quarks and Hadrons	$\sim (150 \oplus 40) \times 10^{-11}$	[Eidelman-Jegerlehner'95, +others]

Dominant uncertainty stems from lowest order hadronic contribution, which cannot be calculated from QCD ("first principles"). Fortunately we can use experiment . . .

$$a_\mu^{\mathrm{had}} = \frac{\alpha^2}{3\pi^2}\int\limits_{4m_\pi^2}^{\infty} ds\,\frac{K(s)}{s}R(s)$$





HADRONIC VACUUM POLARIZATION

Define: photon vacuum polarization function $\Pi_\gamma(q^2)$

$$i\int d^4x\, e^{iqx}\langle 0|T(J^\mu_{\rm em}(x)(J^\nu_{\rm em}(0))^\dagger)|0\rangle \;=\; -\,(g^{\mu\nu}q^2 - q^\mu q^\nu)\,\Pi_\gamma(q^2)$$

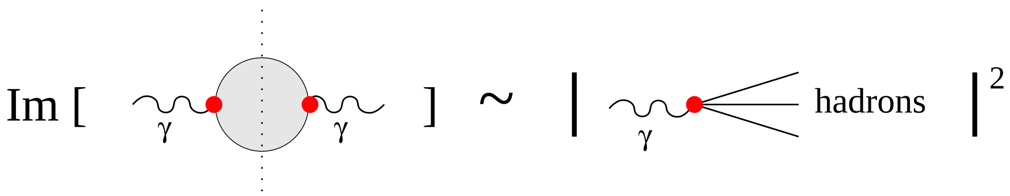
Ward identities: only vacuum polarization modifies electron charge

$$\alpha(s) = \frac{\alpha(0)}{1-\Delta\alpha(s)}\,, \quad \text{and :} \quad \Delta\alpha(s) \;\sim\; -\,\mathrm{Re}[\Pi_\gamma(s) - \Pi_\gamma(0)]$$

Leptonic part $\Delta\alpha_{\rm lep}(s)$ calculable in QED. However, quark loops are modified by long distance hadronic physics not calculable within QCD!

Way out: optical theorem (unitarity) [Born: $\sigma^{(0)}(s) = \sigma(s)(\alpha/\alpha(s))^2$]

$$12\pi\mathrm{Im}\Pi_\gamma(s) = \frac{\sigma^{(0)}(e^+e^- \to \mathrm{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} \equiv R(s)$$



and the subtracted dispersion relation of $\Pi_\gamma(s)$ (analyticity)

$$\Pi_\gamma(s) - \Pi_\gamma(0) = \frac{s}{\pi}\int_0^\infty \frac{\mathrm{Im}\Pi_\gamma(s')}{s'(s'-s) - i\epsilon}\,ds'$$

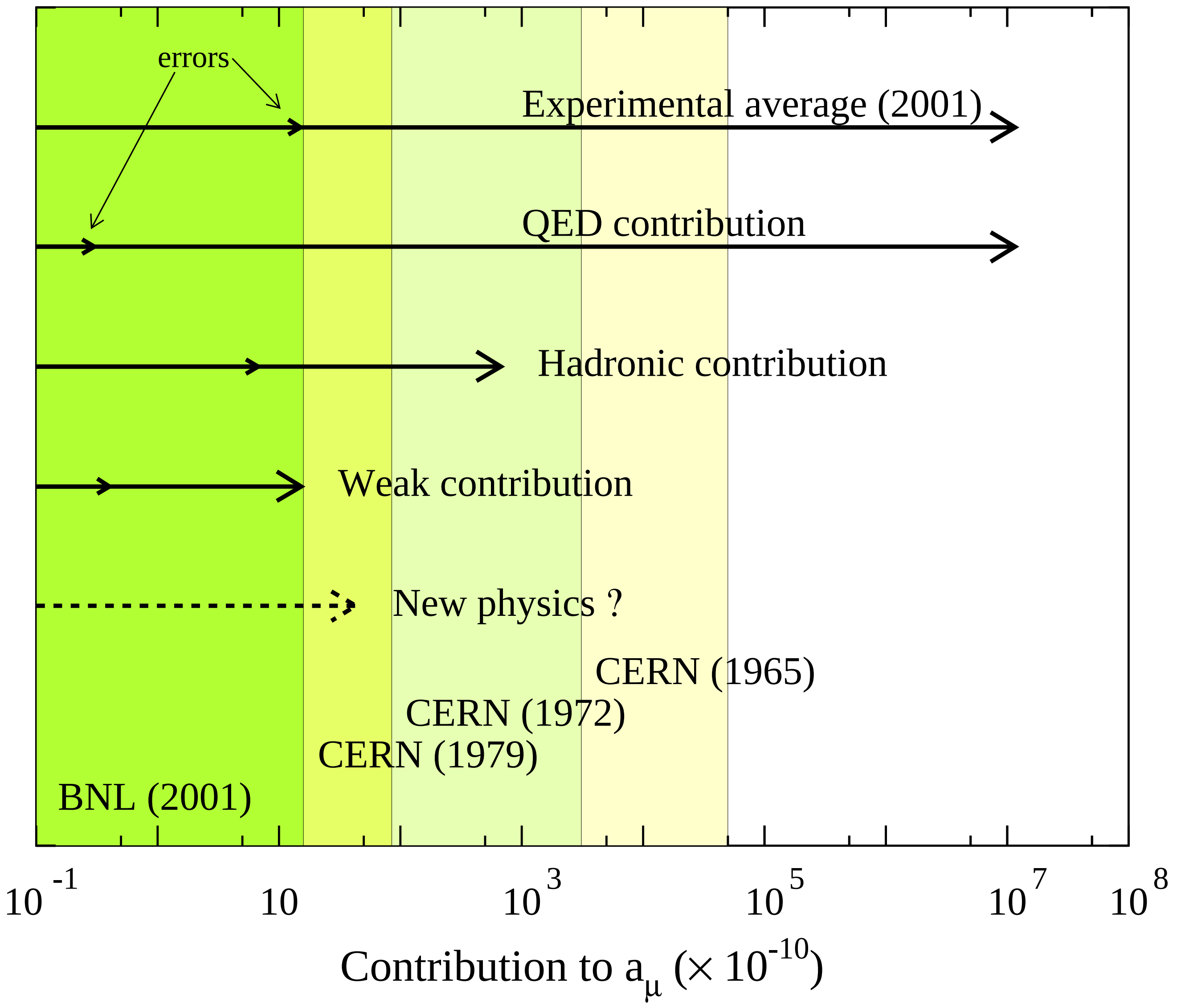
giving

$$\Delta\alpha_{\rm had}(s) = -\frac{\alpha s}{3\pi}\,\mathrm{Re}\int_0^\infty \frac{R(s')}{s'(s'-s) - i\epsilon}\,ds'$$

. . . and equivalently for $a_\mu^{\rm had}$

MOTIVATION

Experimental progress of $(g-2)_\mu$:



$\Longrightarrow$ Aims at: $\sigma(a_\mu(\mathrm{SM})) \simeq \sigma(a_\mu^{\mathrm{had}}) \ll a_\mu^{\mathrm{weak}}$

THE PRECISE BNL RESULT

New value announced last month by E821 at BNL with precision $\times 3$ compared to previous CERN and BNL results combined:

$$a_{\mu^+} \;=\; 11\;659\;202(16) \;\times\; 10^{-10}$$

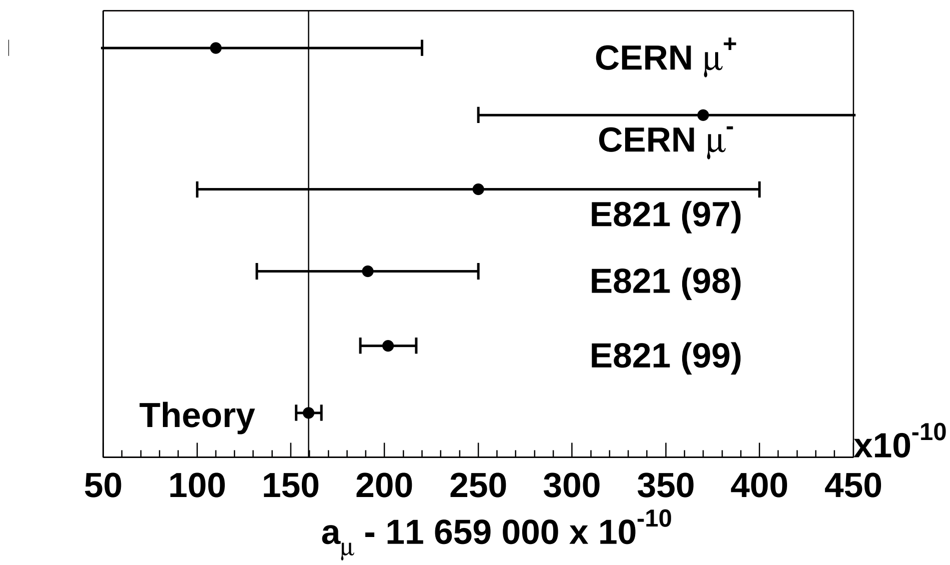
Uncertainty dominated by statistics, small systematic errors:
3.5×10^{-10} from precession frequency, 4.5×10^{-10} from magn. field

BNL compares with theory value

$$a_{\mu^+} \;=\; 11 \;\; 59\;159.6(6.7) \;\times\; 10^{-10}$$

Averaging with previous results gives

$$a_\mu(\mathrm{exp}) - \mathrm{a}_\mu(\mathrm{SM}) \;=\; 43(16) \;\times\; 10^{-10}$$



NEW PHYSICS IN a_μ ?

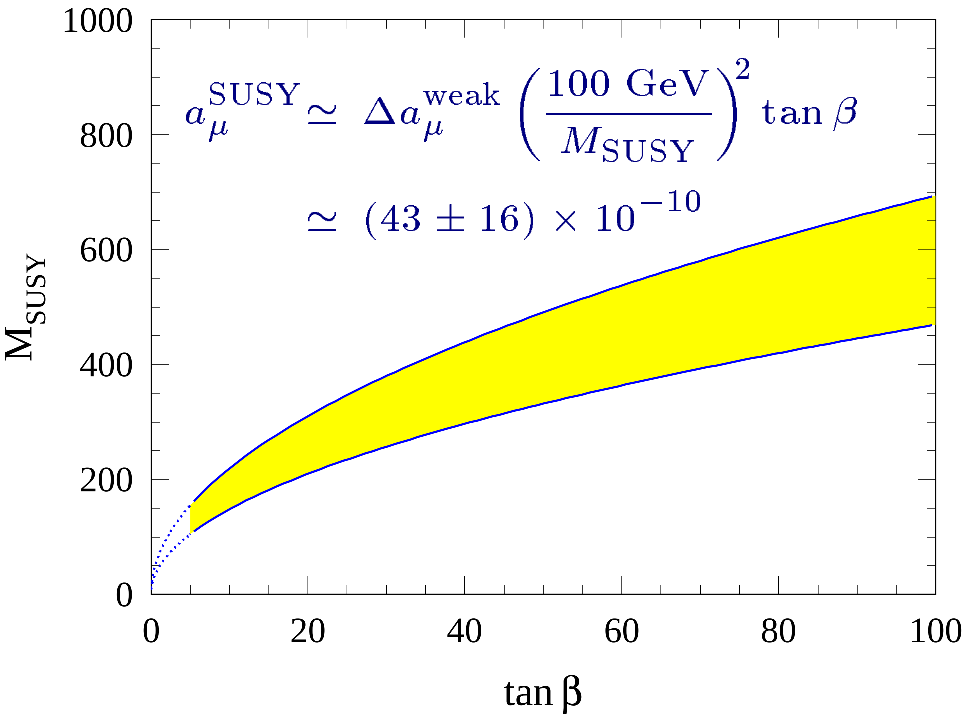
$$2.6\sigma \text{ Discrepancy} \Longrightarrow \begin{cases} (1) \text{ statistical fluctuation?} \\ (2) \text{ experimental systematic effect?} \\ (3) \text{ theory value incorrect } (a_\mu^{\rm had})\text{?} \\ (4) \text{ new physics?} \end{cases}$$

Comments:

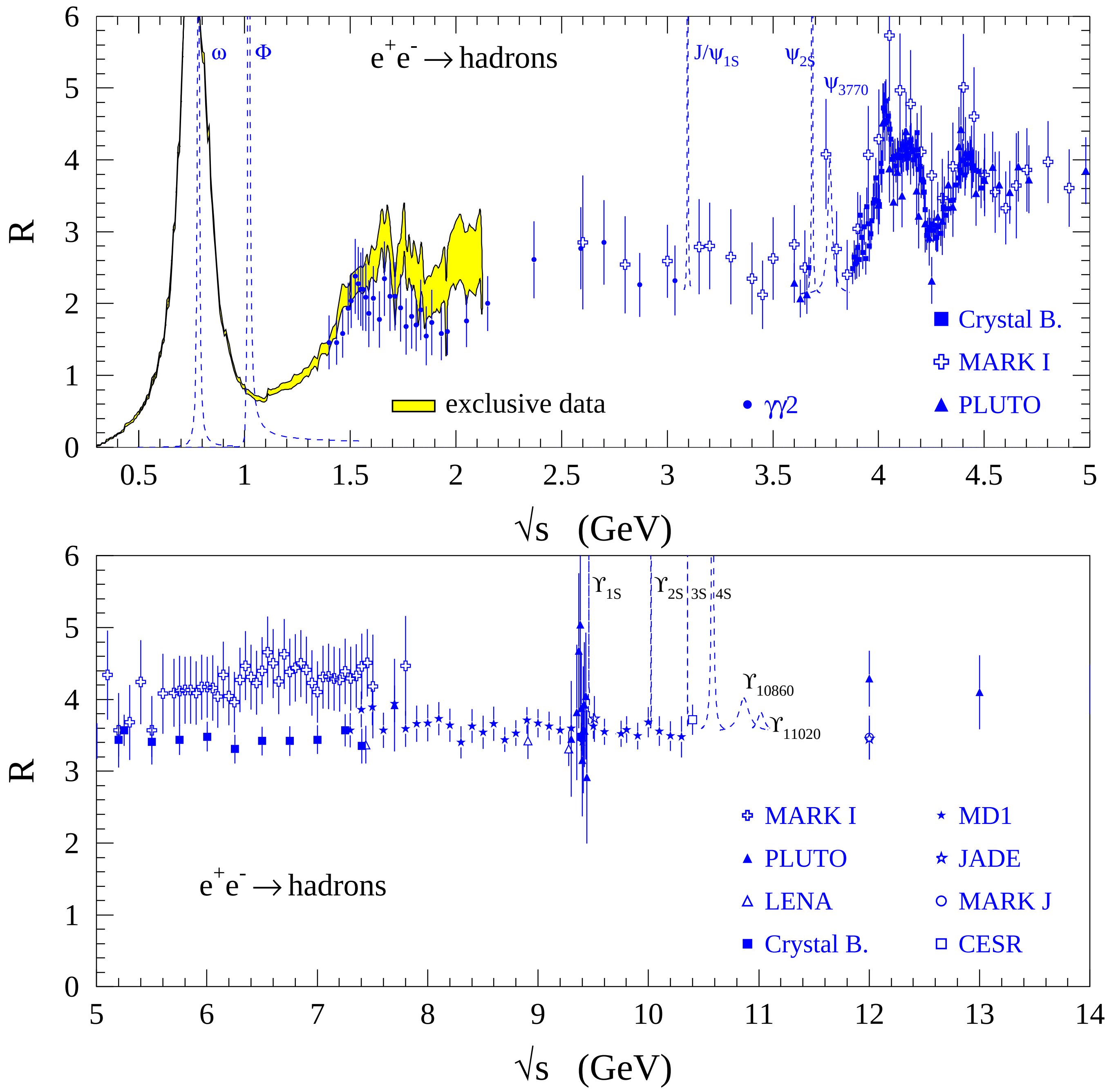
(1) E821 is analysing 2000 data ($4\times$ statistics)
(2) estimated syst. error $\sim \frac{1}{2.5}\times$ stat. error
(3) see later
(4) many possibilities (paper flood in the last two months):
SUSY, muon substructure, anomalous EW couplings, leptoquarks, LFV, . . .

Most fashionable new physics scenario: SUSY [Fayet 80, Grifols-Mendez 82, Ellis-Hagelin-Nanopoulos 82, Barbieri-Maiani 82, . . .]

Involves $(\tilde{\mu},\chi^0)$ and $(\chi,\tilde{\nu}_\mu)$ loops at mass scale $M_{\rm SUSY}$ [Moroi 96, . . .]



THE DATA (around 1995)



Data density and quality unsatisfactory in some crucial energy regions

IMPROVED DETERMINATION OF THE HADRONIC CONTRIBUTION TO $(g-2)_\mu$ AND TO $\alpha(M_Z^2)$

(The analysis is equivalent for both dispersion integrals)

SITUATION 1995 [Eidelman-Jegerlehner]:

Energy [GeV]	Input 1995	Input 1998	Input 2001 (forthcoming)
$2m_\pi$ – 1.8	Data	Data + QCD	Data, Data + QCD
1.8 – J/ψ	Data	QCD	QCD
J/ψ – Υ	Data	Data + QCD	Data, Data + QCD
Υ – 40	Data	QCD	QCD
40 - ∞	QCD	QCD	QCD

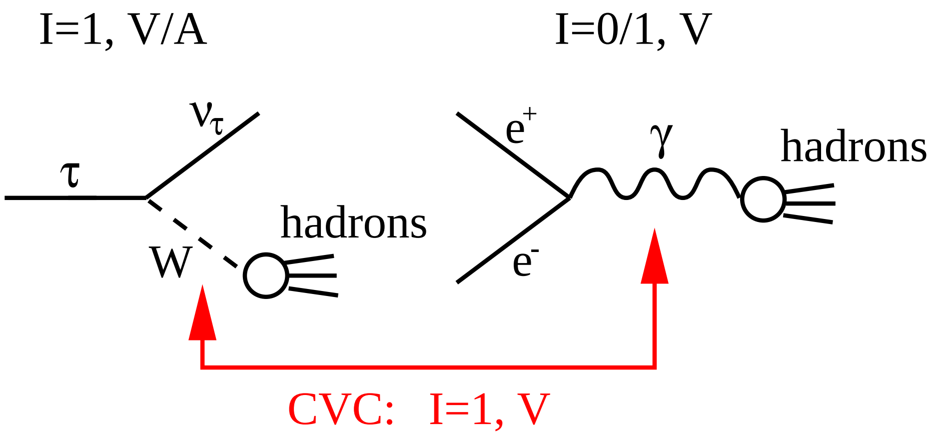
IMPROVEMENTS IN 3 STEPS:

(I) Addition of precise τ data using CVC
[Alemany-Davier-Höcker '97, Narison '01]

(II) More use of pQCD
[Martin-Zeppenfeld '95, Davier-Höcker '97, Kühn-Steinhauser '98, Erler '98, +others]

(III) More theoretical constraints from QCD Sum Rules
[Groote *et al.* '98, Davier-Höcker '98, Martin *et al.* '01, Cvetic *et al.* '01, +others]

(I) CONSERVED VECTOR CURRENT



Hadronic Physics Factorizes: *Spectral Functions*

Isospin Symmetry (CVC) connects e^-e^+ cross sections, $\sigma_{e^+e^-}$, to τ Spectral Functions, v, e.g. :

$$\sigma^{I=1}(e^+e^- \longrightarrow \pi^+\pi^-) = \frac{4\pi\alpha^2}{s}\, v(\tau^- \to \pi^-\pi^0\nu_\tau)$$

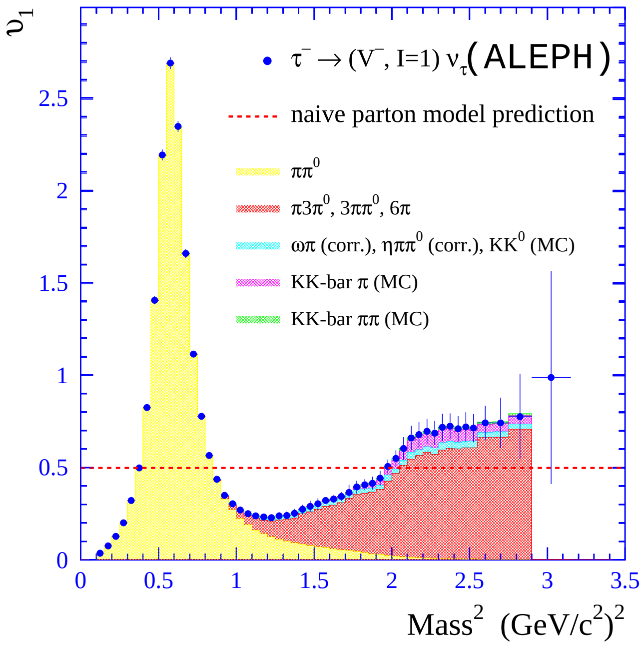
$$v_{\pi\pi^0} \sim \underbrace{\frac{B_{\pi\pi^0}}{B_e}}_{\text{Branching Ratios}} \quad \underbrace{\frac{1}{N_{\pi\pi^0}}\frac{dN_{\pi\pi^0}}{ds}}_{\text{Mass Spectrum}} \quad \underbrace{\frac{M_\tau^2}{(1-s/M_\tau^2)^2(1+2s/M_\tau^2)}}_{\text{Kinematic Factor}}$$

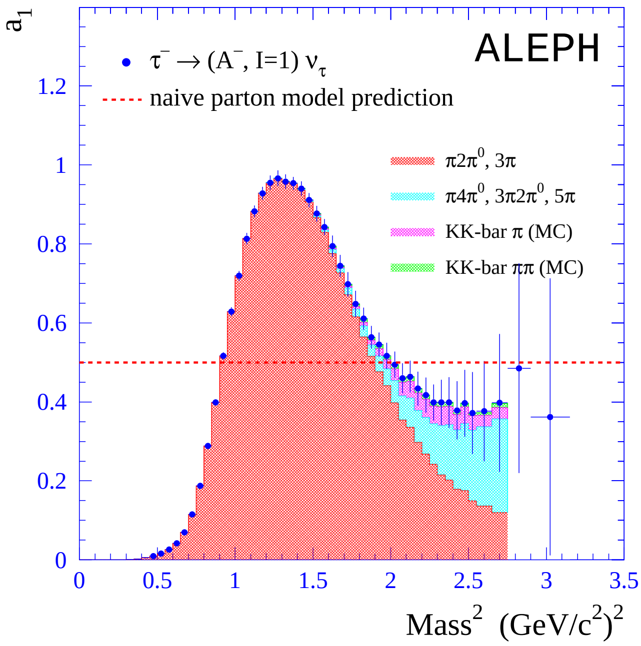
Spectral Functions are the fundamental input relating low-energy (resonances) to high-energy description (QCD)

τ SPECTRAL FUNCTIONS

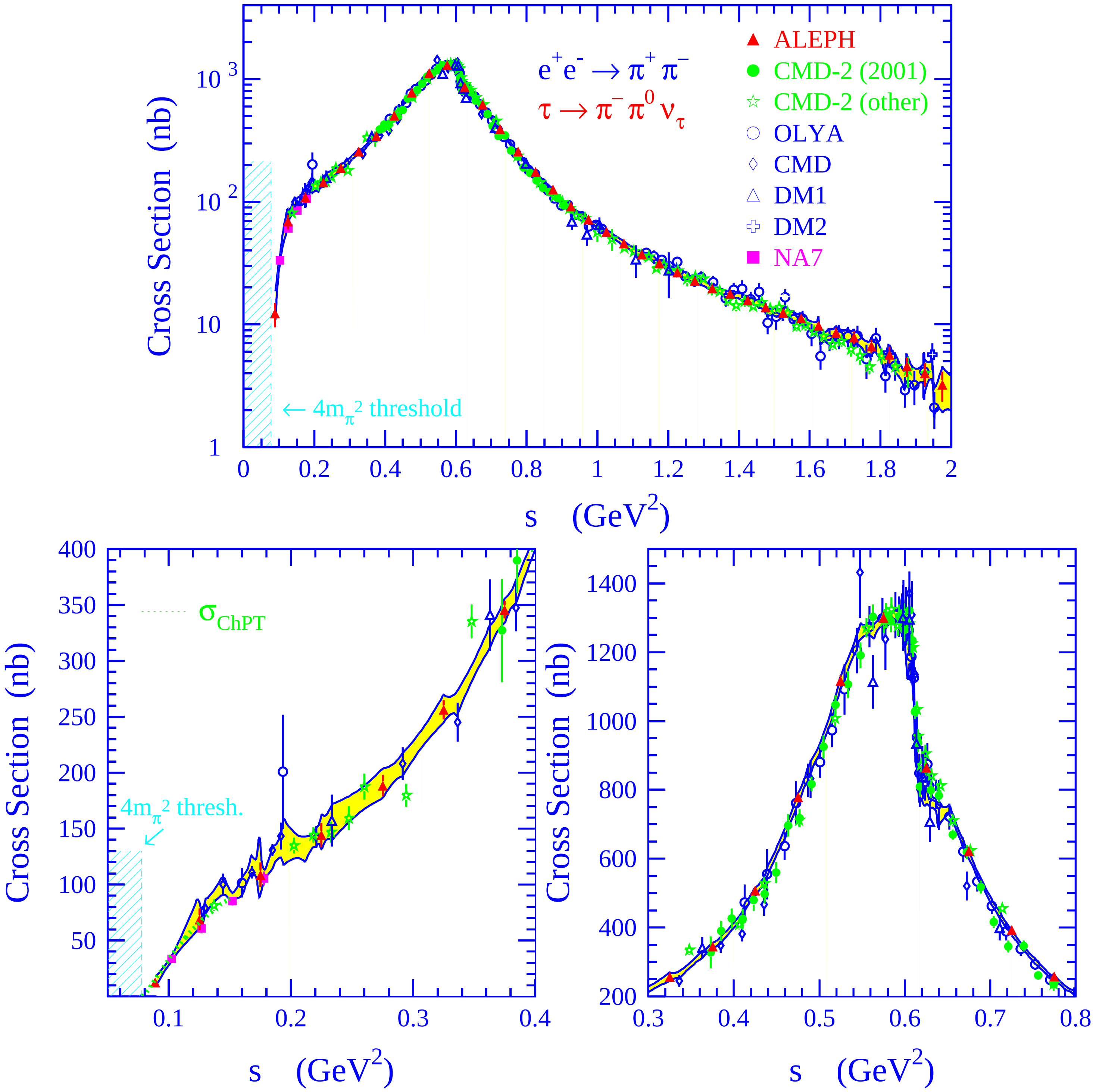
Hadronic τ decays are a clean probe of hadron dynamics
In many ways complementary to $e^+e^- \to \mathrm{hadrons}$

