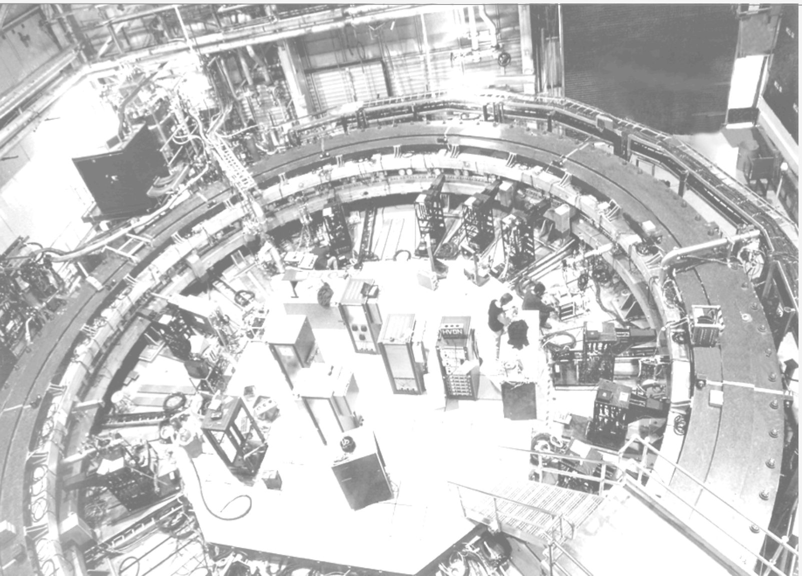
— *Conclusions on the state of the art including new evaluations* —

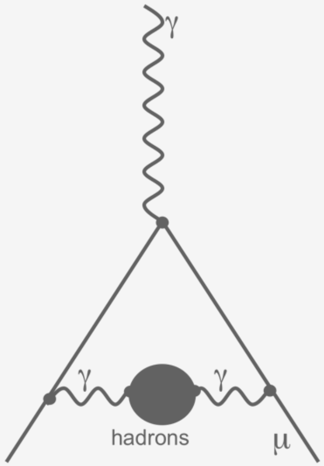
The Muon $g-2$ and its Hadronic Contribution

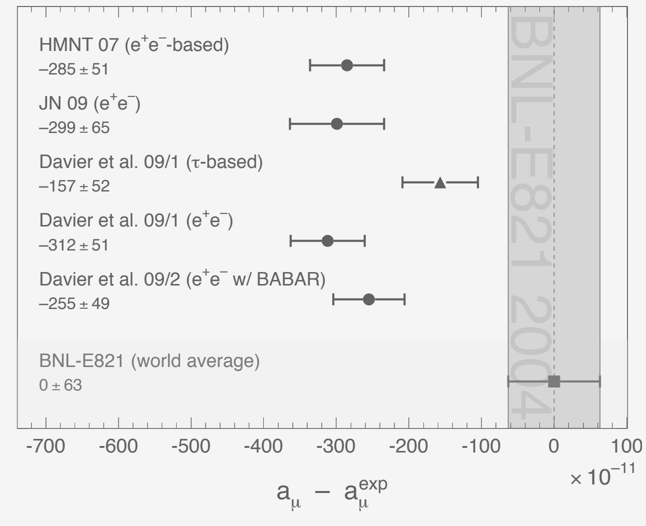
[… and also that of $\alpha_{QED}(M_Z)$]

Andreas Hoecker (CERN)

Tau Workshop, Manchester, UK, Sep 13 – 17, 2010







Hadronic Contribution to Muon $g-2$

$$a_\mu^{\text{SM}} \equiv \left(\frac{g-2}{2}\right)_\mu = a_\mu^{\text{QED}} + a_\mu^{\text{had,LO}} + a_\mu^{\text{had,NLO}} + a_\mu^{\text{weak}}$$

$\sigma^{\text{Exp}} = 6.3$ | $\sigma_{\text{QED}}^{\text{SM}} \simeq 0.02$ | $\sigma_{\text{had,LO}}^{\text{SM}} \approx 4$ | $\sigma_{\text{had,NLO}}^{\text{SM}} \simeq \sigma_{\text{had,LBLS}}^{SM} \approx 3$ | $\sigma_{\text{weak}}^{\text{SM}} \approx 0.2$

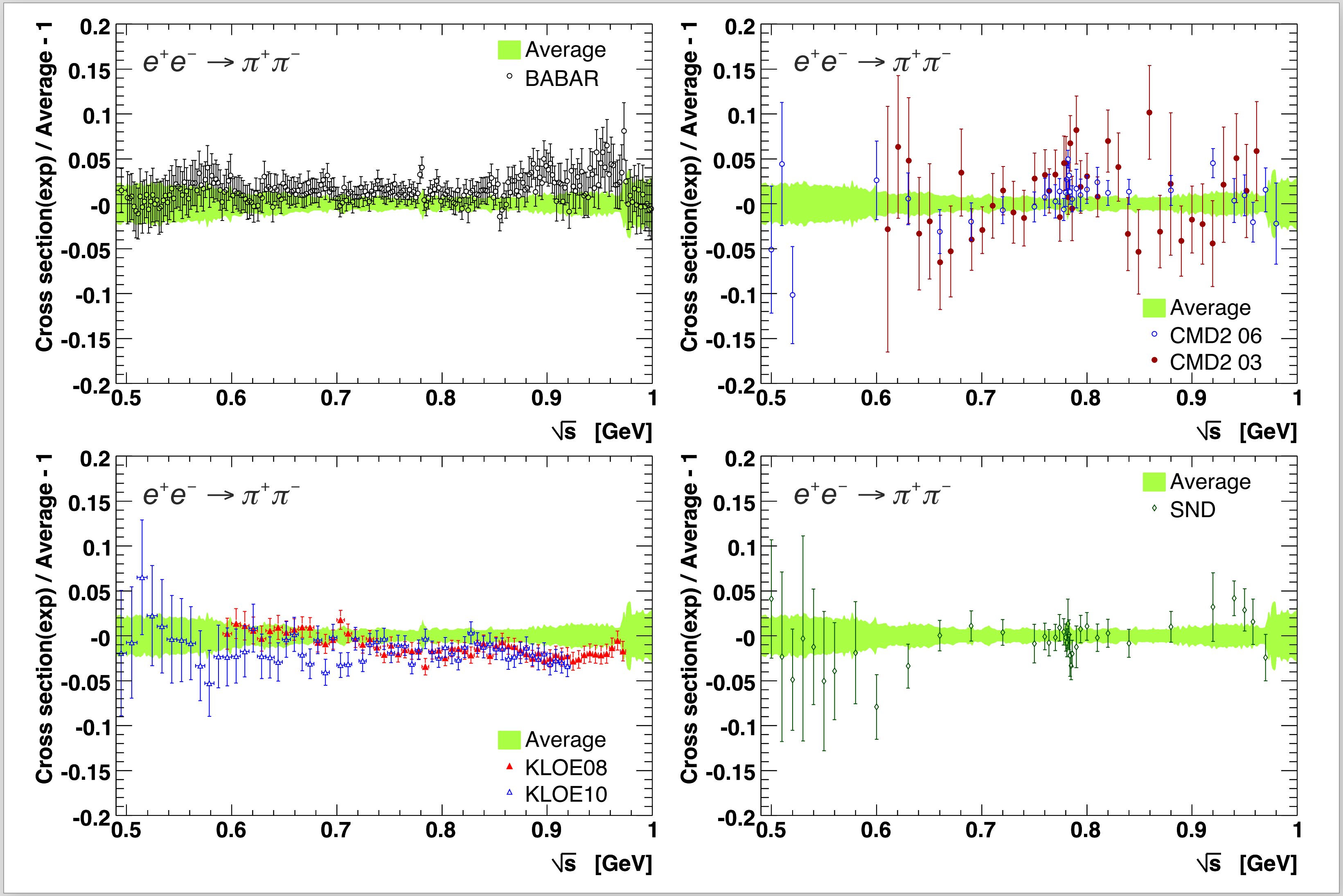
→ SM error on a_μ dominated by hadronic part, ie, by **experimental data** !

