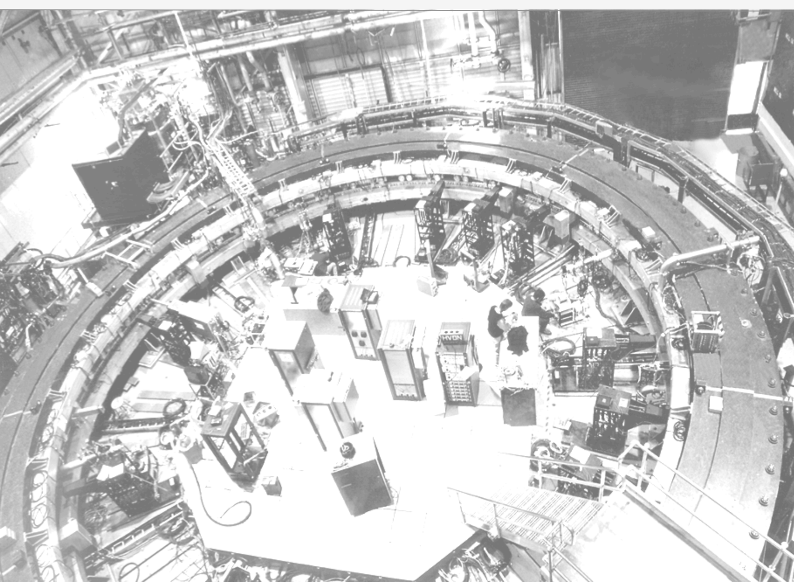
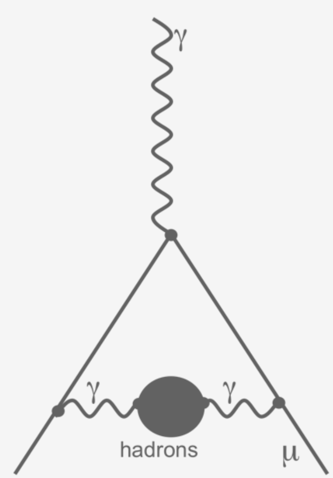
The Muon $g-2$ and its Hadronic Contribution

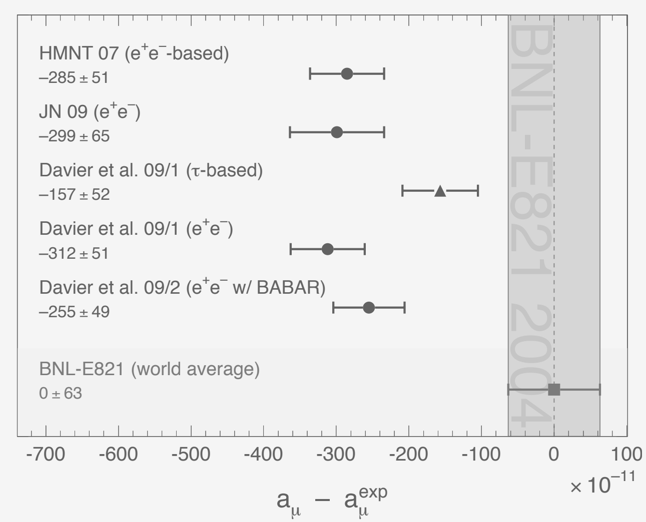
[… and also that of $\alpha_{QED}(M_Z)$]

Andreas Hoecker (CERN)

Tau Workshop, Manchester, UK, Sep 13 – 17, 2010



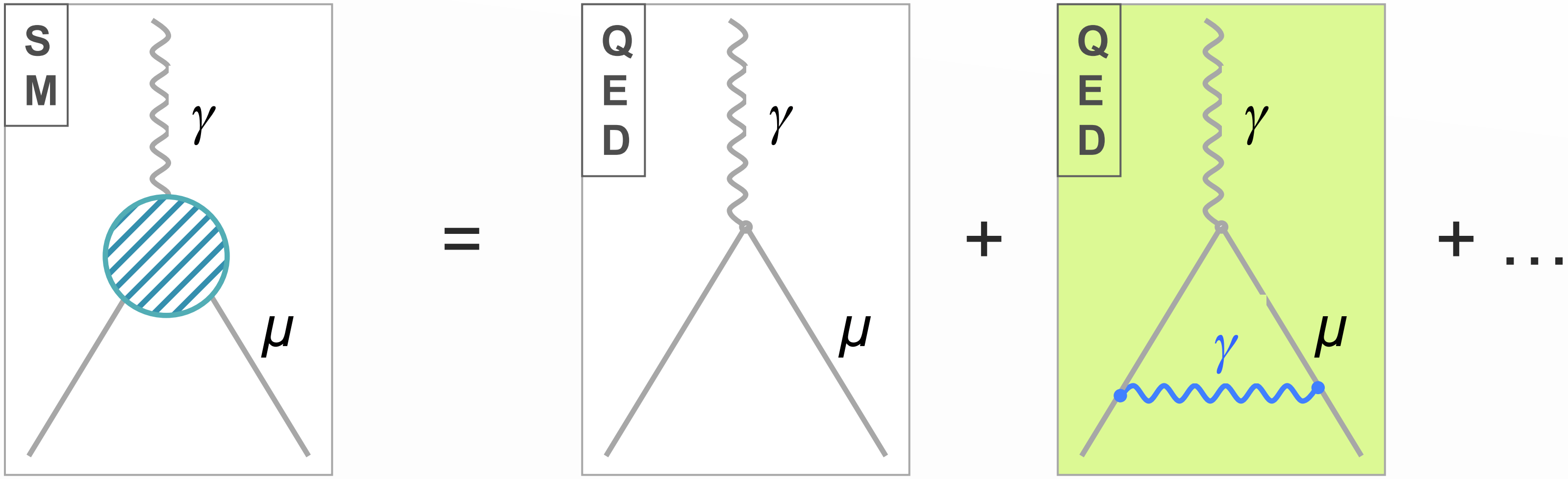




Brief Introduction to Muon $g - 2$

The Anomalous Magnetic Moment

Dirac's gyromagnetic factor g = 2 is modified by virtual gauge boson and fermion exchanges