- CVC relates them to each other
- Different systematics:
  - τ data:
    - Excellent normalization (branching ratios) due to high statistics, large acceptance, small non-τ background
    - Shape subject to bin-to-bin corrections from resolution and fake photons: requires *unfolding*
  - e^+e^- annihilation data:
    - Excellent relative cross sections (correlated systematics)
    - Overal normalization subject to radiative corrections, systematic uncertainties from acceptance and luminosity





Compare $\sigma(e^+e^- \to \pi^+\pi^-)$ for e^+e^- and τ data



Introduce CVC-violating ρ-ω interference from fit to e^+e^- data

SU(2) BREAKING

Correction for isospin violation applied to $\tau^- \to \pi^-\pi^0\tau_\nu$:

Source	$\Delta a_\mu^{\rm had}$ $(\times 10^{-10})$
Radiative corrections to τ decays: $S_{\rm EW} = 1.0194$ [Marciano-Sirlin '88, Braaten-Li '90] (weak final state dependence expected, verified for $\tau^- \to \pi^-\nu_\tau$ [Decker-Finkemeyer '94])	-9.6 ± 2.0
Pion velocity: $\beta_- \neq \beta_0$ (due to EM π^--π^0 mass splitting: affects cross section)	-7.1
Form Factor [Davier-H.'97, Czyż-Kühn'00]:	
∘ ρ-ω interference	$+3.7 \pm 0.6$
∘ π^--π^0 mass splitting (affects Γ_ρ)	$+3.3$
∘ ρ^--ρ^0 mass splitting (affects Γ_ρ)	0 ± 0.2
∘ EM ρ decays: $\pi\gamma$, $\eta\gamma$, $\ell^+\ell^-$, $\pi\pi\gamma$ (affect Γ_ρ)	-0.2 ± 1.2
∘ second class currents e.g., $\tau^- \to \pi^-\eta\nu_\tau \propto (m_u - m_d)^2 \sim 10^{-5}$	negligible
Total correction	-9.9 ± 2.4

TESTING CVC

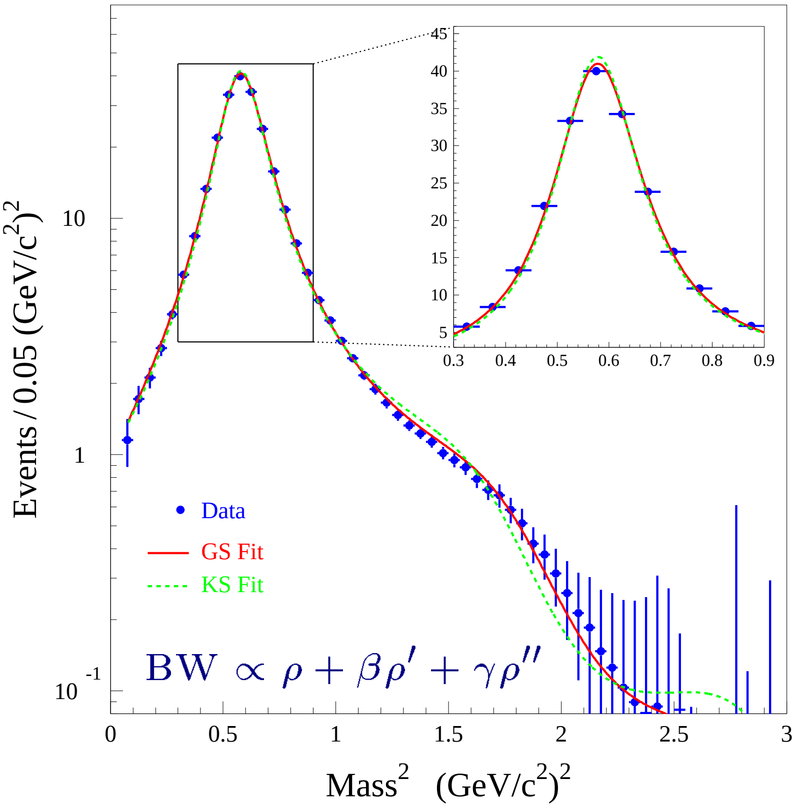
Fit the pion form factor:

$$v_0(s) = \frac{\beta_0^3(s)}{12\pi}|F_\pi^0(s)|^2$$

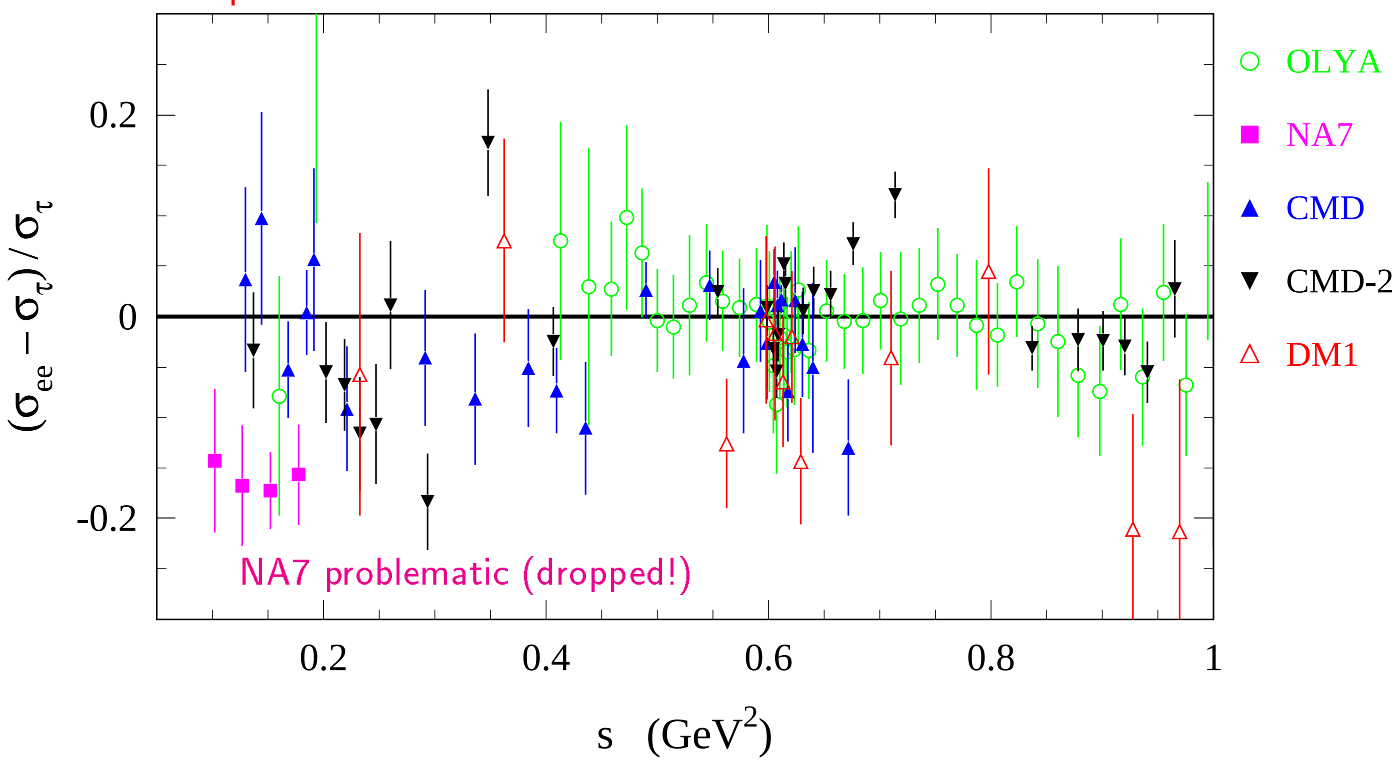
$$v_-(s) = \frac{\beta_-^3(s)}{12\pi}|F_\pi^-(s)|^2$$

SU(2) symmetry (CVC):

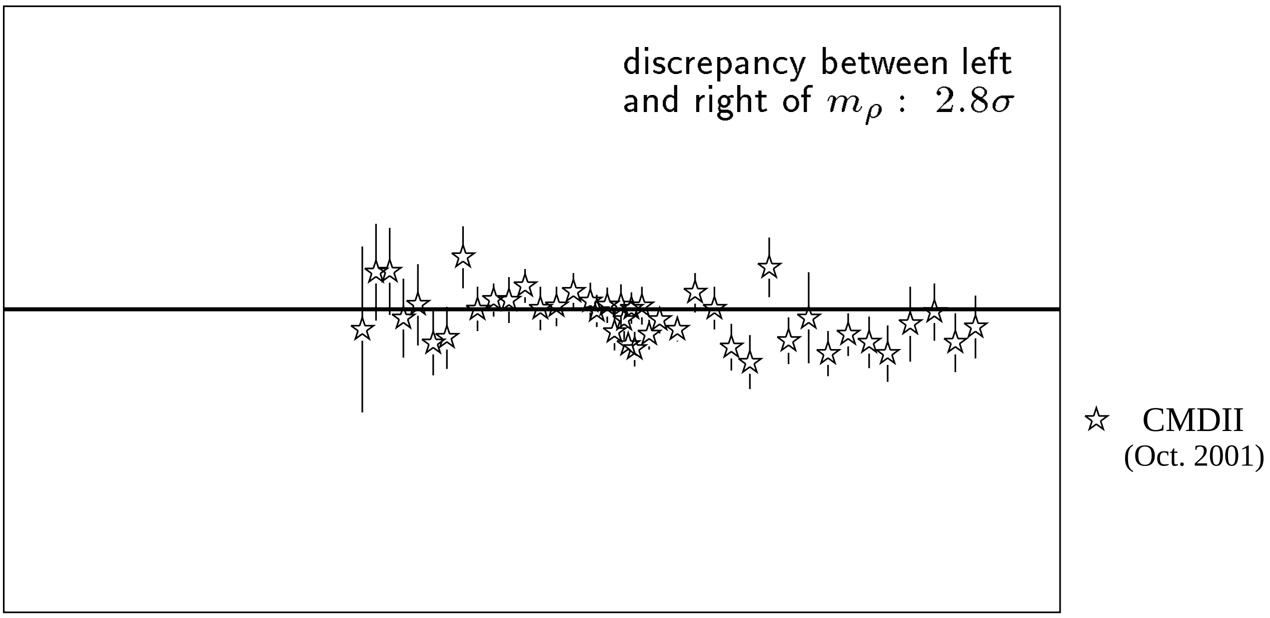
$$v_0(s) = v_-(s)$$



compare τ GS fit with e^+e^- data:



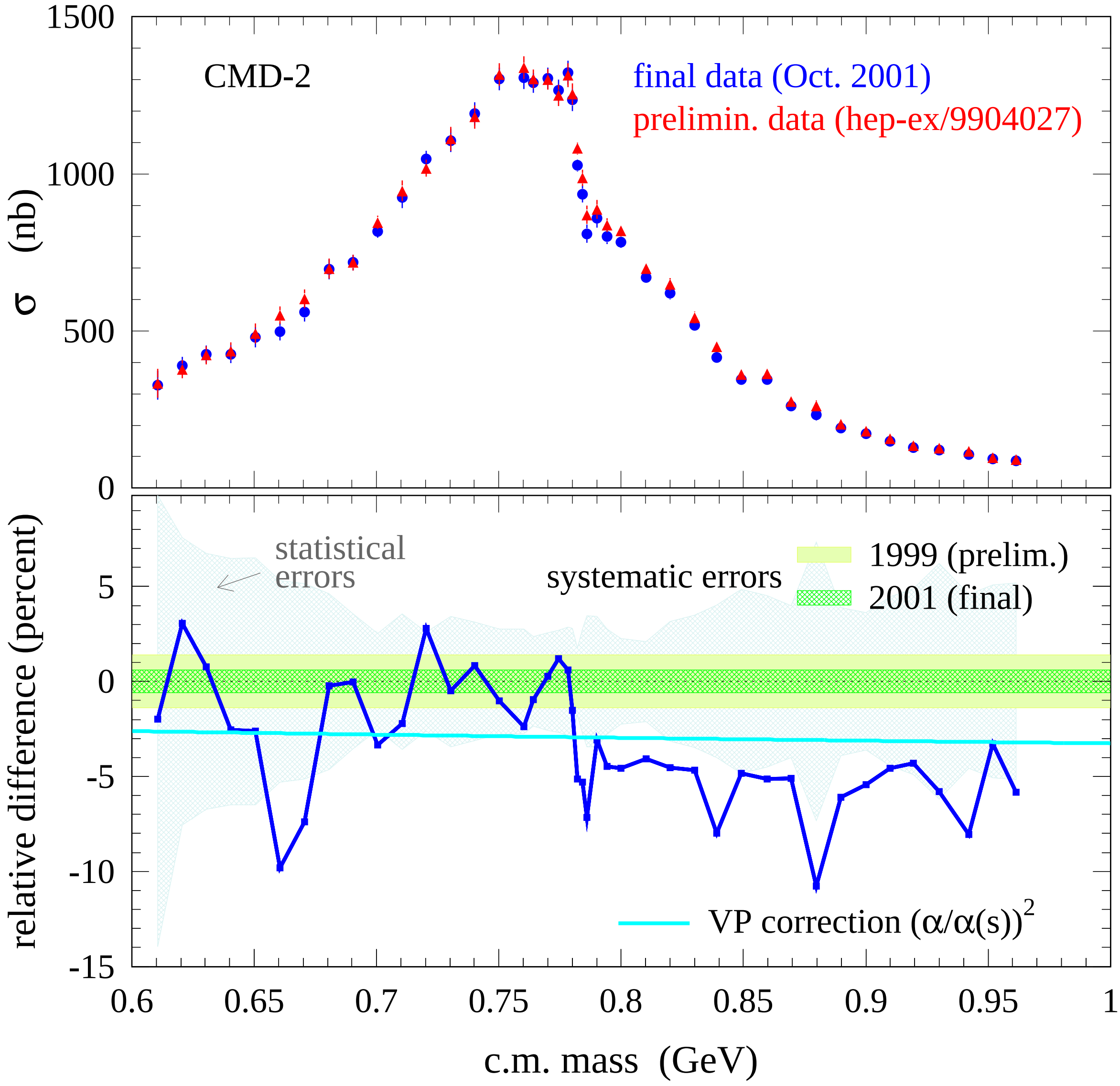
TESTING CVC


discrepancy between left
and right of m_ρ : 2.8σ
☆ CMDII
(Oct. 2001)

TESTING ... e^+e^- DATA ?