In recent years, although not so visible in the final results, significant improvements in the evaluation of the hadronic contribution were obtained

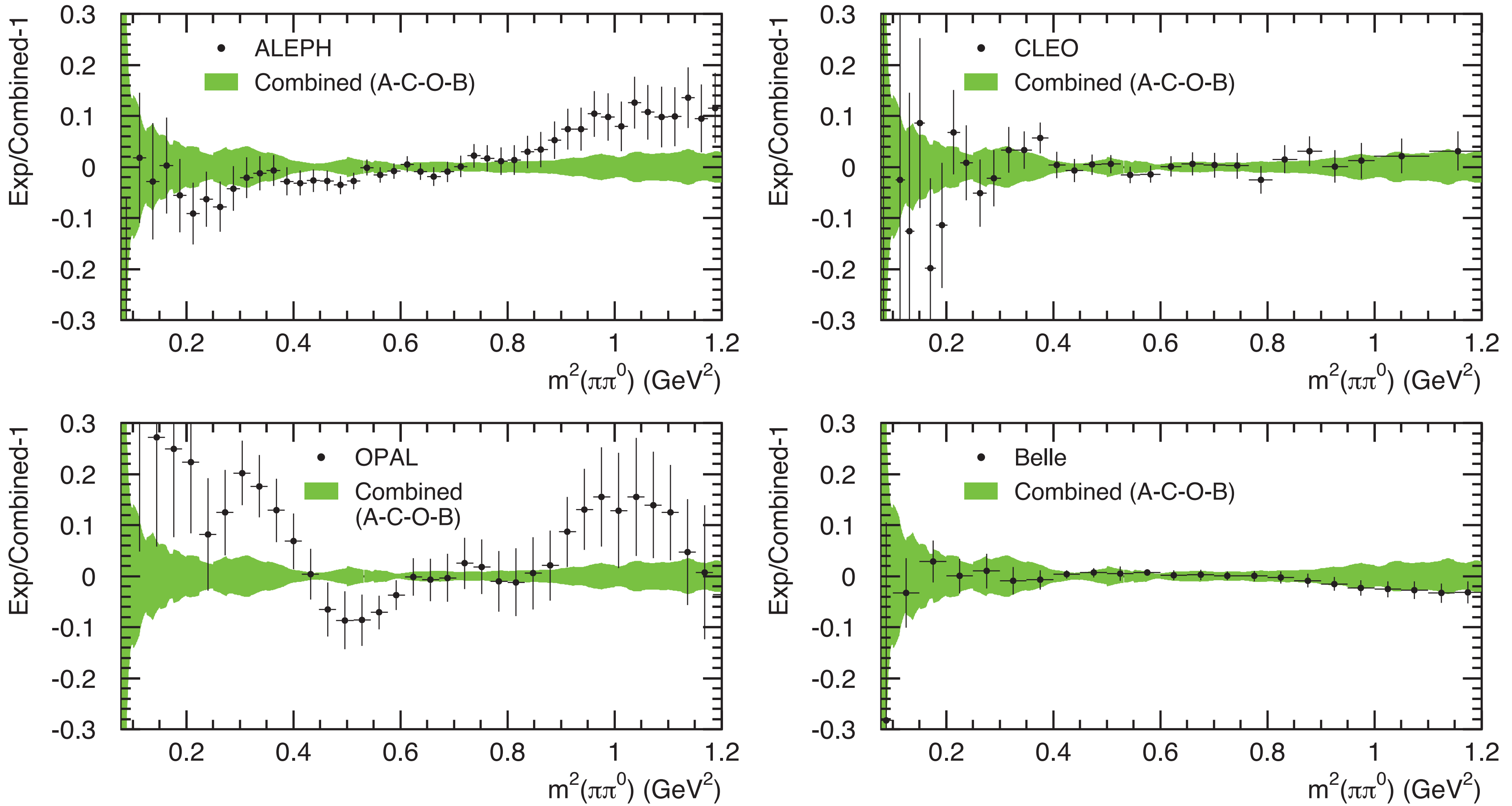
- BABAR and KLOE ISR data increase the redundant information on the $\pi^+\pi^-$ channel
- BABAR data very significantly improved the knowledge of many important final states
- New tau data from Belle increased the redundancy on the $\pi^+\pi^0$ channel and, together with newly evaluated isospin-breaking corrections, the discrepancy with e^+e^- data could be much reduced (if not resolved, as it seems between Belle (tau) and BABAR (e^+e^-)).

e^+e^- and Tau Data Comparisons

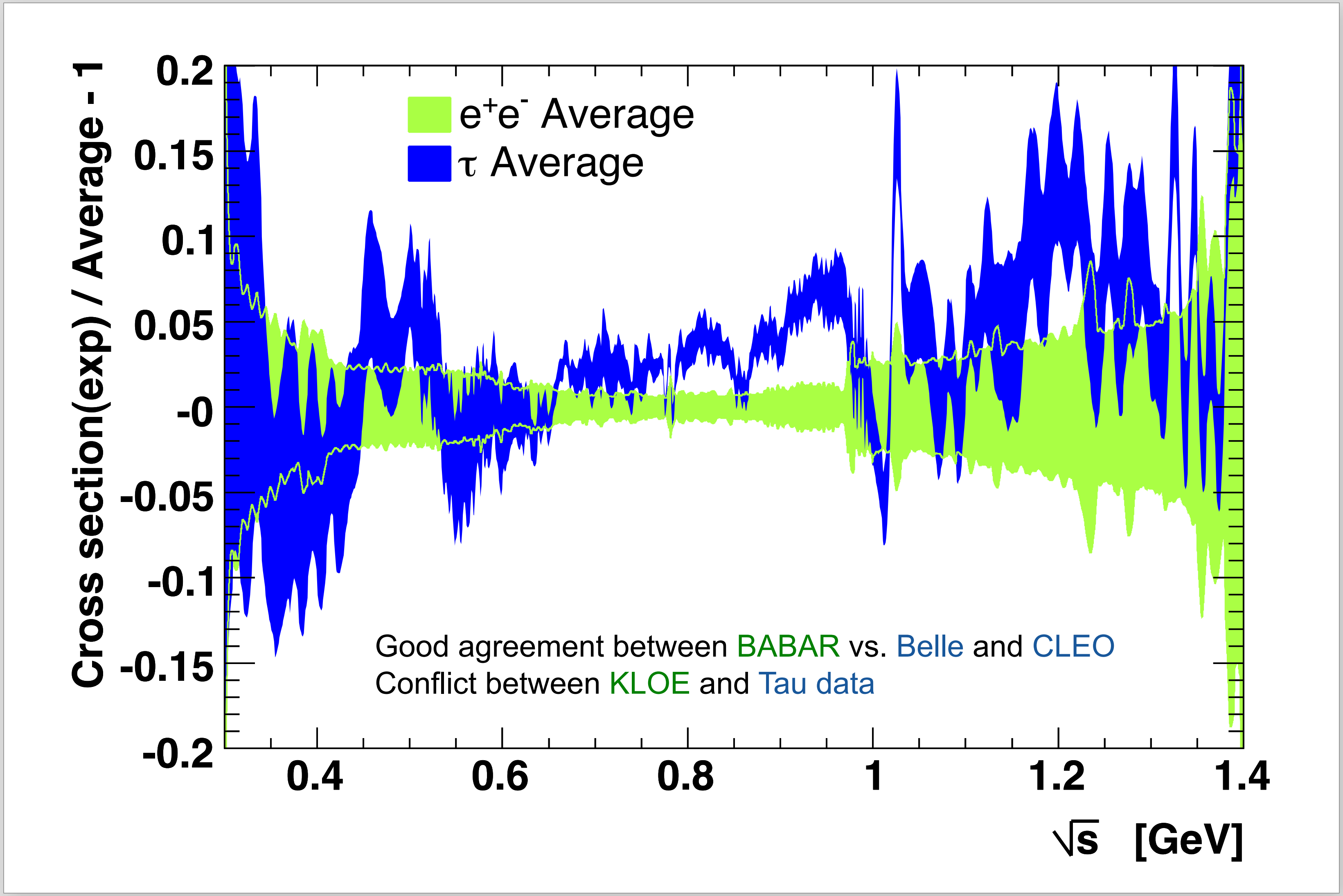
Situation of Two-Pion Channel (e^+e^-)



Situation of Two-Pion Channel (Tau)



Situation in Two-Pion Channel

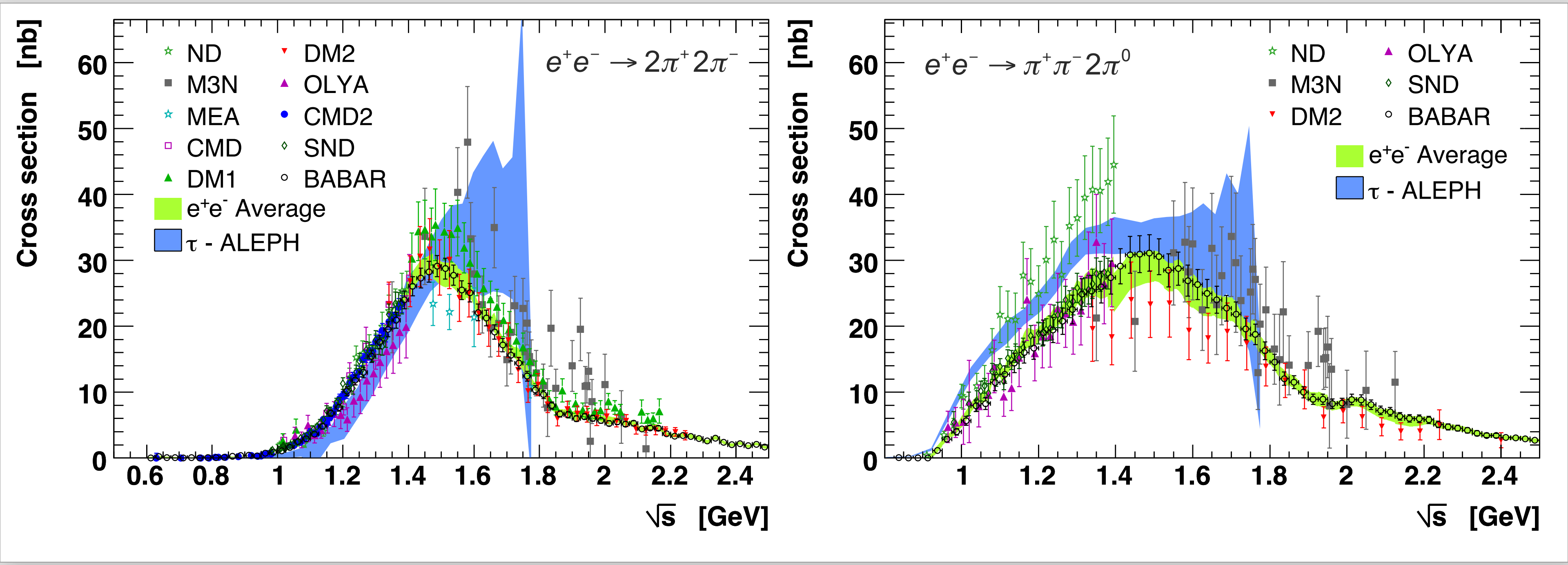


Situation in Four-Pion Channels

Recall 4π isospin relations:

$$\sigma^{(I=1)}_{2\pi^+2\pi^-} = 2\cdot\frac{4\pi\alpha^2}{s}\,\upsilon_{\pi^-3\pi^0\nu_\tau}$$

$$\sigma^{(I=1)}_{\pi^+\pi^-2\pi^0} = \frac{4\pi\alpha^2}{s}\left(\upsilon_{2\pi^-\pi^+\pi^0\nu_\tau} - \upsilon_{\pi^-3\pi^0\nu_\tau}\right)$$