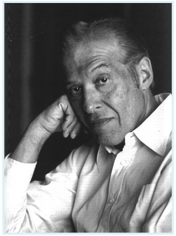
$(g\text{–}2)_e \neq 0$ (full Standard Model)

$(g\text{–}2)_e = 0$ (Dirac)

coupling to virtual fields:
$(g\text{–}2)_e \neq 0$ (1st order QED)

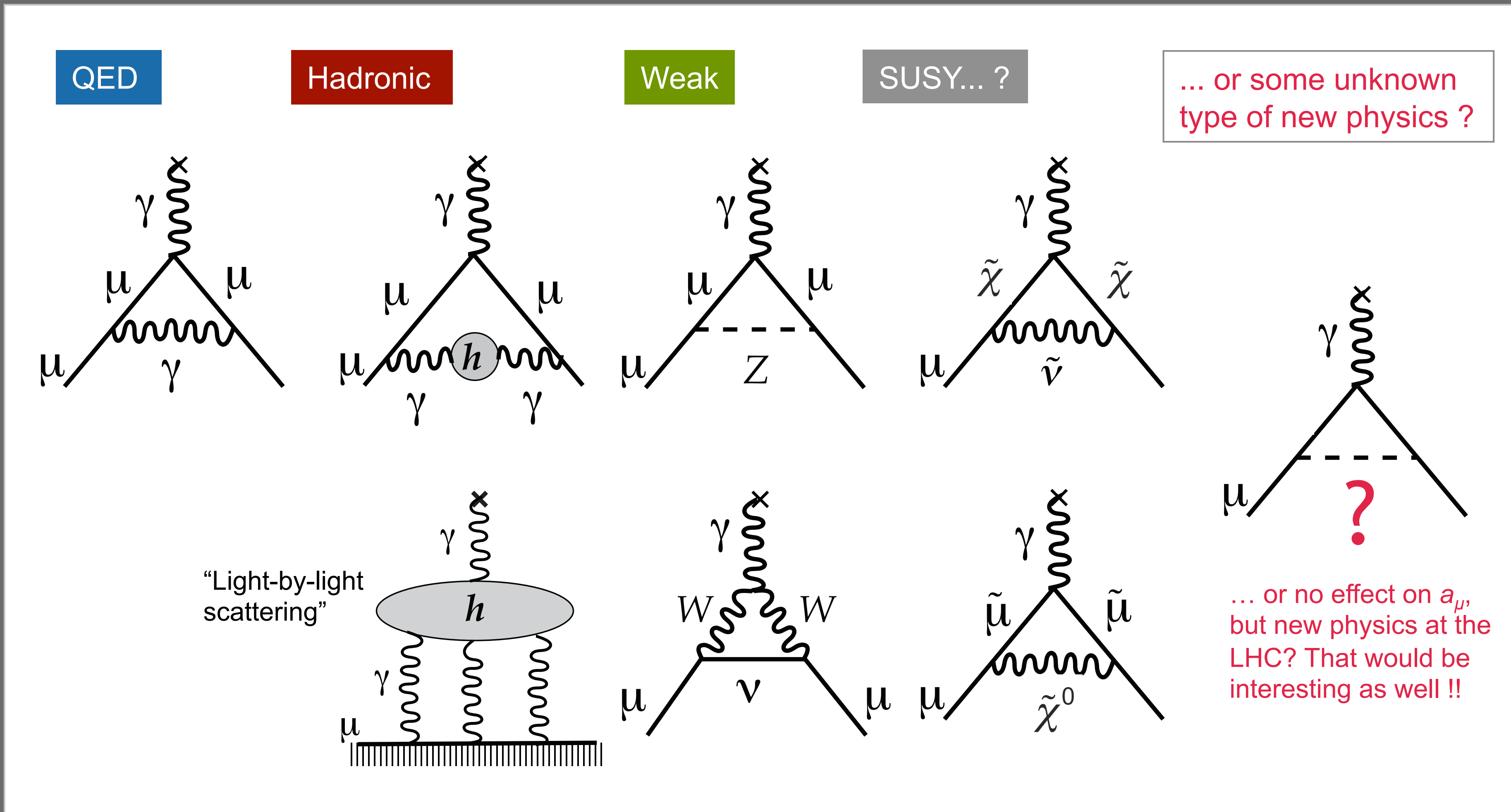
"Anomalous" magnetic moment:

$$a_\ell \equiv \frac{g_\ell - 2}{2} = \frac{\alpha}{2\pi} + \ldots = 0.001161\ldots$$



Julius Schwinger
1-loop calculation: 1948

Contributing diagrams:



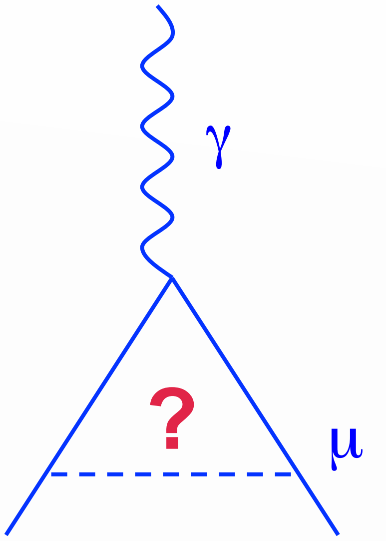
Quest for New Physics

The experimental precision for a_μ will be worse than for a_e, so why do it ?