1999: CMD-2 publishes data *without* vacuum polarization (VP) correction [hep-ex/9904027]

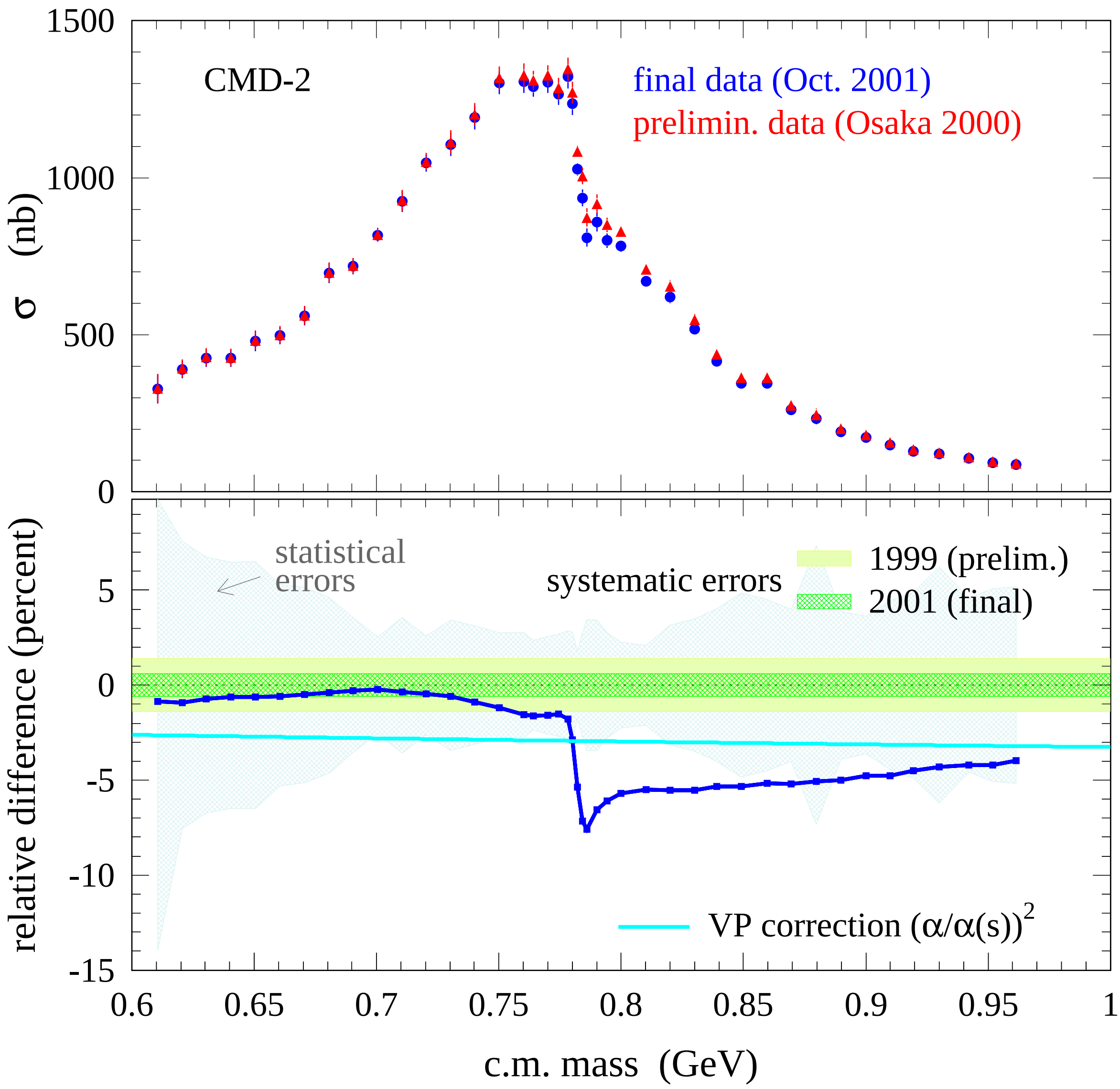
2001: New analysis of similar data sample *including* VP correction. Also ISR corrections revisited.



TESTING . . . e^+e^- DATA ?

2000: CMD-2 provides a preliminary reanalysis for ICHEP'00 (Osaka)

2001: New analysis of similar data sample *including* VP correction. Also ISR corrections revisited.



TESTING CVC (II)

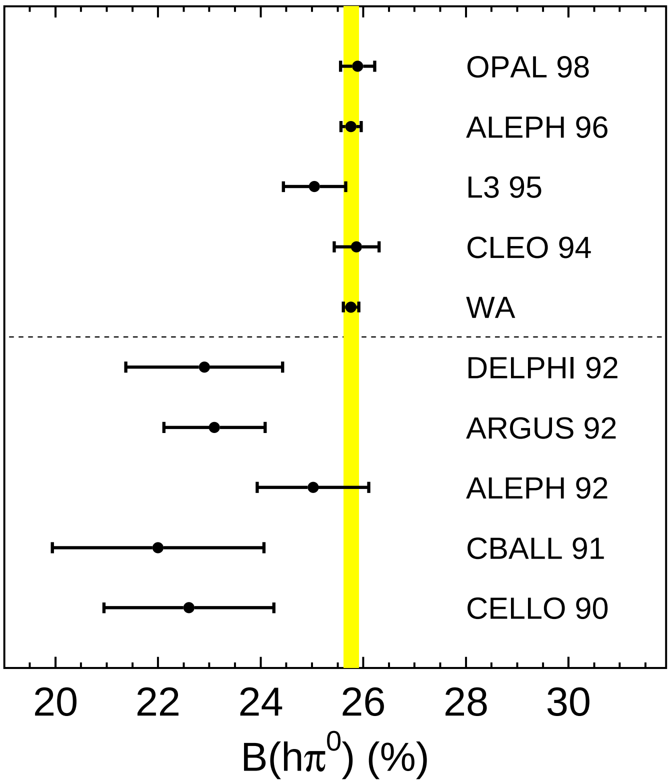
Infer $\mathrm{BR}(\tau^- \to \pi^-\pi^0\nu_\tau)$ ($\equiv \mathrm{BR}_{\pi\pi^0}$) from e^+e^- data:

$$\mathrm{BR}_{\pi\pi^0} = \mathrm{BR}_e \frac{6\pi|V_{ud}|^2 S_{\mathrm{EW}}}{m_\tau^2} \int_0^{m_\tau^2} \mathrm{Kin}(s) v_0^{\mathrm{SU(2)-corr}}(s)\, ds$$

Using *published* data from CMD-2 (1999, not including 2000/2001), OLYA and DM2 and correcting for EM $SU(2)$ breaking one finds:

$$\mathrm{BR}_{\pi\pi^0} = (24.67 \pm 0.39_{e^+e^-} \pm 0.12_{S_{\mathrm{EW}},|V_{ud}|^2, B_e} \pm 0.12_{\Delta\Gamma_{\rho^{\pm 0}}})\%$$

Compilation of $\mathrm{BR}_{h\pi^0}$ measurements



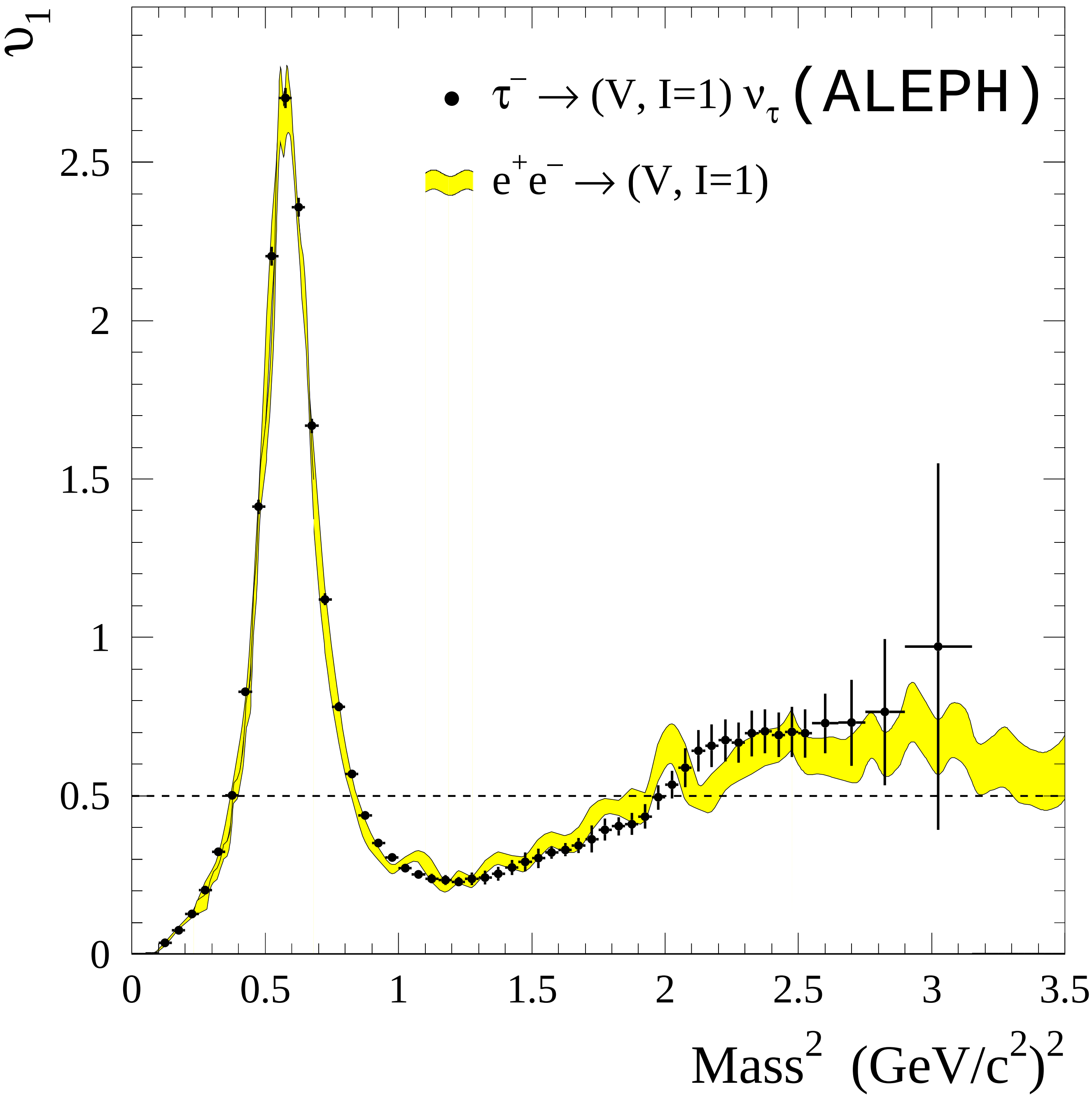
corrected for K^* fraction:
$\mathrm{BR}_{\pi\pi^0}(\mathrm{WA}) = (25.31 \pm 0.15)\%$

Discrepancy:
$\Delta_{\mathrm{CVC}}\mathrm{BR}_{\pi\pi^0} = (0.64 \pm 0.45)\%$

Future improvement expected from ALEPH: $\sigma(\mathrm{BR}_{\pi\pi^0})) \sim 0.12\%$