Isospin Breaking Corrections in $\pi^+\pi^-$ Mode

Electromagnetism does not respect isospin and hence we have to consider isospin breaking when dealing with an experimental precision of 0.5%

- **Radiative corrections:**

  S_{EW} ~ 2% (short distance),
  $G_{EM}(s)$ (long distance)

- **Charged/neutral mass splitting:**

  $m_{\pi^-} \neq m_{\pi^0}$, ρ-ω mixing, $m,\Gamma_{\rho^-} \neq m,\Gamma_{\rho^0}$

  leads to phase space and width differences

- **Electromagnetic decays:**

  $\rho \to \pi\pi\gamma$, $\rho \to \pi\gamma$, $\rho \to \eta\gamma$, $\rho \to l^+l^-$

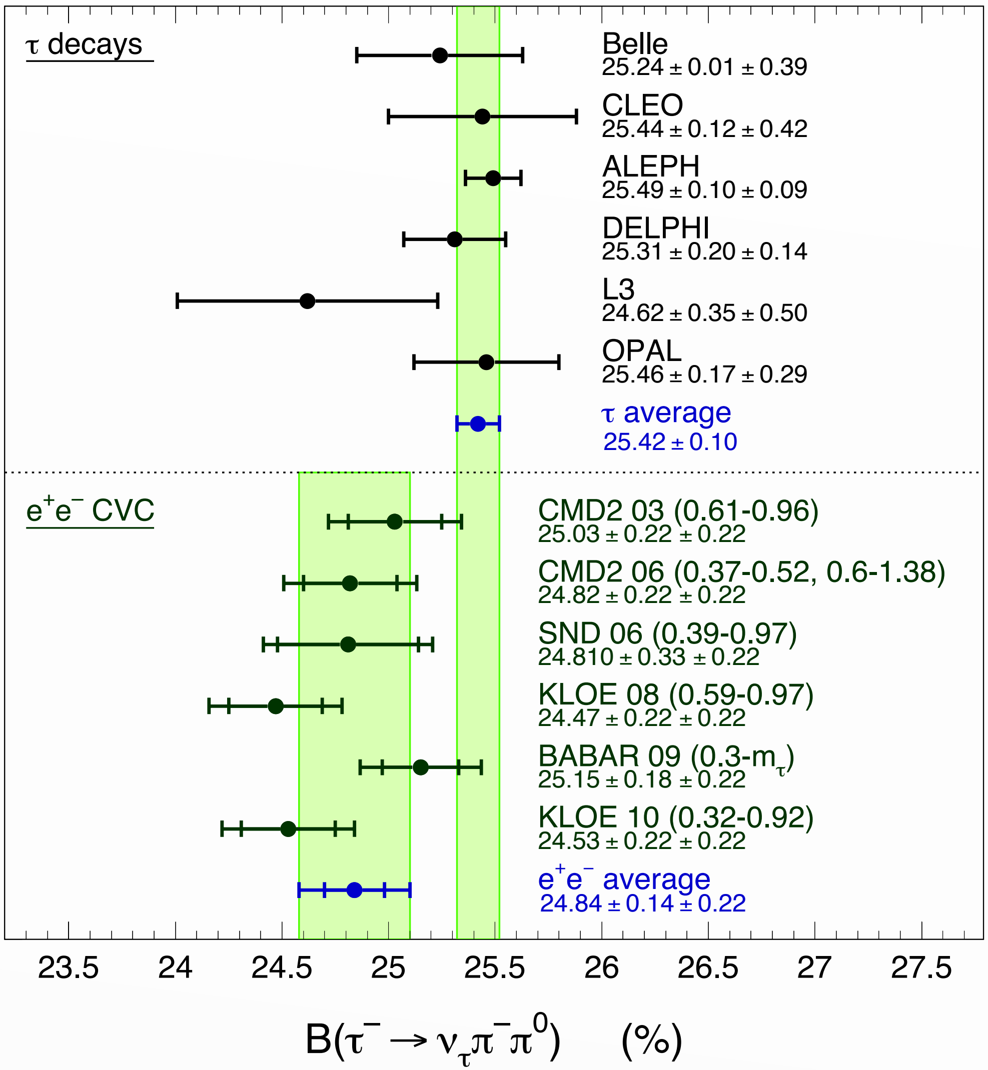
- **Quark mass difference:**

  $m_u \neq m_d$ negligible

Source	$\Delta a_\mu^{\rm had,LO}[\pi\pi,\tau]$ (10^{-10}) GS model	KS model
$S_{\rm EW}$	-12.21 ± 0.15	
$G_{\rm EM}$	-1.92 ± 0.90	
FSR	$+4.67 \pm 0.47$	
ρ–ω interference	$+2.80 \pm 0.19$	$+2.80 \pm 0.15$
$m_{\pi^\pm} - m_{\pi^0}$ effect on σ	-7.88	
$m_{\pi^\pm} - m_{\pi^0}$ effect on Γ_ρ	$+4.09$	$+4.02$
$m_{\rho^\pm} - m_{\rho^0_{\rm bare}}$	$0.20^{+0.27}_{-0.19}$	$0.11^{+0.19}_{-0.11}$
$\pi\pi\gamma$, electrom. decays	-5.91 ± 0.59	-6.39 ± 0.64
Total	-16.07 ± 1.22	-16.70 ± 1.23
	-16.07 ± 1.85	

Davier et al, arXiv:0906.5443

Predict Tau Branching Ratios from e^+e^-



$$\mathrm{BR}^{\mathrm{CVC}}_{\tau^-\to\pi^-\pi^0\nu_\tau} \propto \int_0^{m_\tau^2} ds\ \mathrm{kin}(s)\cdot \sigma^{\text{IB-corrected}}_{e^+e^-\to\pi^+\pi^-}(s)$$

IB corrections of +0.69 ± 0.22 applied for $\pi^+\pi^0$

Difference: BR[τ] – BR[e^+e^- (CVC)]:

Mode	$\Delta(\tau - e^+e^-)$	"Sigma"
$\tau^- \to \pi^-\pi^0\nu_\tau$	+ 0.58 ± 0.28	2.1
$\tau^- \to \pi^- 3\pi^0\nu_\tau$	– 0.03 ± 0.09	0.3
$\tau^- \to 2\pi^-\pi^+\pi^0\nu_\tau$	+ 0.69 ± 0.22	3.2

CVC predictions of $\pi^+\pi^-$ much improved with BABAR data and re-evaluated IB corrections (4.5σ previously) !

Davier *et al*, arXiv:0906.5443 (2009)

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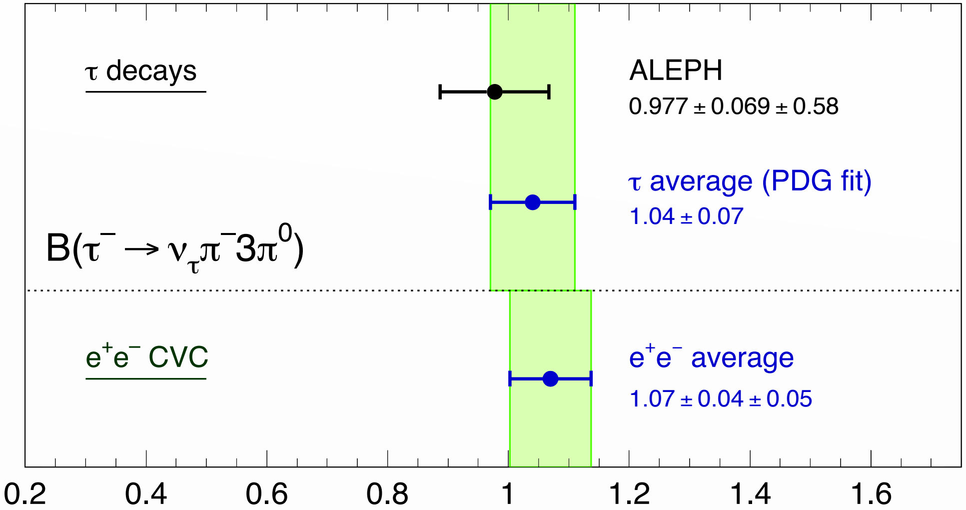