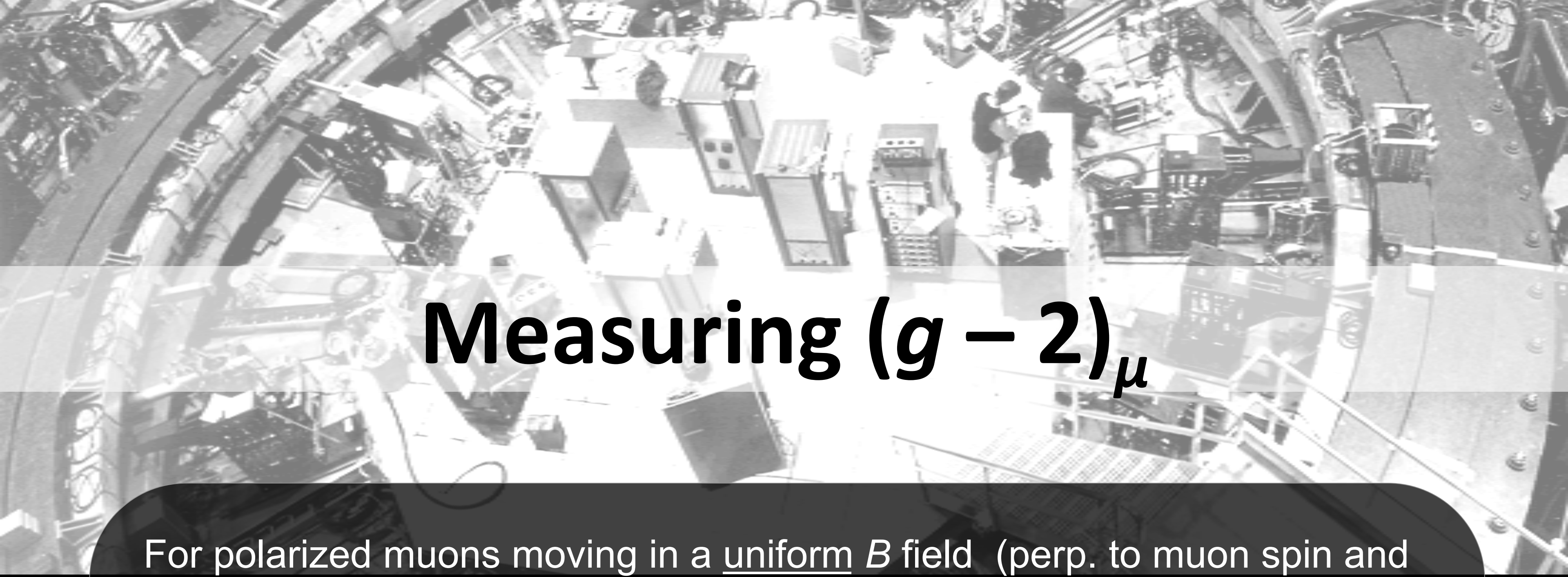
- In lowest order, where mass effects appear, contributions from heavy virtual particles scale as $m^2_{e/\mu}$:

$$a_\ell^{\mathrm{NP}}(\Lambda_{\mathrm{NP}}) \propto \mathcal{O}\left(\frac{m_\ell^2}{\Lambda_{\mathrm{NP}}^2}\right) \quad \Rightarrow \quad \frac{a_\mu^{\mathrm{NP}}}{a_e^{\mathrm{NP}}} \propto \mathcal{O}\left(\frac{m_\mu^2}{m_e^2}\right) \approx 43{,}000$$

- Loose about a factor of 800 in experimental precision



→ a_μ should be roughly 50 times more sensitive to NP than a_e !



Measuring $(g-2)_\mu$

For polarized muons moving in a <u>uniform</u> *B* field (perp. to muon spin and orbit plane), and vertically focused in *E* quadrupole field, the observed difference between spin precession frequency and cyclotron frequency is:

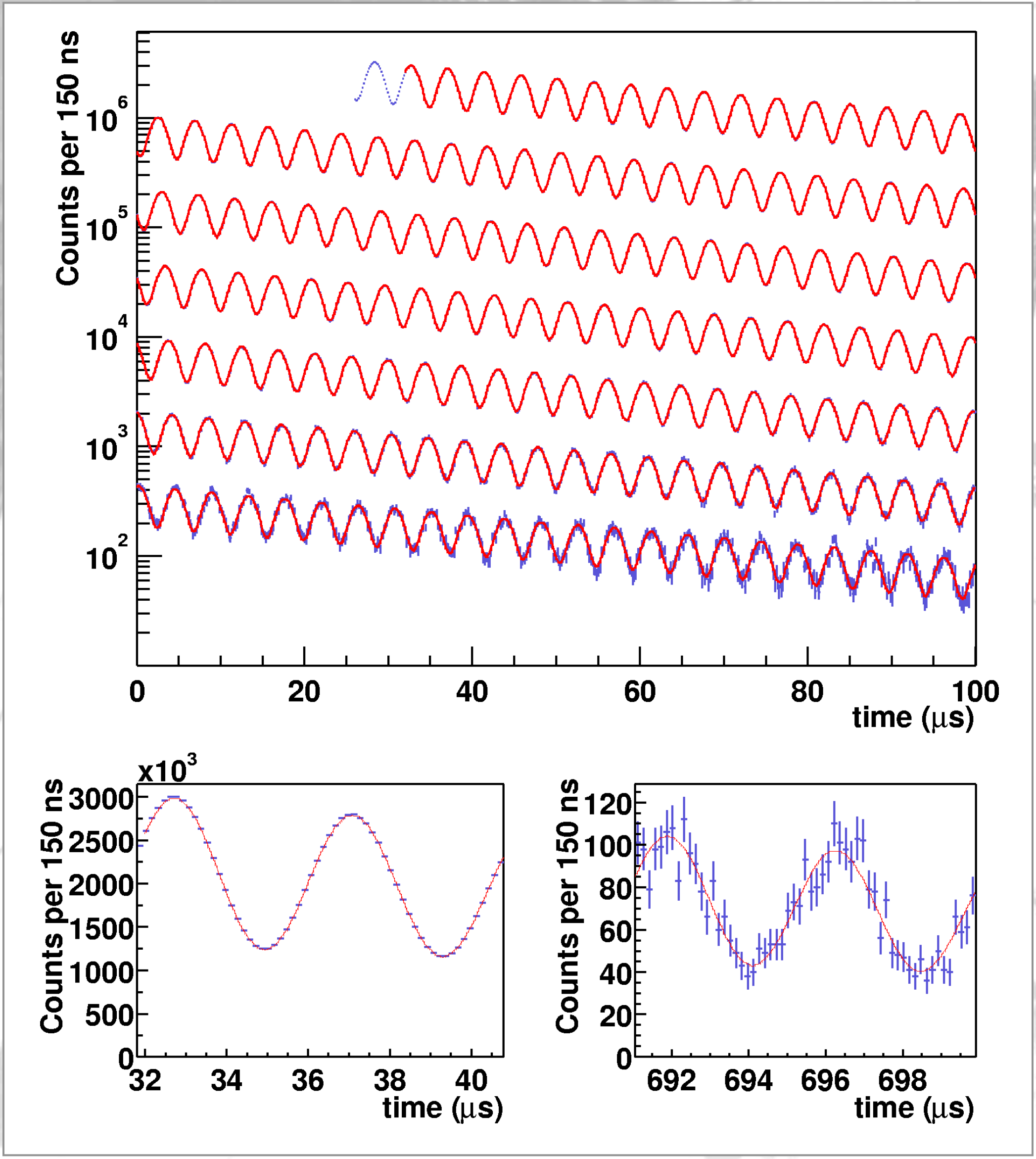
$$\vec{\omega}_a = \frac{e}{mc}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\vec{\beta}\times\vec{E}\right]$$

assuming,
EDM = 0 !

The *E* dependence is eliminated at “magic γ”: $\gamma = 29.3 \rightarrow p_\mu = 3.09$ GeV/c

The experiment measures directly $(g-2)/2$!

The BNL $(g-2)_\mu$ Measurement



Observed positron rate in successive 100μs periods

Difference between spin precession and cyclotron frequency:

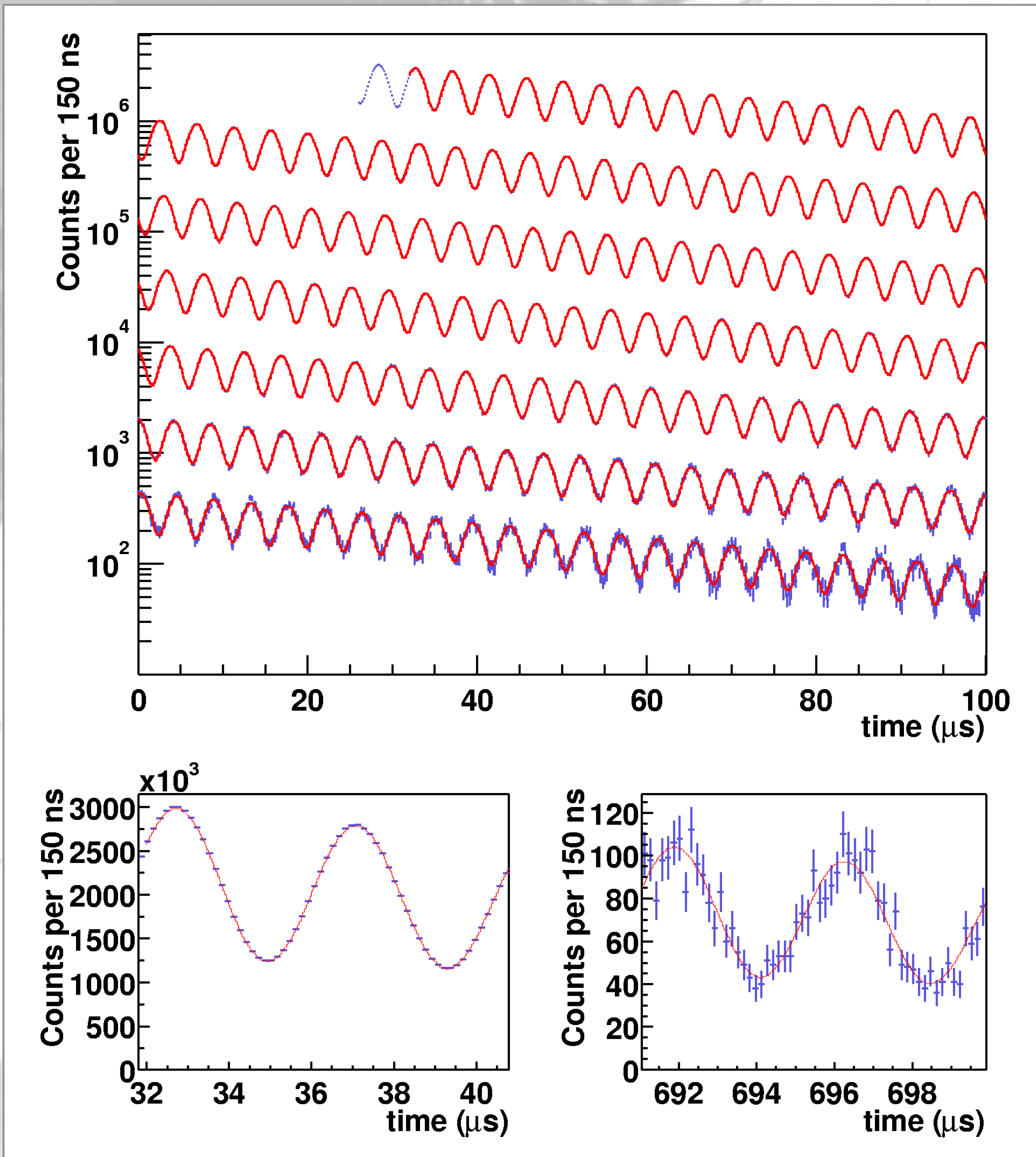
$$\vec{\omega}_a = \frac{e}{m_\mu c} a_\mu \vec{B}$$