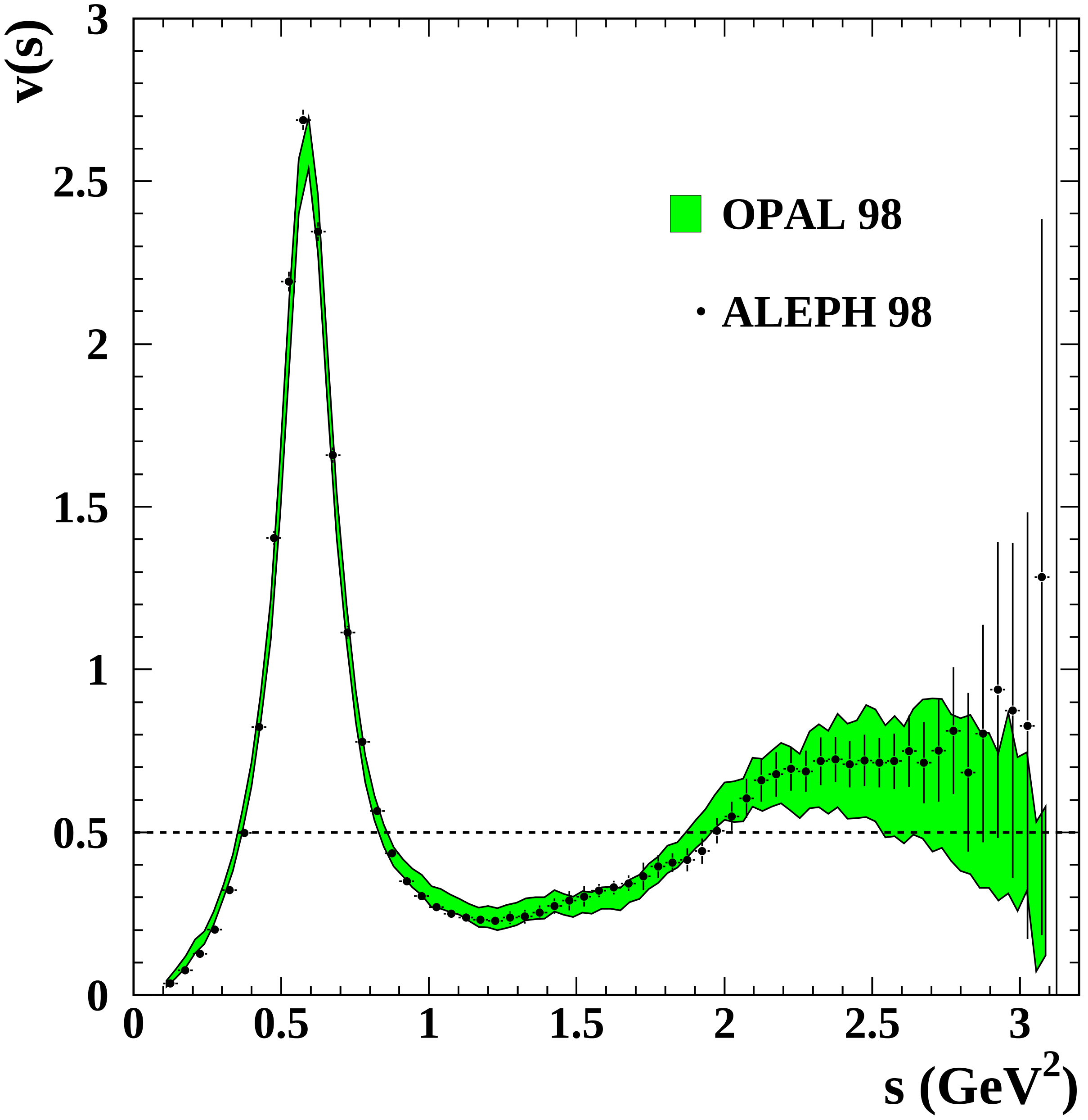
TESTING CVC (III)

Relate isovector e^+e^- cross section to τ vector spectral function:



TESTING CVC (IV)

Agreement between ALEPH'98 and OPAL'99 data



Important correlations between both spectral functions due to common use of branching ratios and MC simulation

LOW ENERGY HADRONIC CONTRIBUTIONS

Final states	$a_\mu^{\rm had}$ $(\times 10^{10})$	$\Delta\alpha_{\rm had}(M_Z^2)$ $(\times 10^4)$	Energy (GeV)
$\pi^+\pi^-$ $(e^+e^-$ & $\tau)$	500.00 $\pm$ 6.03	34.20 $\pm$ 0.38	0.320 – 1.800
$\pi^+\pi^-$ $(e^+e^-$ only)	495.05 $\pm$ 12.46	33.90 $\pm$ 0.87	0.320 – 1.800
Resonances	84.46 $\pm$ 1.52	19.31 $\pm$ 0.70	0.420 – 11.20
$\pi^+\pi^-\pi^0$	5.80 $\pm$ 0.65	0.80 $\pm$ 0.15	0.810 – 1.800
$\pi^+\pi^-2\pi^0$ $(e^+e^-$ & $\tau)$	19.79 $\pm$ 1.32	5.00 $\pm$ 0.48	0.897 – 1.800
$\pi^+\pi^-\pi^+\pi^-$ $(e^+e^-$ & $\tau)$	14.56 $\pm$ 0.87	3.86 $\pm$ 0.25	0.983 – 1.800
$(5\pi)^0$	3.28 $\pm$ 0.51	1.01 $\pm$ 0.15	1.019 – 1.800
$(6\pi)^0$	6.19 $\pm$ 2.43	2.52 $\pm$ 0.99	1.350 – 1.800
$\eta\,\pi^+\pi^-$	0.44 $\pm$ 0.16	0.13 $\pm$ 0.03	1.075 – 1.800
K$\bar{K}$	4.26 $\pm$ 0.58	0.83 $\pm$ 0.10	1.055 – 1.800
K$\bar{K}\pi$	3.39 $\pm$ 0.67	0.92 $\pm$ 0.18	1.340 – 1.800
K$\bar{\rm K}\pi\pi$	2.73 $\pm$ 1.00	0.94 $\pm$ 0.34	1.441 – 1.800
Total error $(e^+e^-$ & $\tau)$	9.4	6.2	$2m_\pi - \infty$
Total error $(e^+e^-$ only)	15.0	6.3	$2m_\pi - \infty$

$$\Longrightarrow \quad \sigma(a_\mu^{\rm had}) = 9.4 \times 10^{-10} \qquad (15.3 \times 10^{-10} \;\; [\mathrm{EJ}\ '95])$$

$$\sigma(\alpha^{-1}(M_Z^2)) = 0.085 \qquad (0.090 \;\; [\mathrm{EJ}\ '95])$$

(II) MORE QCD

HINT: τ decays $(R_{\tau,V+A})$ revealed a safe applicability of QCD at $M_\tau \simeq 1.8$ GeV and below !

$\Longrightarrow$ Why Not For $R_{e^+e^-}$?

TEST: Using $e^+e^- \to \mathrm{hadrons}$ and $\tau \to \mathrm{hadrons}\ (\mathrm{V})\ \nu_\tau$ data $\Longrightarrow$ $\{\ldots$

$\ldots\}$ $\Longrightarrow$ Theoretical Prediction for $R(s_0)$:

$$R(s_0) = \frac{1}{2\pi i} \oint\limits_{|s|=s_0} \frac{ds}{s} D(s)$$

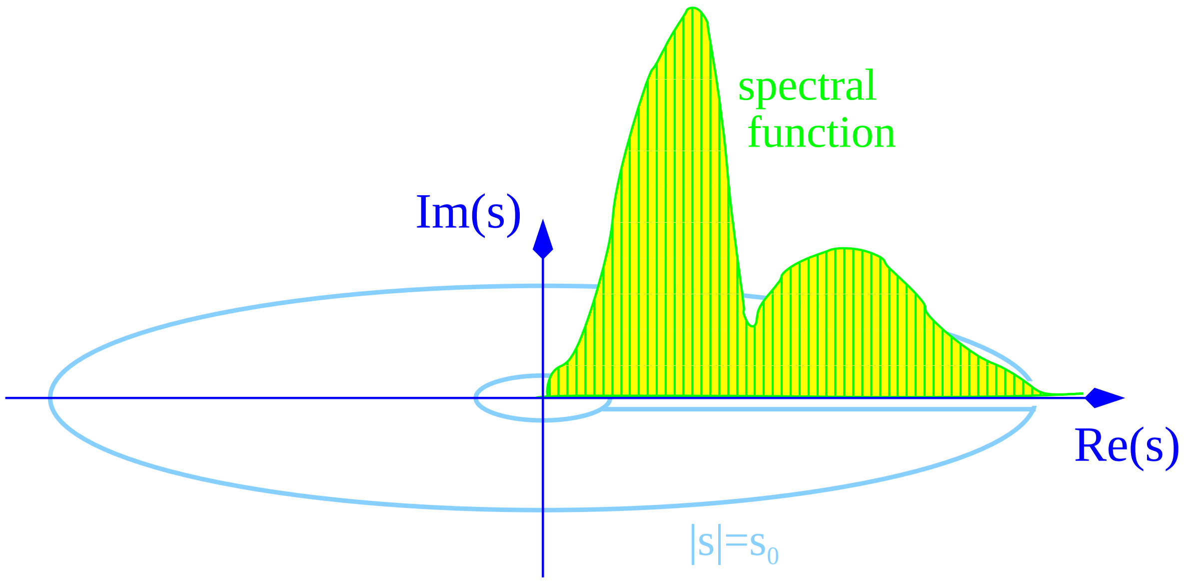
The combined fit of moments to data showed consistent OPE predictions and revealed small non-perturbative contributions at 1.8 GeV !

SPECTRAL FUNCTIONS AND QCD

(1) Optical theorem: $v(s) \propto \mathrm{Im}\Pi(s)$

(2) Apply Cauchy's theorem for "save" (*i.e.*, suffeciently large) s_0:

$$R(s_0) \propto \int\limits_0^{s_0} \underset{\substack{\uparrow\\ \text{kinematic factor}}}{f(s)}\mathrm{Im}\Pi(s)\,ds \quad \Longrightarrow \quad -\frac{1}{2i}\oint\limits_{|s|=s_0} f(s)\Pi(s)\,ds$$



(3) . . . use the *Adler* function to remove unphysical subtractions

$$D(s) = s\frac{d\Pi(s)}{ds}$$

(4) Use *global quark-hadron duality* in the framework of the *Operator Product Expansion* (OPE) to predict $D(s)$

(5) Use analytical moments, $f_n(s) = f(s) \cdot \mathrm{poly}_n(s)$, to fit non-perturbative parameters of the OPE

OPERATOR PRODUCT EXPANSION

$$D(s) = \sum_f \left(D^f_{\rm pert}(s) + D^f_{\rm mass}(s) + D^f_{\rm non-pert}(s) \right)$$

For a given quark flavour f, the OPE contributions are:

$$D^f_{\rm pert}(-s) = 1 + d_0 \frac{\alpha_s(s)}{\pi} + d_1 \left(\frac{\alpha_s(s)}{\pi}\right)^2 + d_2 \left(\frac{\alpha_s(s)}{\pi}\right)^3$$

$$D^f_{\rm mass}(-s) = -6 \frac{m_f^2(s)}{s} \left[1 + b_0 \frac{\alpha_s(s)}{\pi} + b_1 \left(\frac{\alpha_s(s)}{\pi}\right)^2 \right]$$

$$D^f_{\rm non-pert}(-s) = \sum_{D=4,6,8} \frac{1}{(-s)^{D/2}} \sum_{{\rm dim}\mathcal{O}=D} C_D(s,\mu) \langle \mathcal{O}_D(\mu) \rangle$$

The non-perturbative Operators $\langle \alpha_s/\pi GG_{D=4} \rangle$, $\langle O_6 \rangle$ and $\langle O_8 \rangle$ need to be determined from data

$\Longrightarrow$ define *Spectral Moments* :

$$R^{kl}(s_0) = \int_{4m_\pi^2}^{s_0} \frac{ds}{s_0} \left(1 - \frac{s}{s_0}\right)^k \left(\frac{s}{s_0}\right)^l R(s) ,$$

and fit $D(s)$ to weighted data spectra (take care of correlations!)