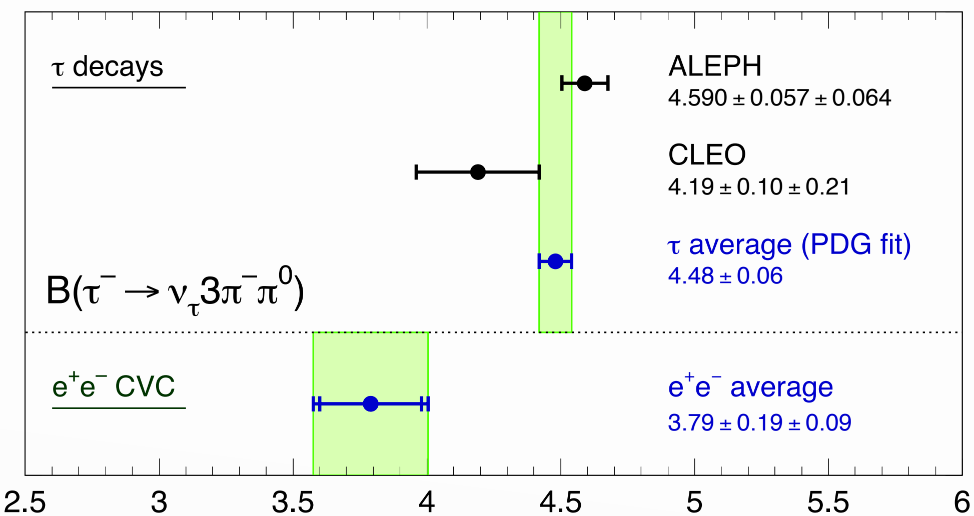

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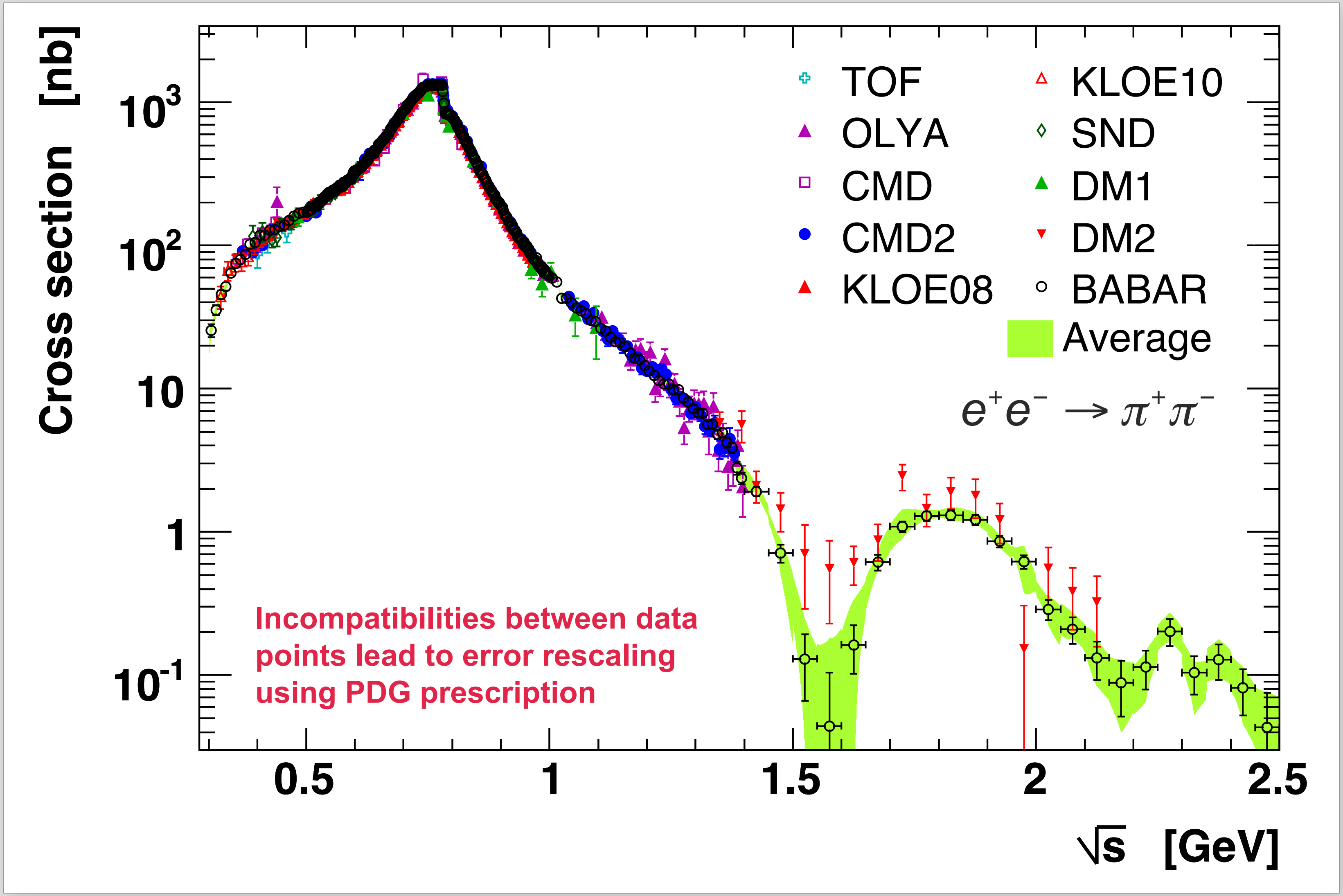
PRELIMINARY

M. Davier, A. Hoecker, B. Malaescu, Z. Zhang (DHMZ) present …

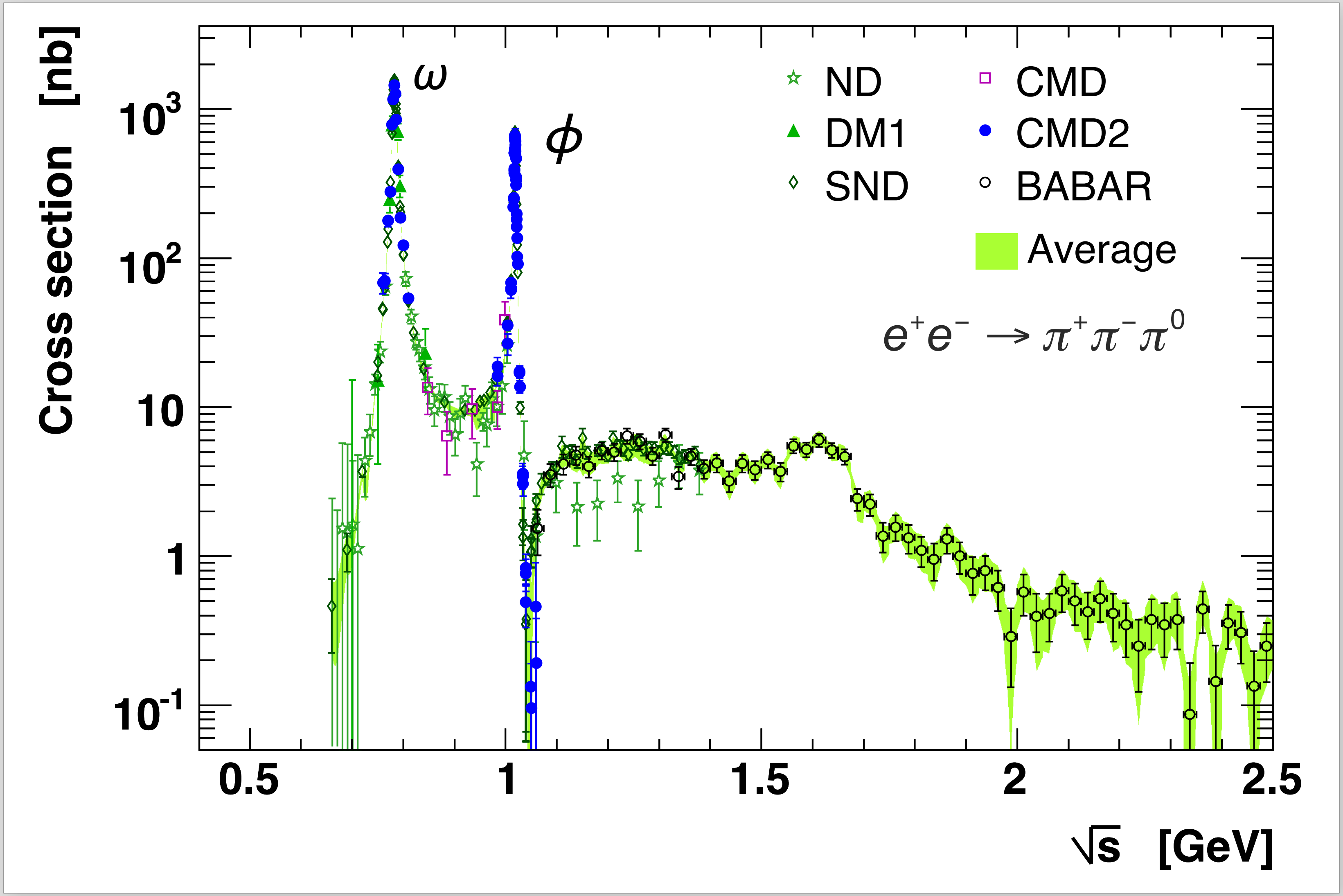
A New Evaluation of $(g-2)_\mu$ and $\alpha(M_Z)$

- Use HVPTools for all channels
- Include KLOE 2010 data (large photon angle)
- Include preliminary $2\pi^\pm 2\pi^0$ data from BABAR
- Re-estimate missing modes using new BABAR data on multi-pion and kaon modes
- Use 4-loop Adler function for pQCD prediction