obtained from fit to:

$$N(t) = N_0 e^{-t/\gamma\tau}\left[1 + A\sin(\omega_a t + \phi)\right]$$

plot taken from:
E821 (*g* –2), hep-ex/0202024

The BNL $(g-2)_\mu$ Measurement



Observed positron rate in successive 100μs periods

These quantities are measured independently *and* blind
→ **doubly blind analysis!**

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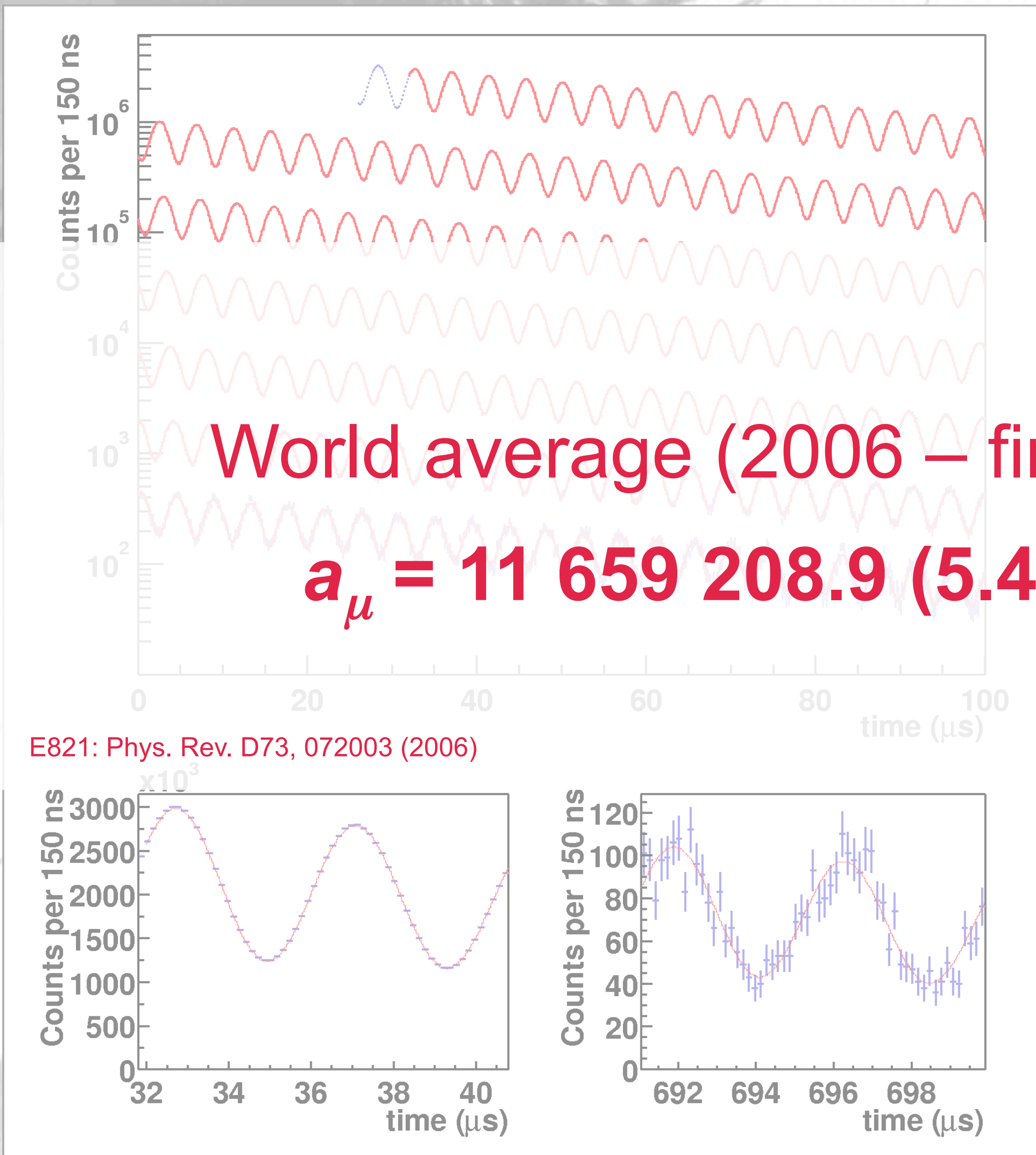


E821: Phys. Rev. D73, 072003 (2006)

Observed positron rate in successive 100μs periods

These quantities are measured independently *and* blind

Doubly blind analysis!