First QCD Analysis of τ Hadronic Width:

ALEPH'93 (*Phys. Lett.* B307 (1993) 209)
CLEO'95 (*Phys. Lett.* B356 (1995) 580)

Full Separation of Vector and Axial-Vector Spectral Functions and QCD Analysis:

ALEPH'97 (*Z. Phys.* C76 (1997) 15)
ALEPH'98 (*Eur. Phys. J.* C4 (1998) 409)
OPAL'99 (*Eur. Phys. J.* C7 (1999) 571)

COMBINED FIT RESULTS (ALEPH'98)

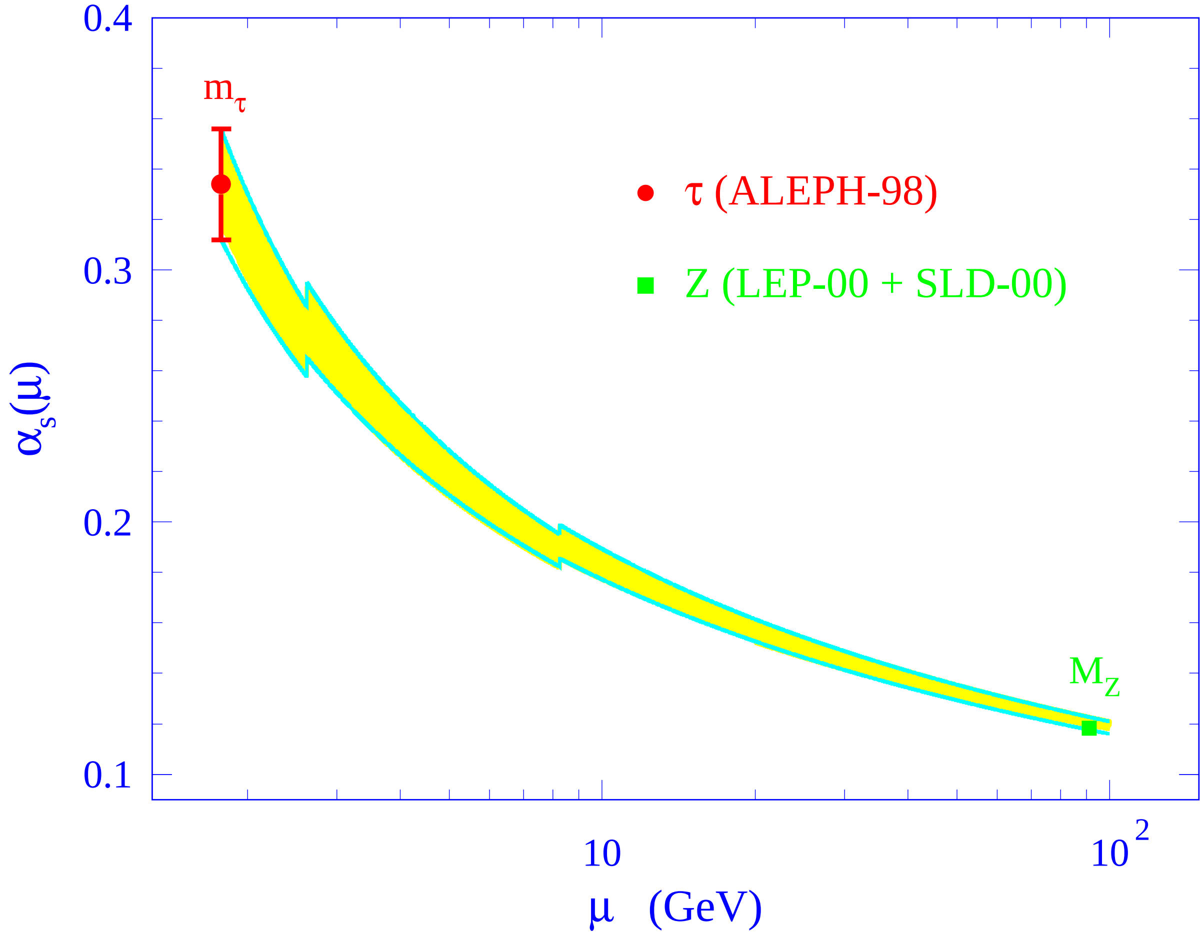
$\mathrm{FOPT}_{\mathrm{CI}}$	Vecteur (V)	Axial-Vecteur (A)	$V + A$
$\alpha_s(m_\tau^2)$	$0.340 \pm 0.016 \pm 0.017$	$0.349 \pm 0.015 \pm 0.017$	$0.345 \pm 0.007 \pm 0.017$
$\delta^{(0)}$	0.198 ± 0.017	0.206 ± 0.018	0.202 ± 0.013
$\delta^{(2)}$	$-(0.3 \pm 0.3)\times 10^{-3}$	$-(0.6 \pm 0.3)\times 10^{-3}$	$-(0.4 \pm 0.2)\times 10^{-3}$
$\delta^{(4)}$	$(0.6 \pm 0.8)\times 10^{-3}$	$(-5.7 \pm 0.9)\times 10^{-3}$	$-(2.5 \pm 0.8)\times 10^{-3}$
$\delta^{(6)}$	0.029 ± 0.004	-0.029 ± 0.004	0.001 ± 0.004
$\delta^{(8)}$	-0.009 ± 0.001	0.008 ± 0.001	-0.001 ± 0.001
δ_{NP} total	0.020 ± 0.004	-0.027 ± 0.004	-0.003 ± 0.004
χ^2/d.o.f.	0.1/1	0.1/1	0.2/1

$$\alpha_s(M_Z) = 0.1202 \pm \underbrace{0.0008_{\mathrm{exp}} \pm 0.0024_{\mathrm{theo}} \pm 0.0010_{\mathrm{evol}}}_{0.0027}$$

TESTING QCD (I)

Evolution of $\alpha_s(m_\tau^2)$ to M_Z^2 using RGE gives
(4-loop QCD β-function with 3-loop quark flavor matching)

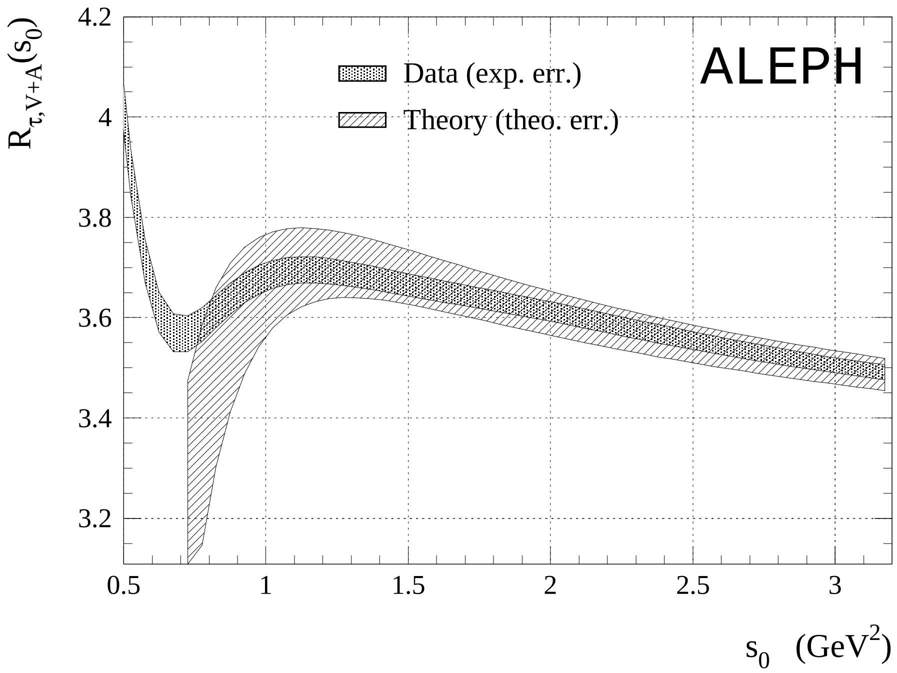
$$\alpha_s(M_Z^2) = 0.1202 \pm 0.0027 \ (\mathrm{ALEPH'98, theory\ dominated})$$
$$\alpha_s(M_Z^2) = 0.1183 \pm 0.0027 \ (\mathrm{LEP, ICHEP'2000})$$



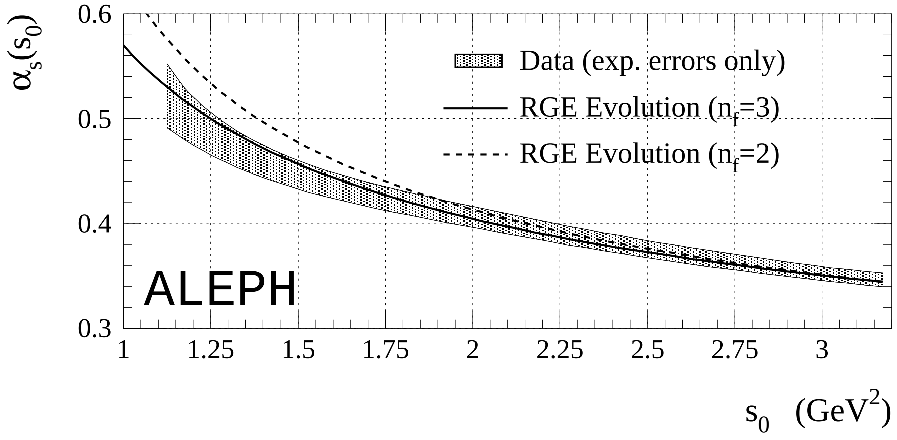
TESTING QCD (II)

RGE Test (I): Vary the τ Mass, $s_0 \leq m_\tau^2$, Using Spectral Functions:

$$R_{\tau,V+A}(s_0) = \int_0^{s_0} ds \, \frac{dR_{\tau,V+A}(s,s_0)}{ds}$$



Prediction of Perturbative QCD for R_τ at $O(\alpha_s^3)$ works down to $s \sim 1$ GeV^2 with an accuracy of 1-3%.

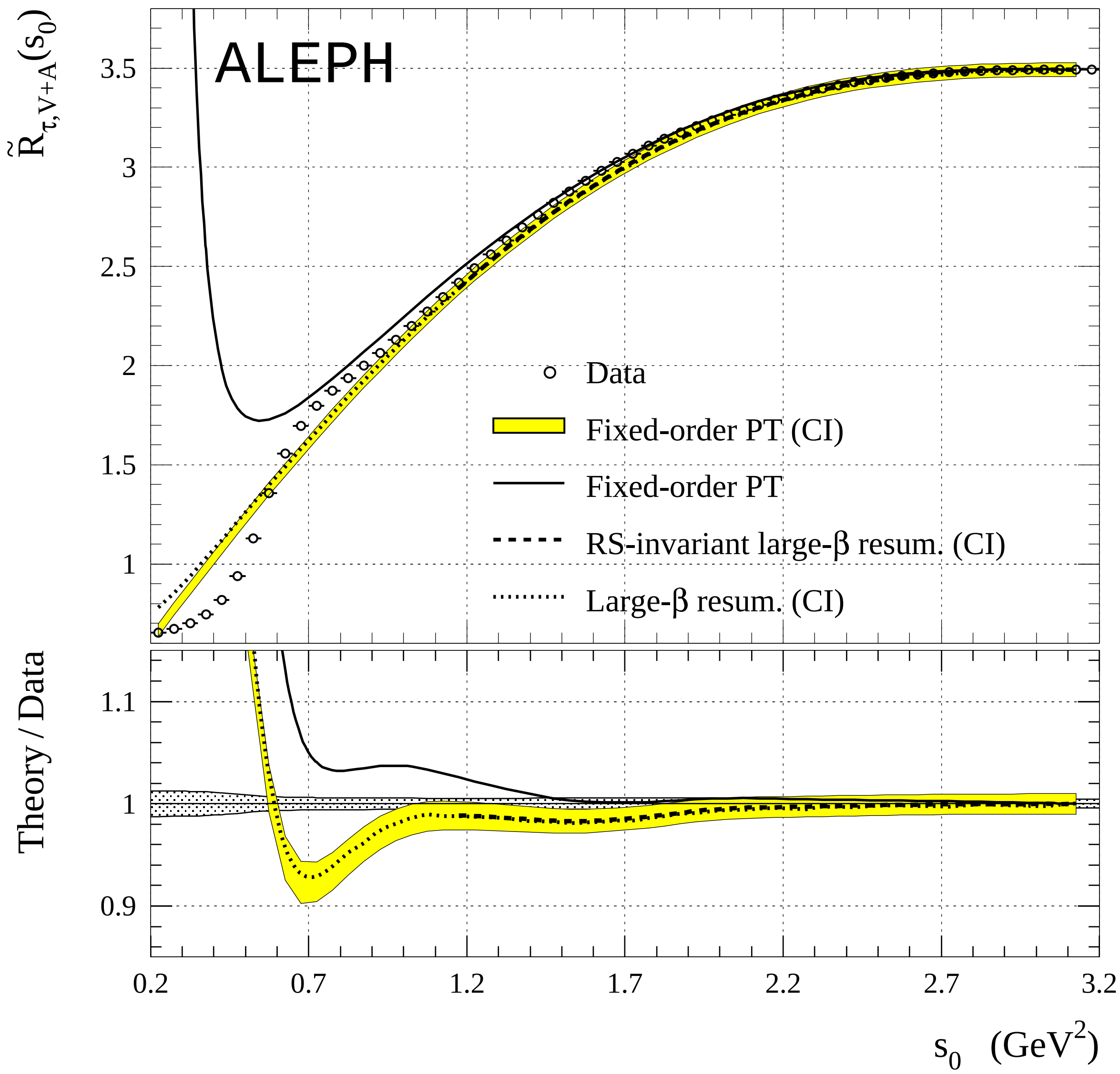


Clear experimental evidence for running coupling $\alpha_s(s_0)$ from τ data only

TESTING QCD (III)

RGE Test (II): Running Via Variation of Cauchy Integration Range:

$$\tilde{R}_{\tau,V+A}(s_0) = \int_0^{s_0} ds \, \frac{dR_{\tau,V+A}(s, m_\tau^2)}{ds}$$



What did we learn ?

$\Longrightarrow$ that the τ and its lighter "ghosts" like pQCD!