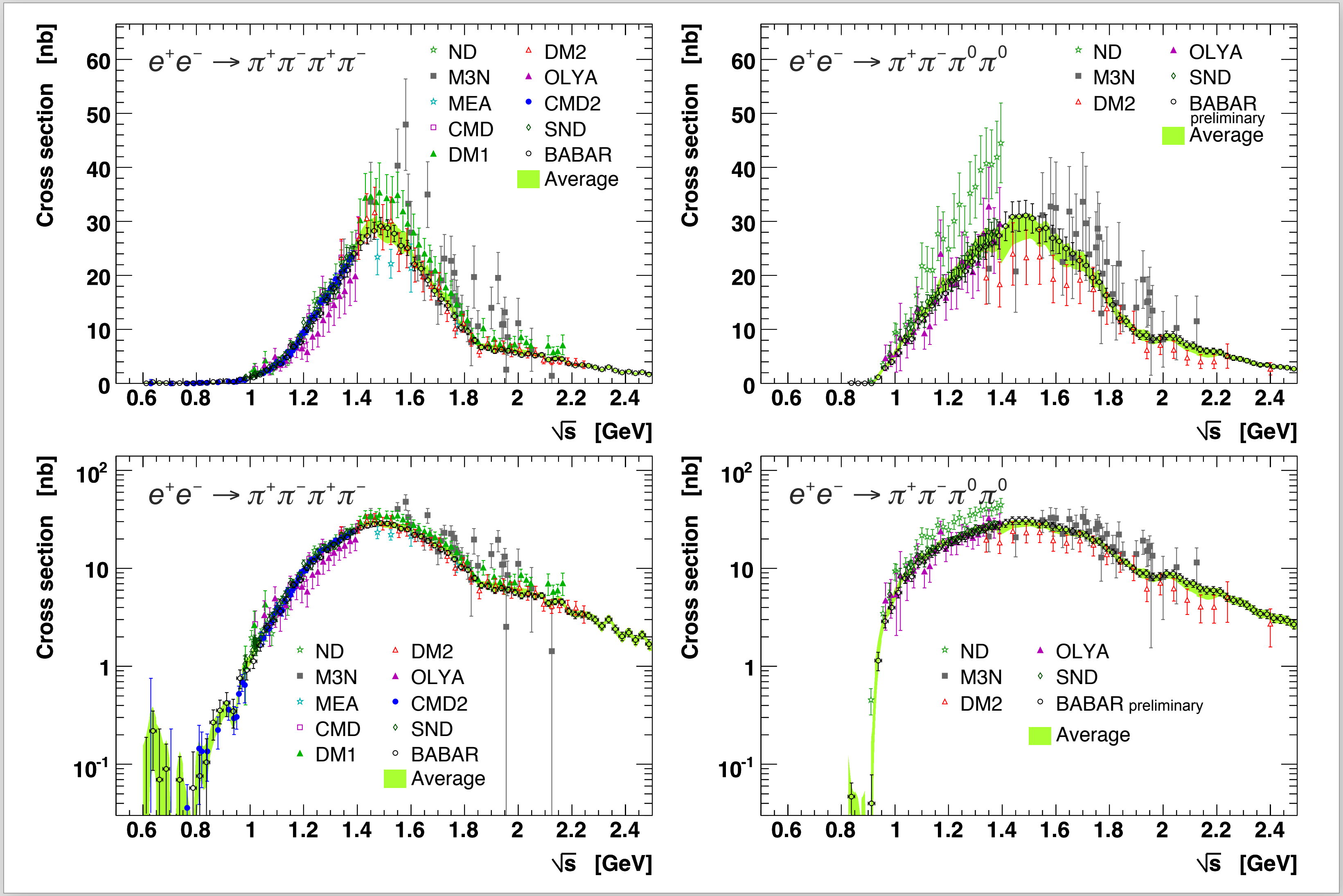
$e^+e^- \rightarrow \pi^+\pi^-$ Cross Section



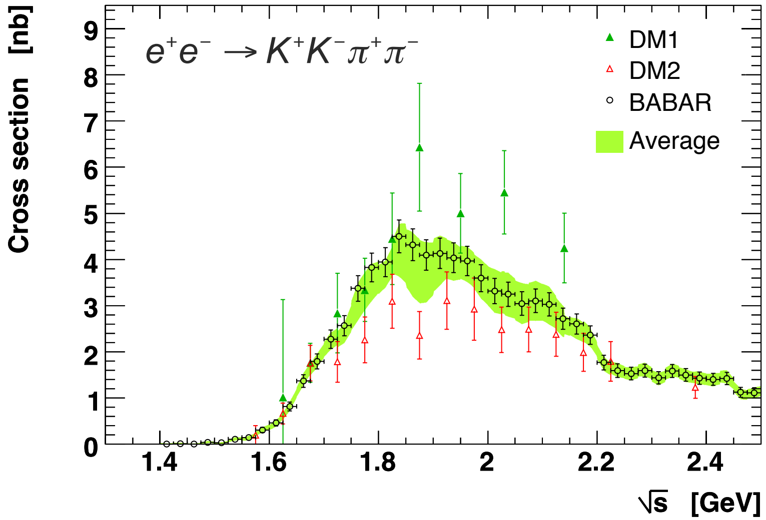
$e^+e^- \to \pi^+\pi^-\pi^0$ Cross Section

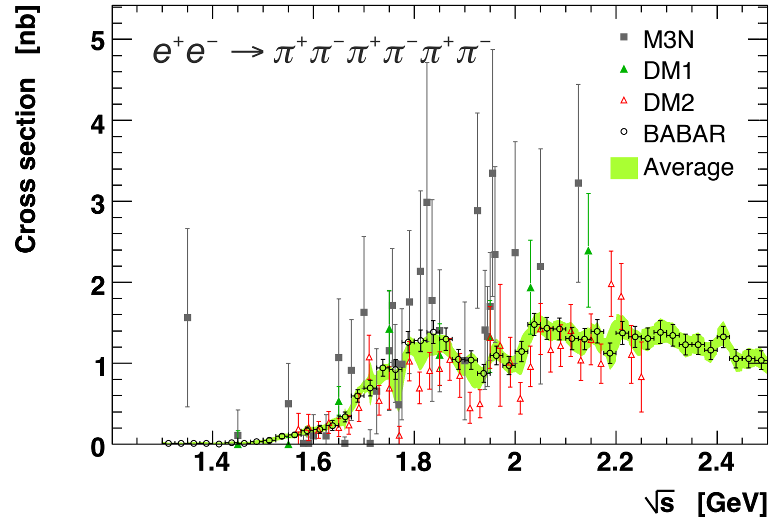


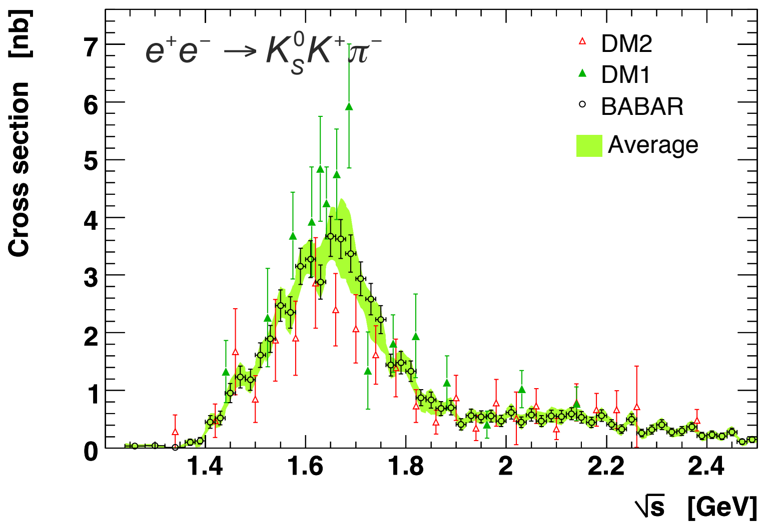
$e^+e^- \rightarrow 4\pi$ Cross Sections