World average (2006 – final E821 report):

$$\boldsymbol{a_\mu = 11\,659\,208.9\,(5.4)\,(3.3) \times 10^{-10}}$$

Difference between spin precession and cyclotron frequency:

$$\omega_a = \frac{e}{m_\mu c} a_\mu \vec{B}$$

obtained from fit to:

$$N(t) = N_0 e^{-t/\gamma\tau}\left[1 + A\sin(\omega_a t + \phi)\right]$$

plot taken from:
E821 (*g* –2), hep-ex/0202024

Confronting Experiment with Theory

The Standard Model prediction of a_μ is decomposed in its main contributions:

$$a_\mu^{SM} \equiv \left(\frac{g-2}{2}\right)_\mu = a_\mu^{QED} + a_\mu^{had} + a_\mu^{weak}$$

of which the hadronic contribution has the largest uncertainty!

The Muon $g - 2$ in the Standard Model

Using α from latest α_e
[Gabrielse et al. PRL 97, 030802, 2006]

QED contribution

$$a_\mu^{QED} \approx 11{,}658{,}471.809(0.015) \times 10^{-10}$$

Computed up to 4th order
(5th order estimated)

$$= \left(11{,}614{,}097.3 + \underbrace{41{,}321.8 + 3{,}014.2}_{} + 38.1 + 0.4\right) \times 10^{-10}$$

1st order known since 1948
[J. Schwinger, PR73(48)416]

Up to 3rd order
known analytically

4th order known numerically
[T. Kinoshita et al, 1980's]

5th order estimated recently, T. Kinoshita & M. Nio, PRD 73, 053007, 2006

The Muon $g-2$ in the Standard Model

Electroweak contribution

Computed up to 2nd order

a_μ^{weak} suppressed by $\dfrac{\alpha}{\pi}\dfrac{m_\mu^2}{m_W^2} \sim 10^{-9}$ (!)

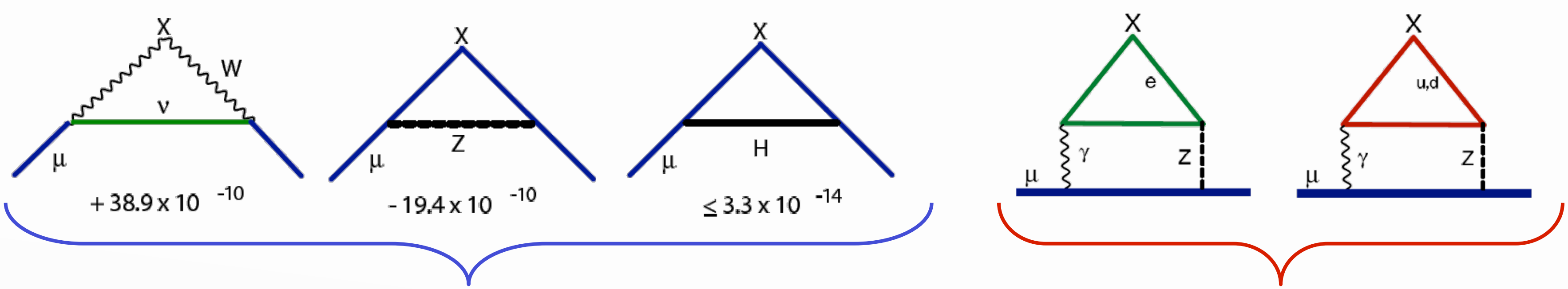
$$\underbrace{a_\mu^{\text{weak}}}_{\text{1-loop}} = \frac{G_\mu m_\mu^2}{8\sqrt{2}\pi^2}\left(\frac{5}{3}+\frac{1}{3}\left(1-4\sin^2\theta_W\right)+\mathcal{O}\left(\frac{m_\mu^2}{m_W^2}\right)+\mathcal{O}\left(\frac{m_\mu^2}{m_H^2}\right)\right) = +19.5\times10^{-10}$$

Czarnecki *et al.*, PRD 52, 2619 (1995); PRL 76, 3267 (1996)

2nd order contribution surprisingly large: $\underbrace{a_\mu^{\text{weak}}}_{\text{2-loop}} = -4.1(0.2)\times10^{-10}$

(due to large logs: $\ln[m_Z/m_\mu]$)

Note that between a_μ and a_e, the same sensitivity factor as for "new physics" applies here



1-loop [computed in 1972]