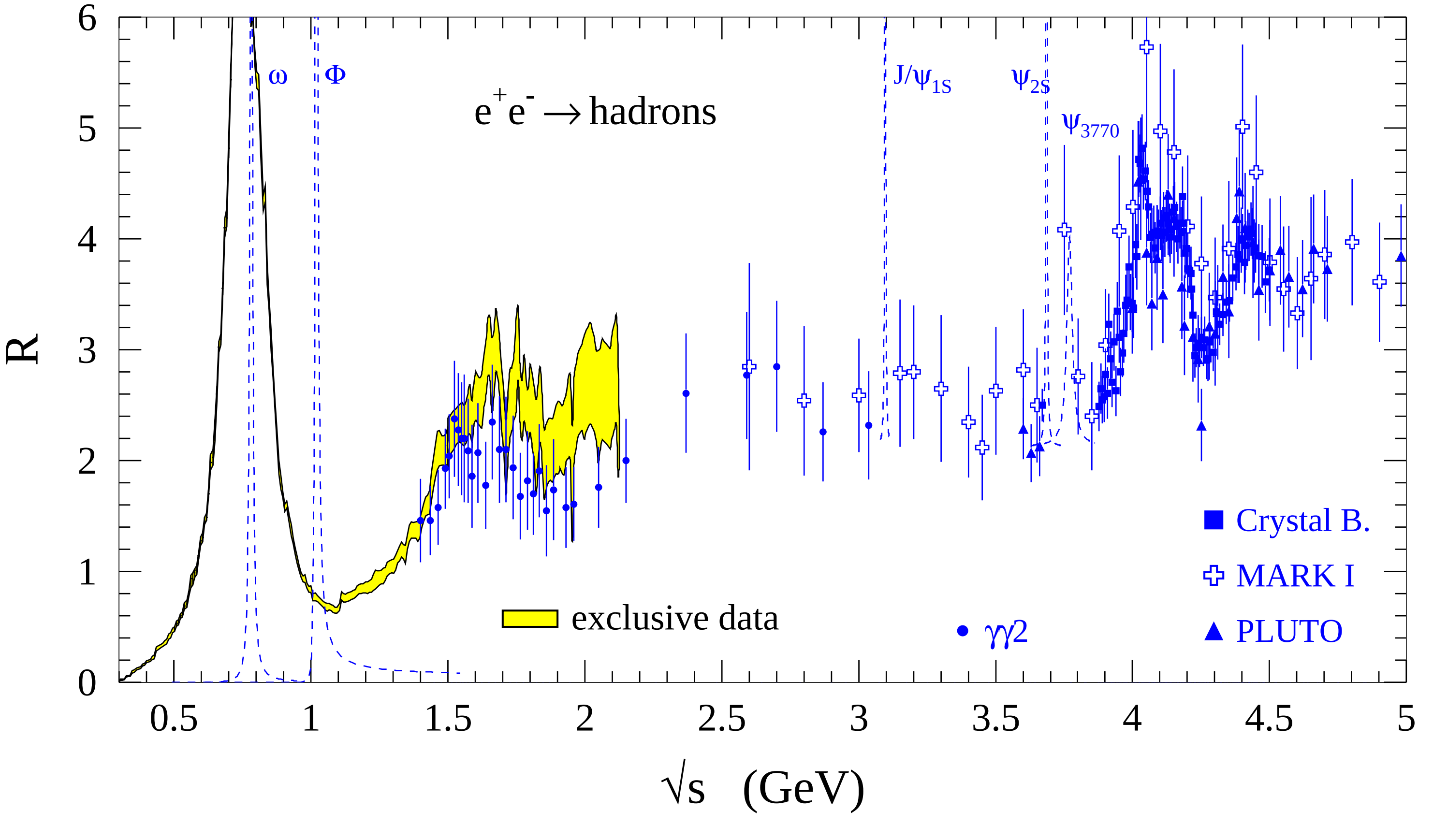
Can we use this knowledge for e^+e^- data?

- $R_{e^+e^-}$ from $e^+e^- \to \mathrm{hadrons}\ (V,\, I=0,1)$ less inclusive than $R_{\tau,\,V+A}\ (I=1)$
- but: more inclusive than $R_{\tau,\,V}\ (I=1)$ for which the OPE worked at M_τ^2

The OPE should therefore work for $R_{e^+e^-}$, but we need to be sure:
$\Longrightarrow$ repeat the analysis using e^+e^- data ...

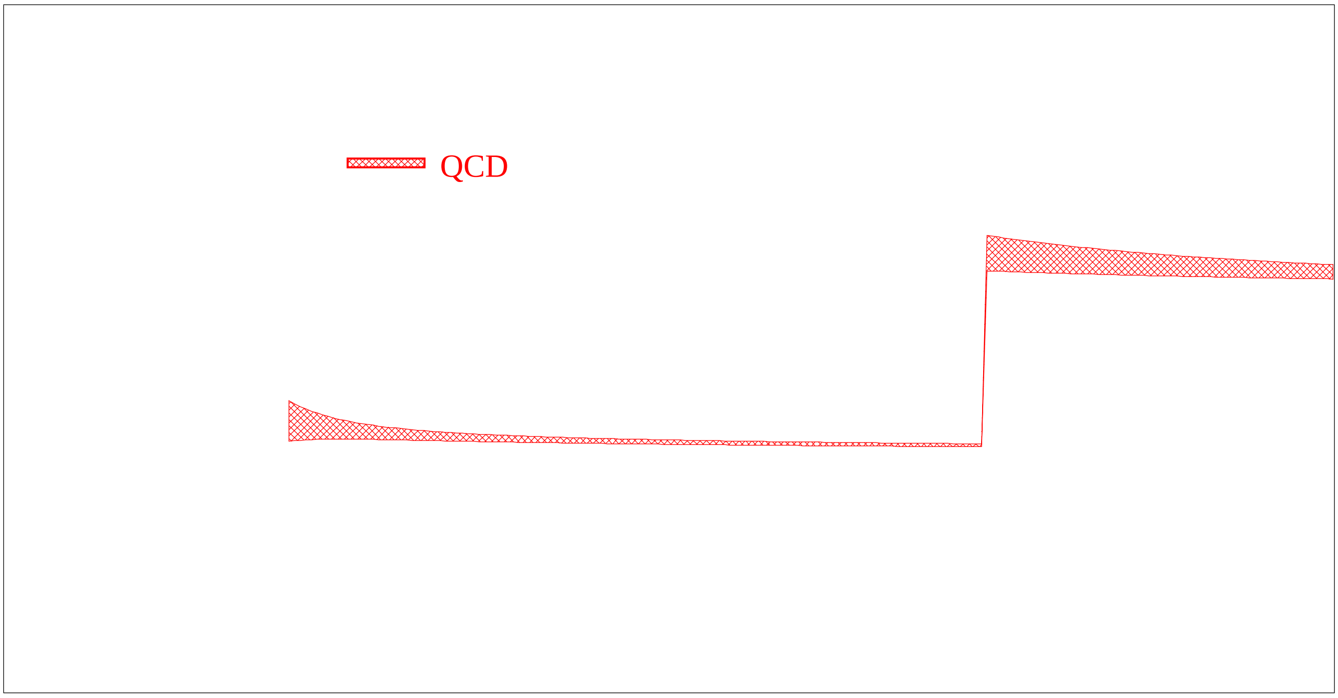
R DATA SPECTRUM

- Available data around 1995



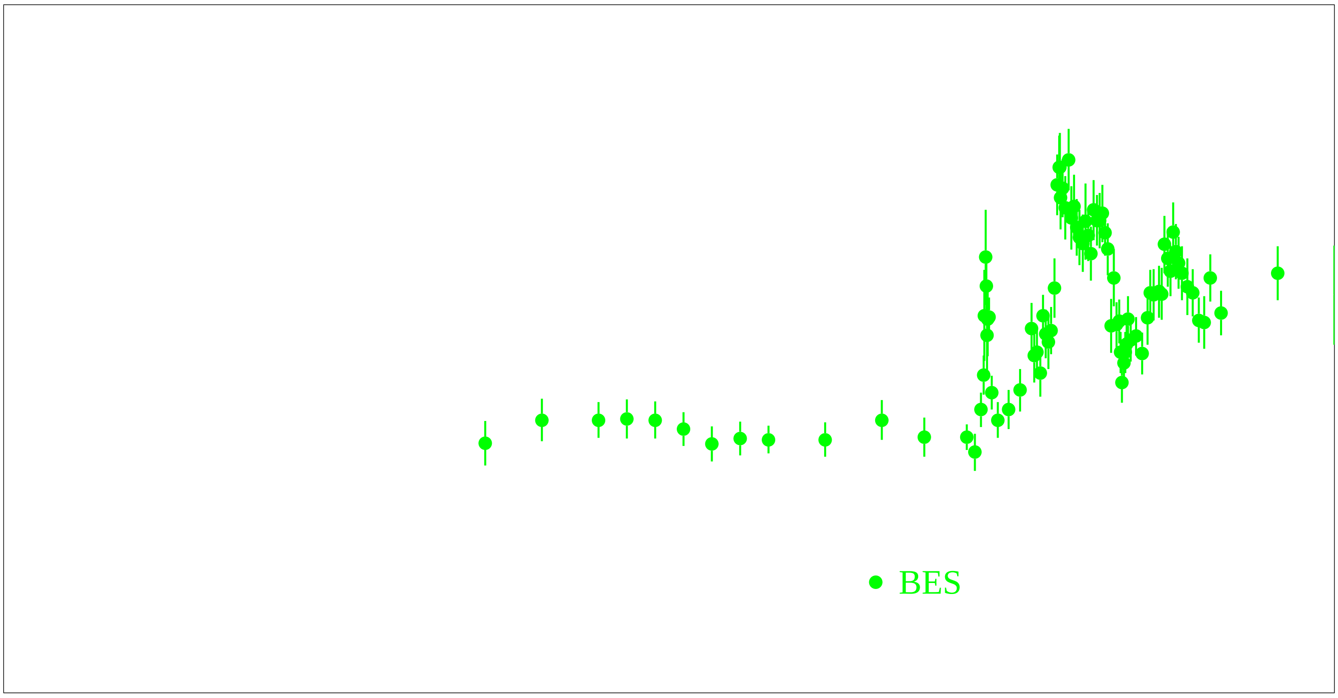
& QCD

- QCD (with generous errors) has much better precision than data

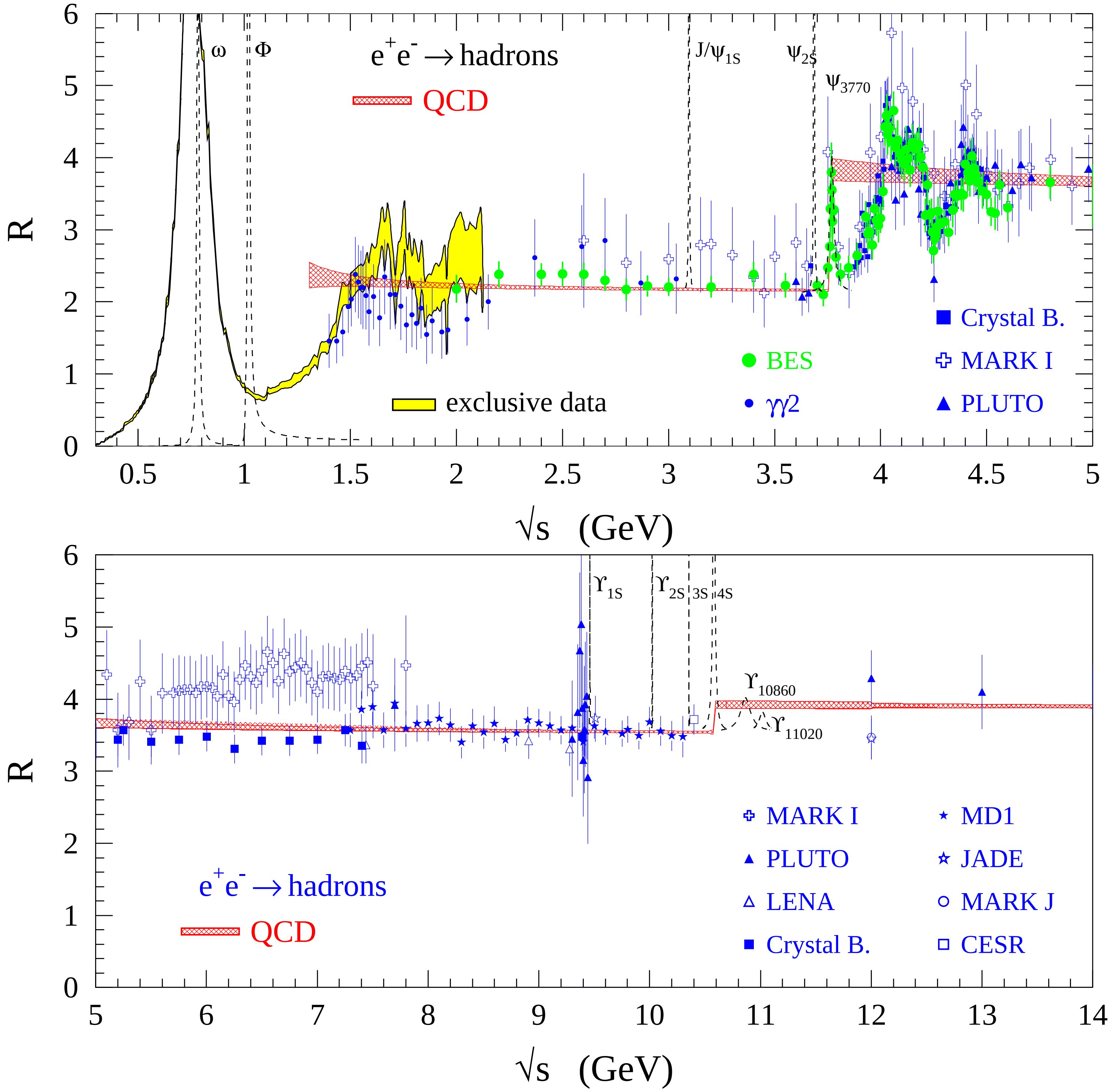


& BES

- BES confirms QCD prediction



R DATA SPECTRUM & QCD & BES



$$\Longrightarrow \quad \Delta a_\mu^{\rm had} = 7.5 \times 10^{-10} \qquad (15.3 \times 10^{-10} \ \ [\text{EJ } '95])$$

$$\Delta\alpha(M_Z^2) = 0.036 \qquad (0.0023 \ \ [\text{KS '98}],\ 0.090 \ \ [\text{EJ '95}])$$