Rarer and High-Multiplicity Modes



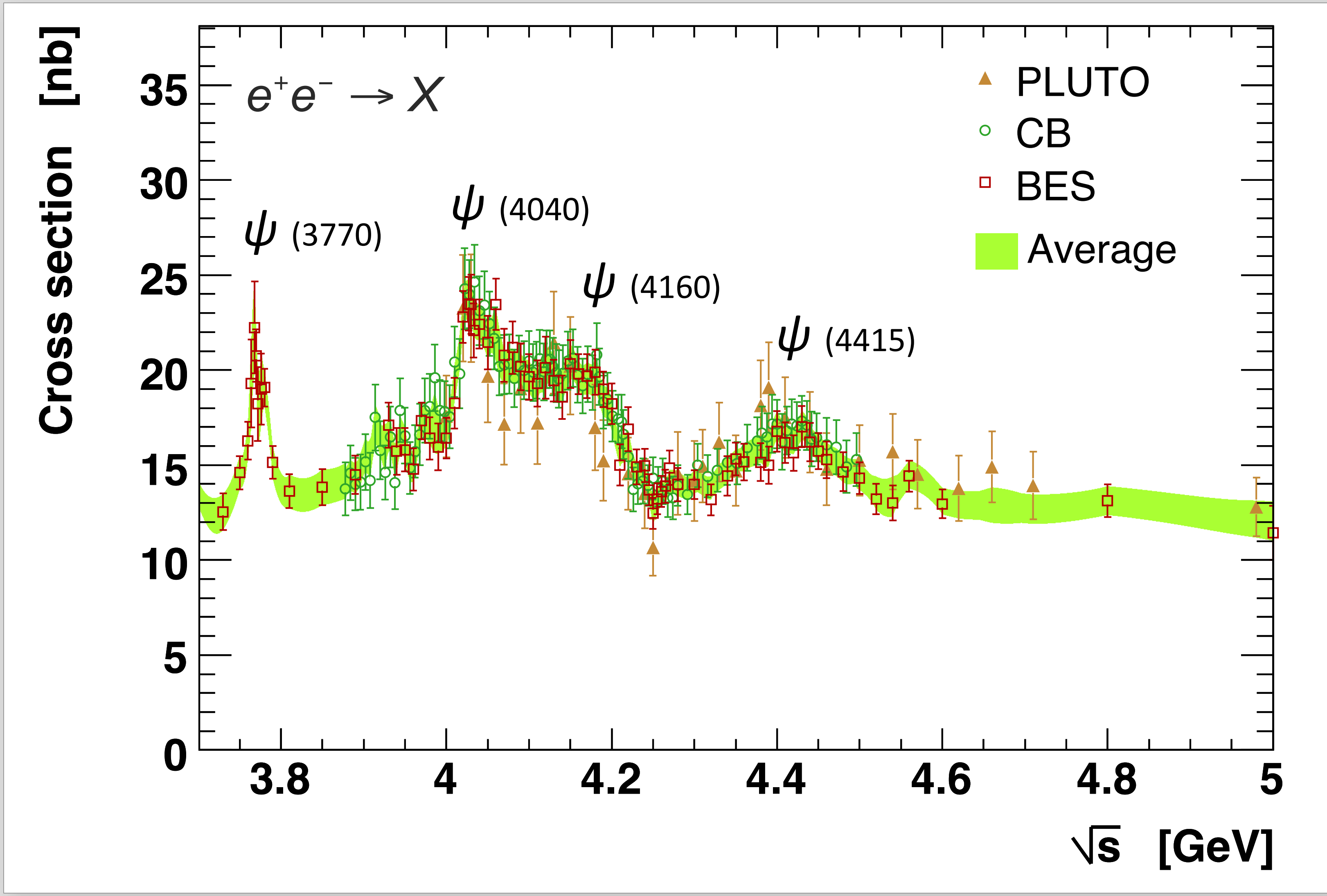




Many rare(r) modes have been measured by BABAR (using the ISR technique), thus greatly reducing uncertainties in the determination of the dispersion integral.

Some rare modes are not known or incomplete, and must be estimated from known modes using isospin symmetry.

Inclusive Cross Section after $D\bar{D}$ Opening



Integrating Data

The integration of data points belonging to different experiments, with different within-experiment and inter-experiment correlated systematic errors, and with different data densities requires a **careful treatment**

It is mandatory to test the accurateness of the integration procedure in terms of central value and error using representative models with known truth.

DHMZ approach:

- Quadratic interpolation of the data points/bins for each experiment
- Local weighted average between interpolations performed in infinitesimal bins
- Full covariance matrices: correlations between data points of an experiment (systematic errors), between experiments and channels (VP, luminosity, …)
- Consistent error propagation using pseudo experiments
- Possible bias tested in 2π channel using a GS model: negligible for quadratic interpolation, but not for linear model (trapezoidal rule)

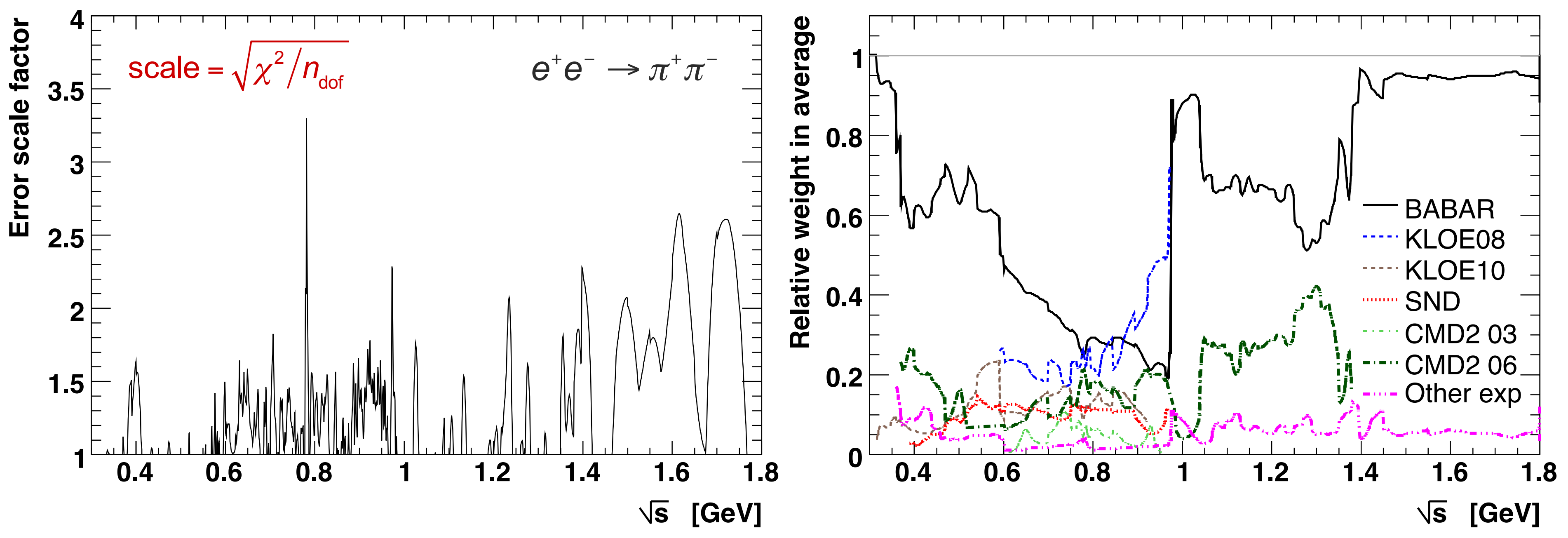
Integrating Data

Incompatibilities between data points lead to error rescaling

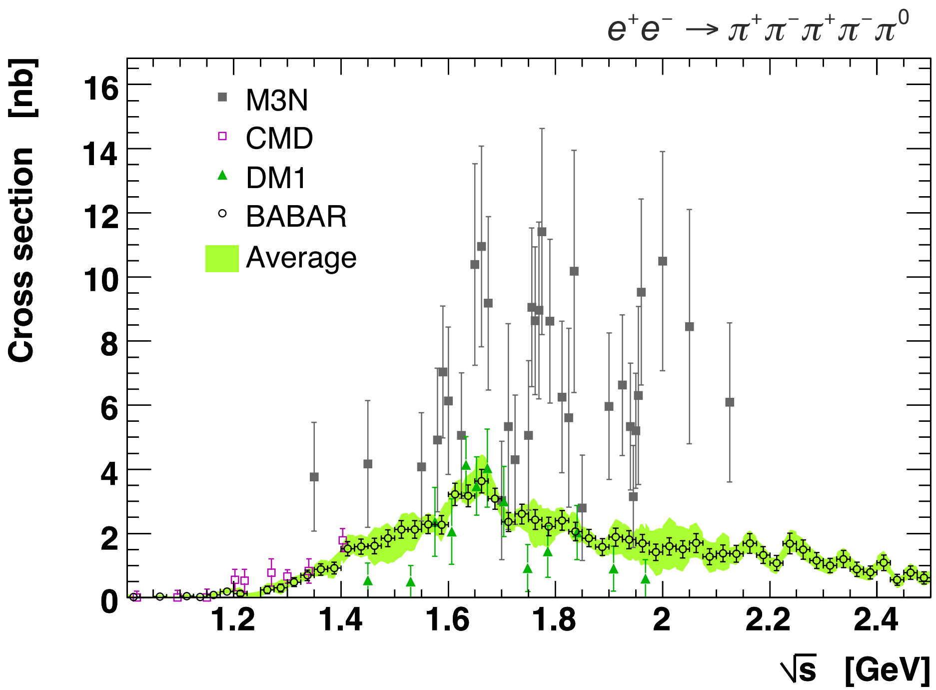
Performed using PDG prescription

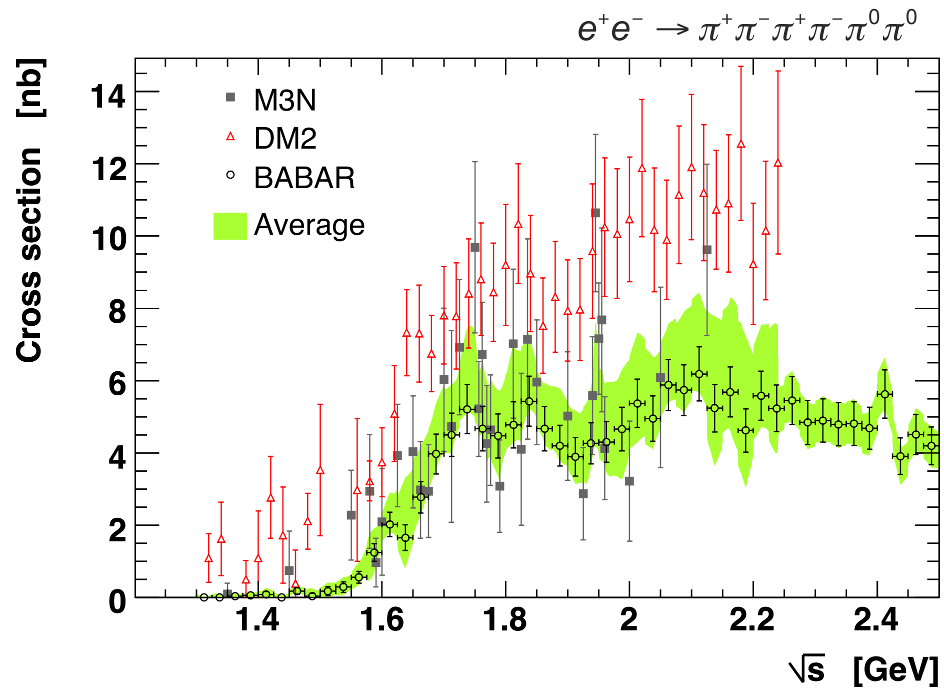
Weights of experiments in average versus mass