2-loop

Hadronic Contribution

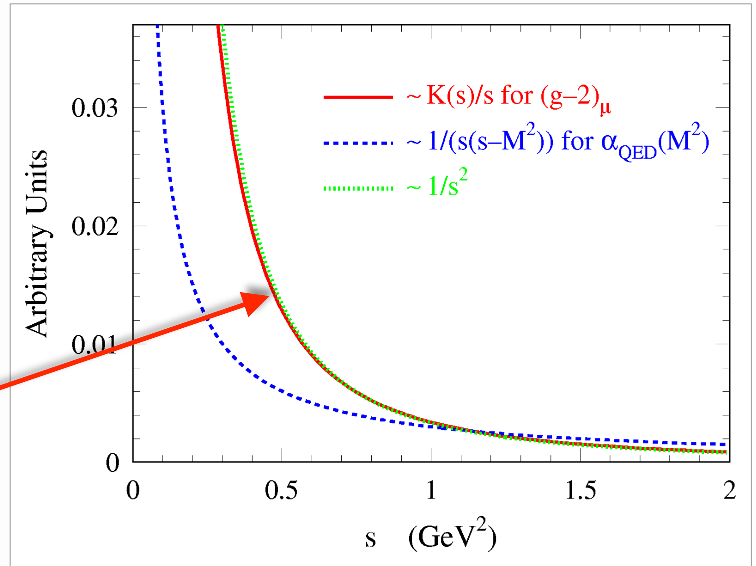
The Muon *g* – 2 in the Standard Model

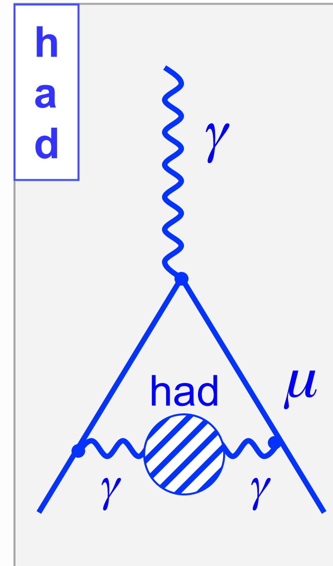
Hadronic contribution provides the by far largest uncertainty to a_μ

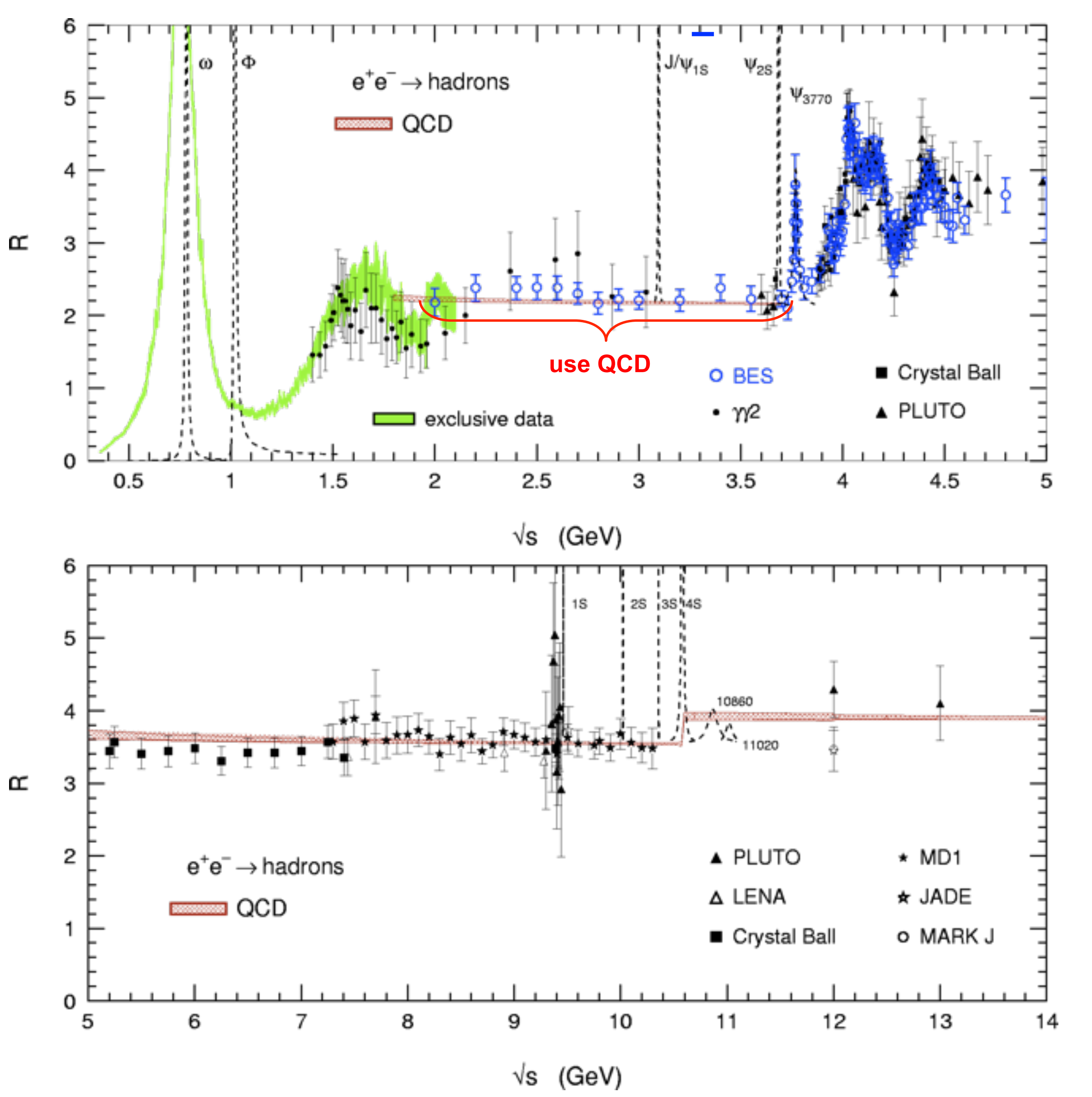
- Cannot be computed from first principles (quark loops) due to low-energy hadronic effects
- Fortunately, one can use analyticity and unitarity to obtain real part of photon polarisation function from dispersion relation over total hadronic cross section data (or theory)