(II) QCD SUM RULES

TWO APPROACHES : (note that: $D(s) \sim s\, d\Pi(s)/ds$)

$$\int\limits_{4m_\pi^2}^{s_0} ds\, f(s) {\rm Im}\Pi = \underbrace{\int\limits_{4m_\pi^2}^{s_0} ds\, [f(s) - P_n(s)]\, {\rm Im}\Pi}_{\text{Data}} + \underbrace{\frac{i}{2} \oint\limits_{|s|=s_0} ds\, P_n(s) \Pi^{\rm QCD}}_{\text{Theory}}$$

Polynomials $P_n(s)$ approximate the $f(s)$ (assumes global duality)

Instead of polynomials, one can use the dispersion relation

$$D_f(Q^2) = Q^2 \int\limits_{4m_f^2}^{\infty} ds\, \frac{R_f(s)}{(s+Q^2)^2}$$

to approximates the kernel $f(s)$ (this assumes local duality)

Thoroughly evaluate theoretical uncertainties:

Sources are: $\alpha_s(s)$, $\mu_{\overline{\rm MS}}$, FOPT vs. ${\rm FOPT_{CI}}$, $m_q(s)$, $\langle \mathcal{O}_D \rangle$

RESULTS: (I) + (II) + (III)

Contributions to $a_\mu^{\rm had}$ from the different energy regions [DH '98]:

Energy (GeV)	$a_\mu^{\rm had} \times 10^{10}$
$(2m_\pi - 1.8)_{uds}$	$634.3 \pm 5.6_{\rm exp} \pm 2.1_{\rm theo}$
$(1.8 - 3.700)_{uds}$	$33.87 \pm 0.46_{\rm theo}$
$\psi(1S, 2S, 3770)_c + (3.7 - 5)_{udsc}$	$14.31 \pm 0.50_{\rm exp} \pm 0.21_{\rm theo}$
$(5 - 9.3)_{udsc}$	$6.87 \pm 0.11_{\rm theo}$
$(9.3 - 12)_{udscb}$	$1.21 \pm 0.05_{\rm theo}$
$(12 - \infty)_{udscb}$	$1.80 \pm 0.01_{\rm theo}$
$(2m_t - \infty)_t$	≈ 0
$(2m_\pi - \infty)_{udscbt}$	$692.4 \pm 5.6_{\rm exp} \pm 2.6_{\rm theo}$

The subscripts denote the quark flavours involved in the calculation.

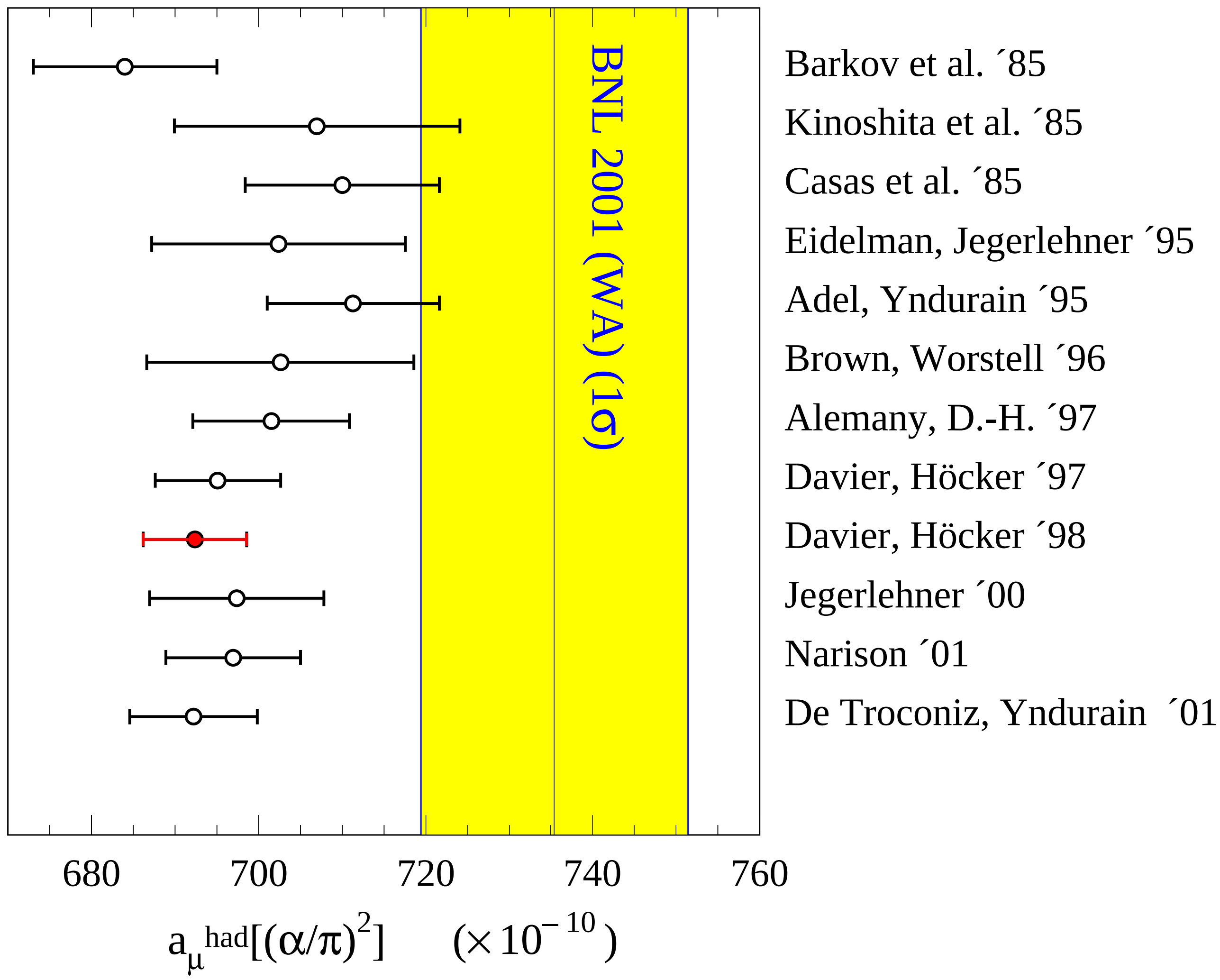
Total contribution [DH '98]:

		$(\Delta\ [\text{EJ '95}])$
$a_\mu^{\rm had}$	$= (692.4 \pm 6.2) \times 10^{-10}$	(15.3×10^{-10})
$a_\mu^{\rm SM}$	$= (11\,659\,159.6 \pm 6.7) \times 10^{-10}$	
$\Delta\alpha_{\rm had}(M_Z^2)$	$= (276.3 \pm 1.6) \times 10^{-4}$	(6.5×10^{-4})
$\alpha^{-1}(M_Z^2)$	$= 128.933 \pm 0.021$	(0.090)

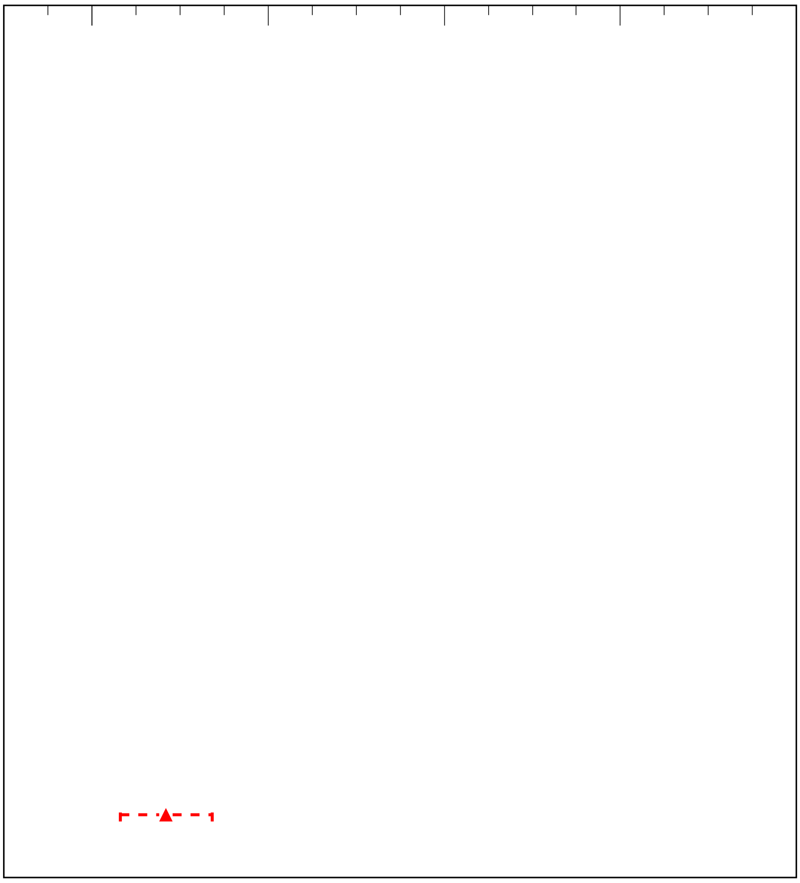
Hadronic contribution from higher orders: $a_\mu^{\rm had}[(\alpha_s/\pi)^3] = -(10 \pm 0.6) \times 10^{-10}$

Contribution from light-by-light scattering: $a_\mu^{\rm had}(\text{LBLS}) = -(8.5 \pm 2.5) \times 10^{-10}$

COMPILATION OF RESULTS ON $a_\mu^{\rm had}$



COMPILATION OF RESULTS ON $a_\mu^{\rm had}$



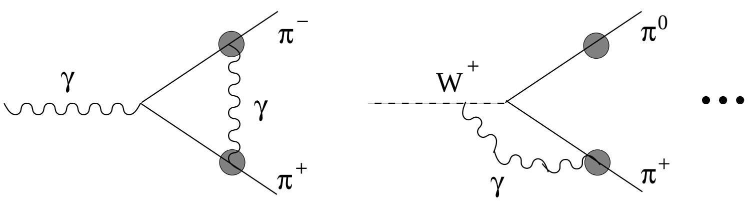
Davier et al. (incl. CMD-2 (01)

. . . discrepancy with experiment would increase !

POSSIBLE SHOWSTOPPERS ?

- No hint for problems from pQCD (+D=4,6,8) and QCD Sum Rules. Their relative weight in the final error is weak (17%).

- Dominant contribution from $\rho^0 \to \pi^+\pi^-$ resonance: 84% of total error. Large impact from τ data. CVC corrections applied:
  - EW radiative corrections (S_{EW} [Marciano-Sirlin '88, Braaten-Li '90])
  - $m_{\pi^\pm} \neq m_{\pi^0}$ corrections applied
  - EM loops of $e^+e^- \to \rho^0 \to \pi^+\pi^-$ wrt. $\tau^+ \to \rho^+ \to \pi^+\pi^0\nu_\tau$ decays [Singer'63, errat.'67 (by part)]



$\Longrightarrow$ this urgently needs attention from theorists !!!

- e^+e^- data: experimental systematics and radiative corrections ?!

$\Longrightarrow$ new, independent data needed (ISR events from e^+e^- factories ?)

- $\tau^+ \to \pi^+\pi^0\nu_\tau$ Spectral Function measurement:
  - BR: Agreement between ALEPH'96 and CLEO'94 (domin. by ALEPH'96)
  - Unfolding of mass spectra: fair agreement between ALEPH'97 and OPAL'99

- Hadronic Light-by-Light Scattering: $-8.5 \pm 2.5 \times 10^{-10}$ (?)
[Hayakawa, Kinoshita; Bijnens *et al.*]

CONCLUSIONS AND PERSPECTIVES

- (I)-(III) Analysis steps improved more than twice the accuracy on $a_\mu^{\rm had}$ and more than four times the precision on $\Delta\alpha_{\rm had}(M_Z^2)$
- Largest impact for $a_\mu^{\rm had}$ from inclusion of τ data
- Largest impact for $\Delta\alpha_{\rm had}(M_Z^2)$ from ext. use of QCD
- More sudies on EM SU(2) breaking needed, in particular, radiative corrections affecting differently $\rho^\pm$ and ρ^0
- No hints for severe problems in the hadronic sector . . . so far
- New τ results from ALEPH and reanalysis of CMD-2 data forthcoming
- Ongoing collaboration of D.-H. with S. Eidelman to prepare detailed data, as well as theory driven analysis with new τ data and new ϵ^+e^- data from CMD-2
- Future Experimental Projects:
  - PEP-N (SLAC): $e^+e^- \rightarrow \mathrm{hadrons}$ at 1.4-3.1 GeV
  - ISR Radiative Decays $e^+e^- \rightarrow \mathrm{hadrons} + \gamma$:
    DAΦNE, BABAR, BELLE, CLEO and BES in τ/charm mode (?)