BABAR dominates everywhere, except for KLOE 08 between 0.8 and 0.93 GeV



More Modes that Changed with BABAR





In several cases, bad measurements lead to overestimated cross sections

Determining Missing Channels

Use Pais isospin I=0,1 classification of N-pion states to derive relations between isospin-symmetric final states to constrain unmeasured channels

- **5-pion channels**: isospin relation reads $\sigma(2\pi^+2\pi^-\pi^0) = 2\times\sigma(\pi^+\pi^-3\pi^0)$, but isospin-breaking (IB) $\eta\pi\pi$ contributions must be subtracted and treated apart

- **6-pion channels**: the unknown $\sigma(\pi^+\pi^-4\pi^0)$ is determined from $\sigma(3\pi^+3\pi^-)$ and $\sigma(2\pi^+2\pi^-2\pi^0)$ together with an upper limit from tau 6π data on two unconstrained Pais classes, and using the experimental fact that these modes are dominated by $\omega3\pi$. IB contributions from $\eta\omega$ are subtracted and treated separately.

- ***KK*π(π) channels**: the missing modes $K_SK_L\pi^0$ and $K_SK^+\pi^-\pi^0$, $K_SK_S\pi^+\pi^-$, $K_SK_L\pi^+\pi^-$ are determined from I=0,1 isospin relations using K^*K dominance, and correcting for small $\phi\pi$ and $KK\rho$ contributions. The $KK\pi\pi\pi$ contribution is determined from the measured $K^+K^-\pi^+\pi^-\pi^0$ using observed $K^+K^-\omega$ dominance.

- **$\eta4\pi$ channels**: the total contribution is estimated as twice the measured $\sigma(\eta2\pi^+2\pi^-)$

Determining Missing Channels

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- **5-pion channels**: isospin relation reads $\sigma(2\pi^+2\pi^-\pi^0) = 2\,\sigma(\pi^+\pi^-3\pi^0)$, but isospin-breaking

All these estimates strongly benefit from BABAR's measurements of rare high-multiplicity modes with and without kaons

together with an upper limit from tau 6π data on two unconstrained Pais classes, and using the experimental fact t ... IB contributions from $\eta\omega$ are subtracted and treated separately.

Overall the new estimates **reduce** the contributions from unknown modes

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- **$\eta 4\pi$ channels**: the total contribution is estimated as twice the measured $\sigma(\eta 2\pi^+2\pi^-)$

Full List of Contributions

Channel	a_μ^{had} (e^+e^-)	a_μ^{had} (τ)
$\pi^0\gamma$	$4.4 \pm 0.1_{\mathrm{stat}} \pm 0.2_{\mathrm{syst}}$	
$\pi^+\pi^-$	$507.2 \pm 1.2 \pm 2.6$	$515.2 \pm 3.0 \pm 1.9_{\mathrm{IB}}$
$\pi^+\pi^-\pi^0$	$46.0 \pm 0.4 \pm 1.4$	
$2\pi^+2\pi^-$	$13.4 \pm 0.1 \pm 0.5$	$21.4 \pm 1.3 \pm 0.6_{\mathrm{IB}}$
$\pi^+\pi^-2\pi^0$	$18.0 \pm 0.1 \pm 1.2$	$12.3 \pm 1.0 \pm 0.4_{\mathrm{IB}}$
KK	$34.6 \pm 0.3 \pm 0.9$	
$KKn\pi$	$3.7 \pm 0.1 \pm 0.4$	
Other exclusive modes	$6.0 \pm 0.3 \pm 0.3$	
Charm region (3.7 – 5 GeV)	$7.0 \pm 0.1 \pm 0.3$	
J/ψ, $\psi(2S)$	7.9 ± 0.2	
R [QCD]	$43.3 \pm 0.3_{\mathrm{theo}}$	*Error includes size of 4-loop term + FOPT/CIPT ambiguity*
Sum	$691.4 \pm 1.4 \pm 4.2$	

All numbers given in units of 10^{-10}

Note: systematic errors partly correlated between channels !

New Results for Muon $g - 2$

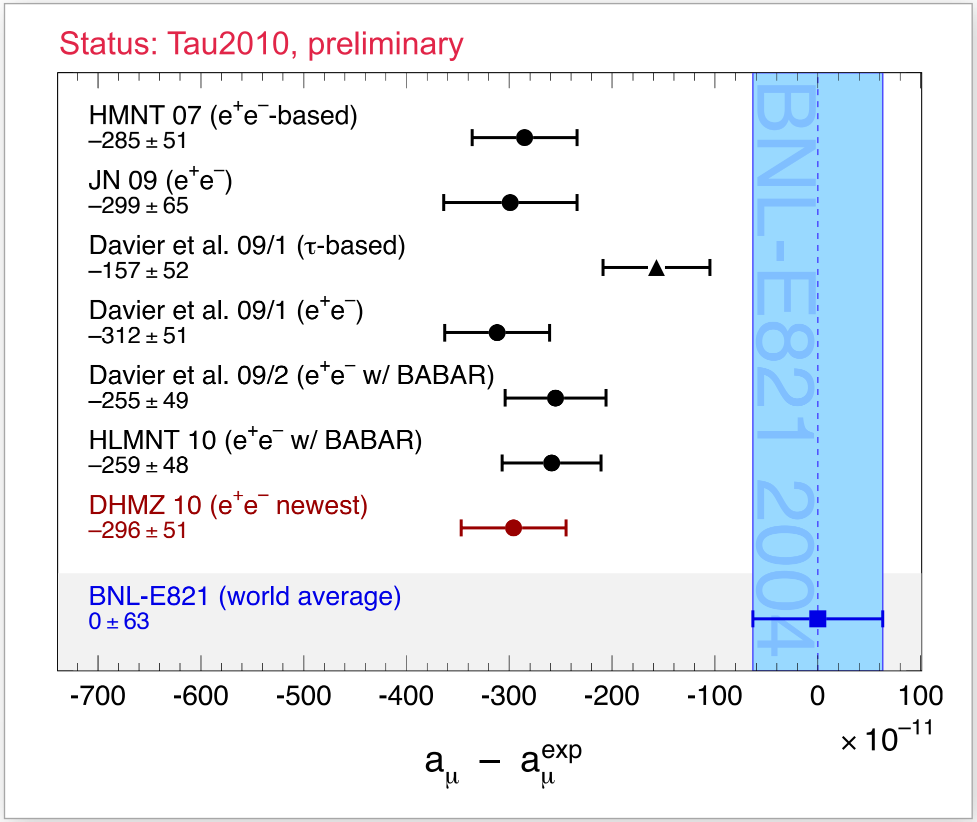
PRELIMINARY

$$a_\mu^{SM}[e^+e^-] = (11\,659\,179.3 \pm 4.4_{\text{had,LO}} \pm 2.6_{\text{NLO}} \pm 0.2_{\text{QED+weak}}) \times 10^{-10}$$

DHMZ, Tau 2010

$$a_\mu^{SM}[e^+e^-] = (11\,659\,183.0 \pm 4.5_{\text{had,LO}} \pm 2.6_{\text{NLO}} \pm 0.2_{\text{QED+weak}}) \times 10^{-10}$$

HLMNT Tau 2010



Observed Difference with Experiment:

$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (29.6 \pm 8.1) \times 10^{-10}$$

➡ 3.6 "standard deviations"

DHMZ, Tau 2010

Note that the re-evaluated tau result would follow large part of shift (though not that due to KLOE …)

… →

Discussion

The Tau-2010 DHMZ result is -4.1×10^{-10} smaller than that of 2009
(the latter of which is compatible with that of HLMNT 2010)

Origins of main changes:

$\pi^+\pi^-$	–1.2	New KLOE 2010 data
$\pi^+\pi^-\pi^0\pi^0$	+0.4	Use preliminary BABAR data
Other exclusive modes and resonances	–2.9	New BABAR data, improved isospin constraints on unknown modes
QCD	–0.5	4-loop pQCD coefficient

Many modes also have been computed for the first time with HVPTools
featuring a more precise interpolation, and better error propagation than our previous software

New Result for α_{QED} at M_Z

Also the hadronic contribution to $\alpha_{QED}(M_Z)$ has been re-evaluated:

$$\Delta\alpha_{had}^{(5)}(M_Z^2) = (274.23 \pm 0.17_{stat} \pm 0.87_{syst} \pm 0.54_{QCD} \; [1.0_{tot}]) \times 10^{-4}$$

DHMZ, Tau 2010

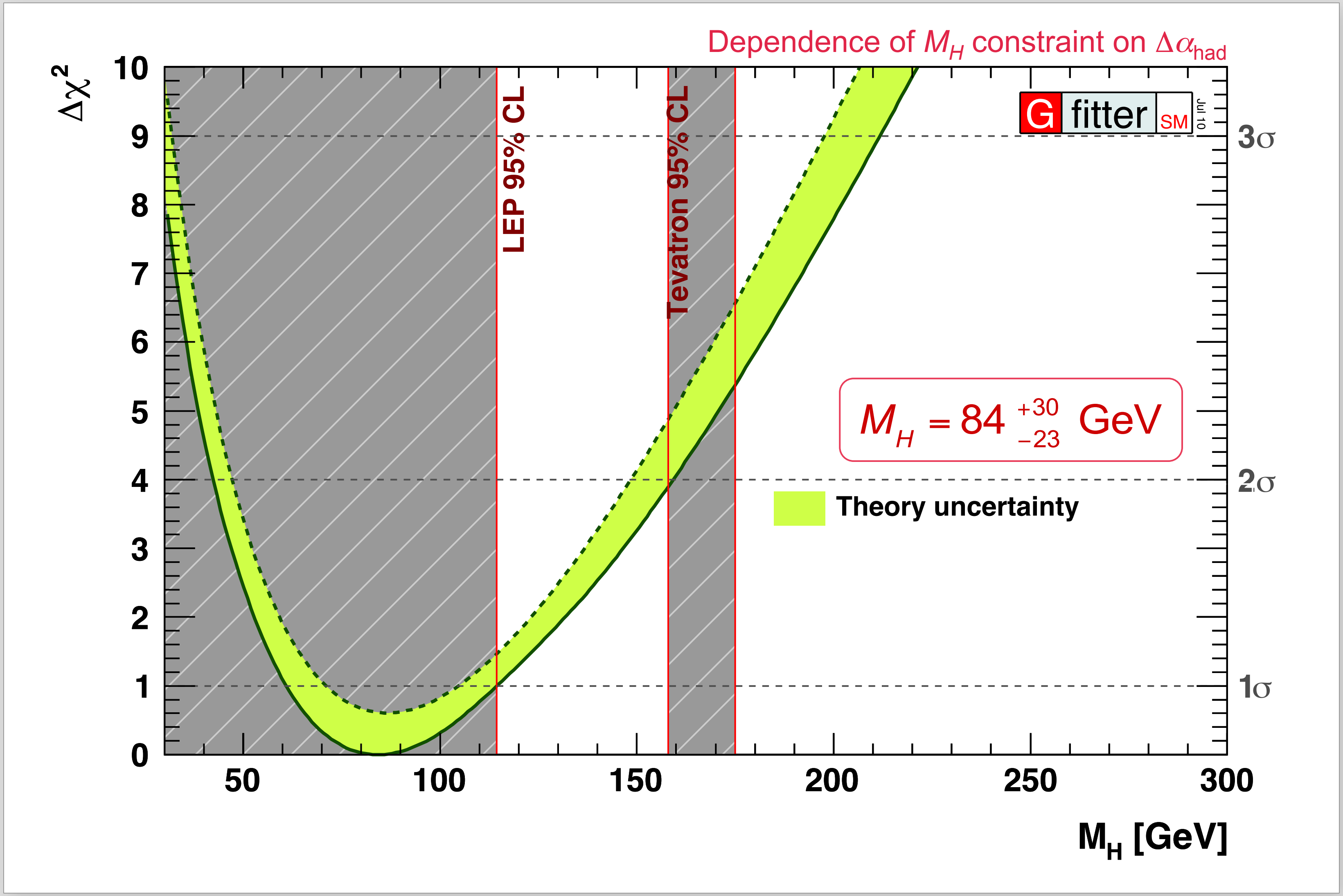
$$\Delta\alpha_{had}^{(5)}(M_Z^2) = (275.9 \pm 1.5_{tot}) \times 10^{-4}$$

HLMNT, Tau 2010

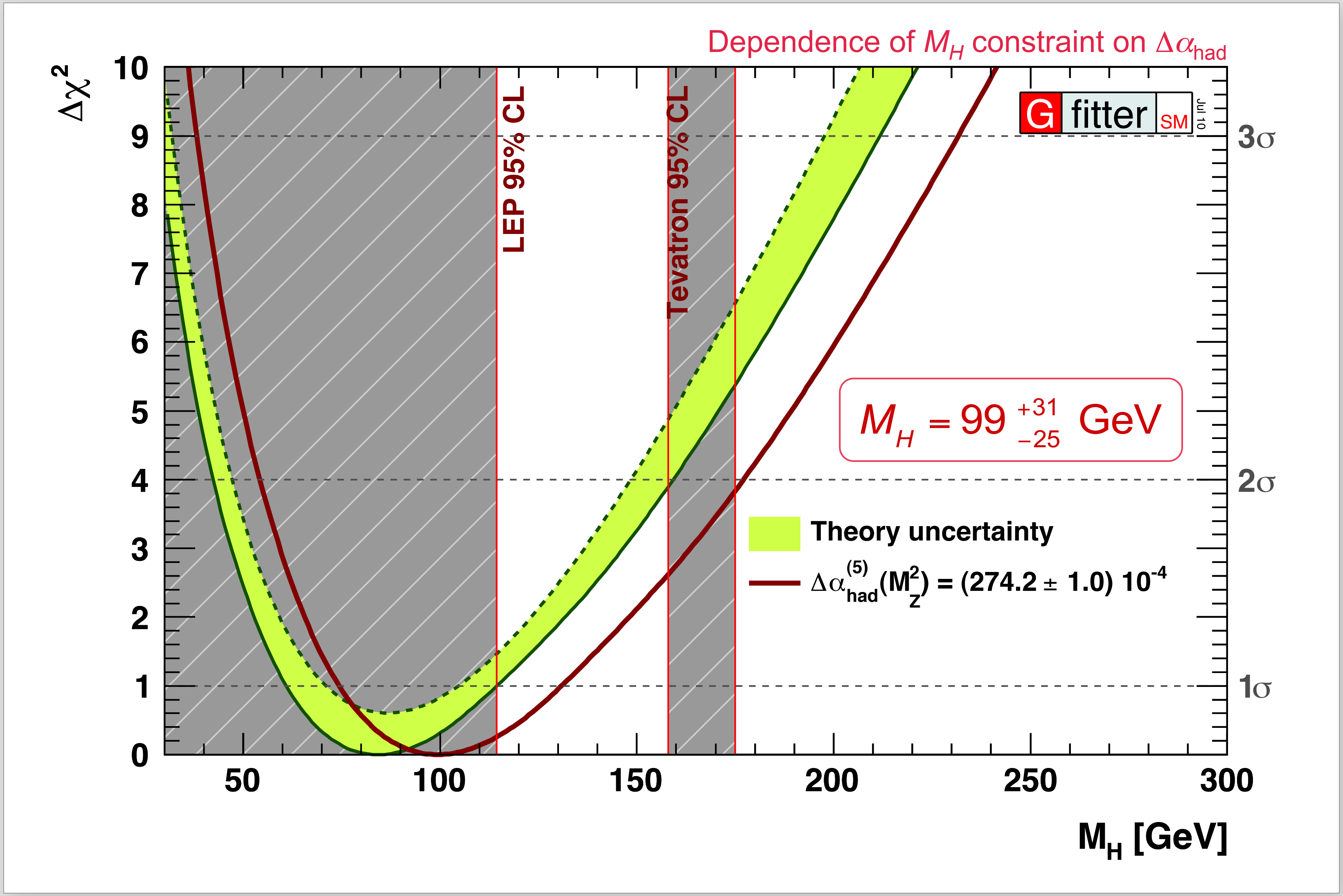
The better precision of the DHMZ value with respect to HLMNT is because of the use of QCD instead of BES data between 1.8 and 3.7 GeV
(HLMNT employs QCD central values, but with BES errors)

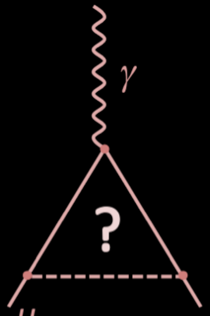
Due to the –40% correlation between $\Delta\alpha_{had}(M_Z)$ and M_H in the global electroweak fit, the change in the central value should increase M_H and reduce tension between fit and direct searches !

Global Electroweak Fit



Global Electroweak Fit





Conclusions and Perspectives

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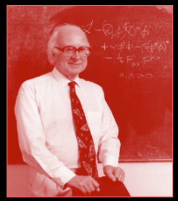
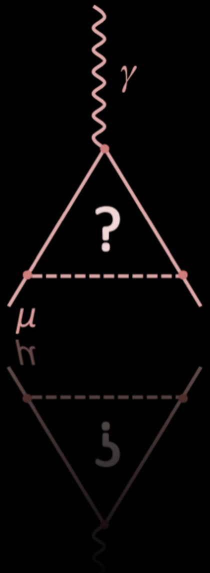


- New evaluation increases SM vs. Exp discrepancy to 3.6σ
- Agreement between e^+e^- and tau data improved, but still unsatisfactory in 2π and 2π2π^0 channels
  - Full agreement between BABAR (e^+e^-) and Belle (tau)
  - KLOE disagrees with BABAR or tau data
- New $\Delta\alpha_{\mathrm{had}}(M_Z)$ value leads to +15 GeV M_H shift in EW fit
- KLOE vs. BABAR problem must be solved
  - Eagerly awaiting ππγ / μμγ ratio analysis from KLOE
  - VEPP 2000 precision data also forthcoming
- Must not forget to work/progress on LBLS contribution !

Error of 2.6×10^{-10} not uncontroversial; may be up to two times larger ?!



Conclusions and Perspectives

And (!) must further reduce experimental error !

→ See Lee Robert's and Tsutomu Mibe's talks !