$$12\pi\,\mathrm{Im}\Pi_\gamma(s) = \frac{\sigma^{(0)}[e^+e^- \to \text{hadrons}]}{\sigma^{(0)}[e^+e^- \to \mu^+\mu^-]} \equiv R(s)$$

$$a_\mu^{\text{had,LO}} = \frac{\alpha^2}{3\pi^2}\int_{m_{\pi^0}^2}^{\infty} ds\, \frac{K(s)}{s} R(s)$$







$$a_\mu^{\mathrm{had,LO}} = \frac{\alpha^2}{3\pi^2} \int_{4m_\pi^2}^{\infty} ds \frac{K(s)}{s} R(s)$$

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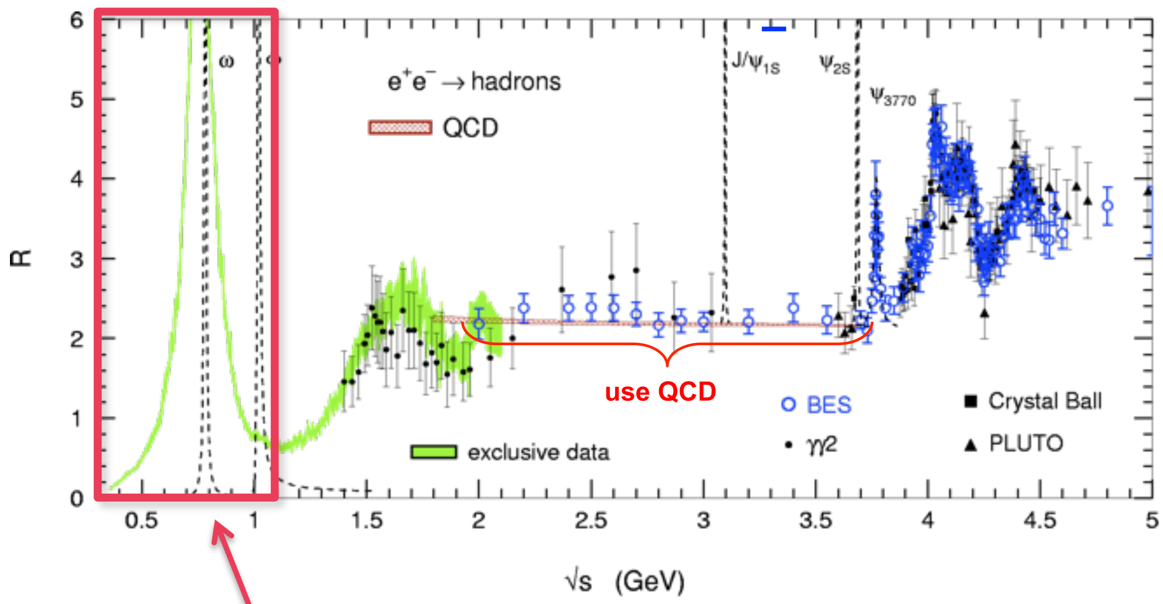
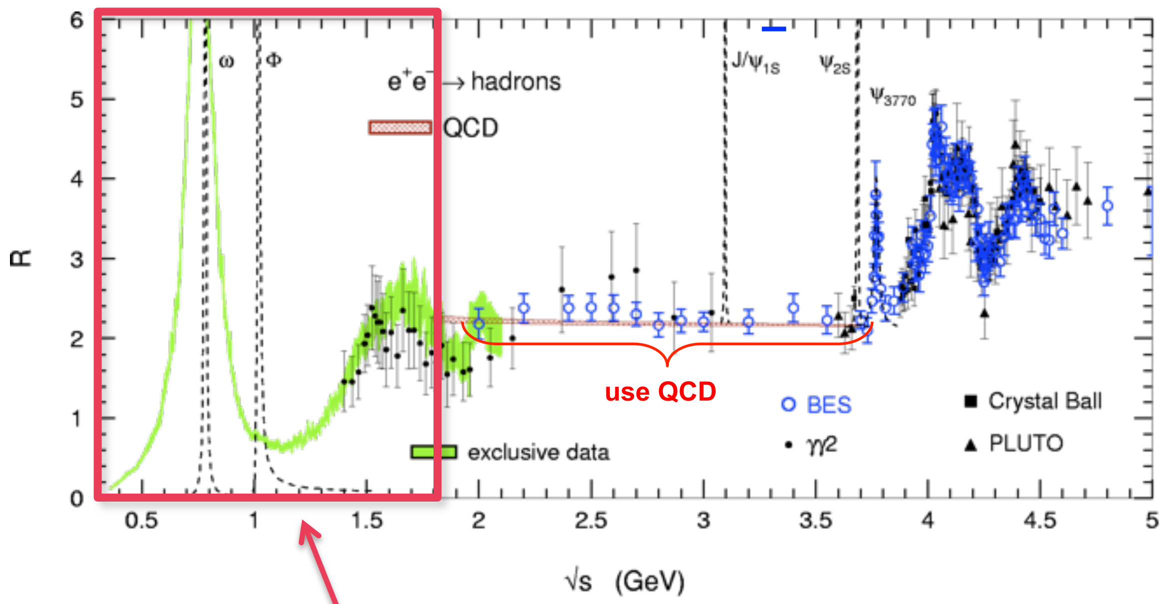
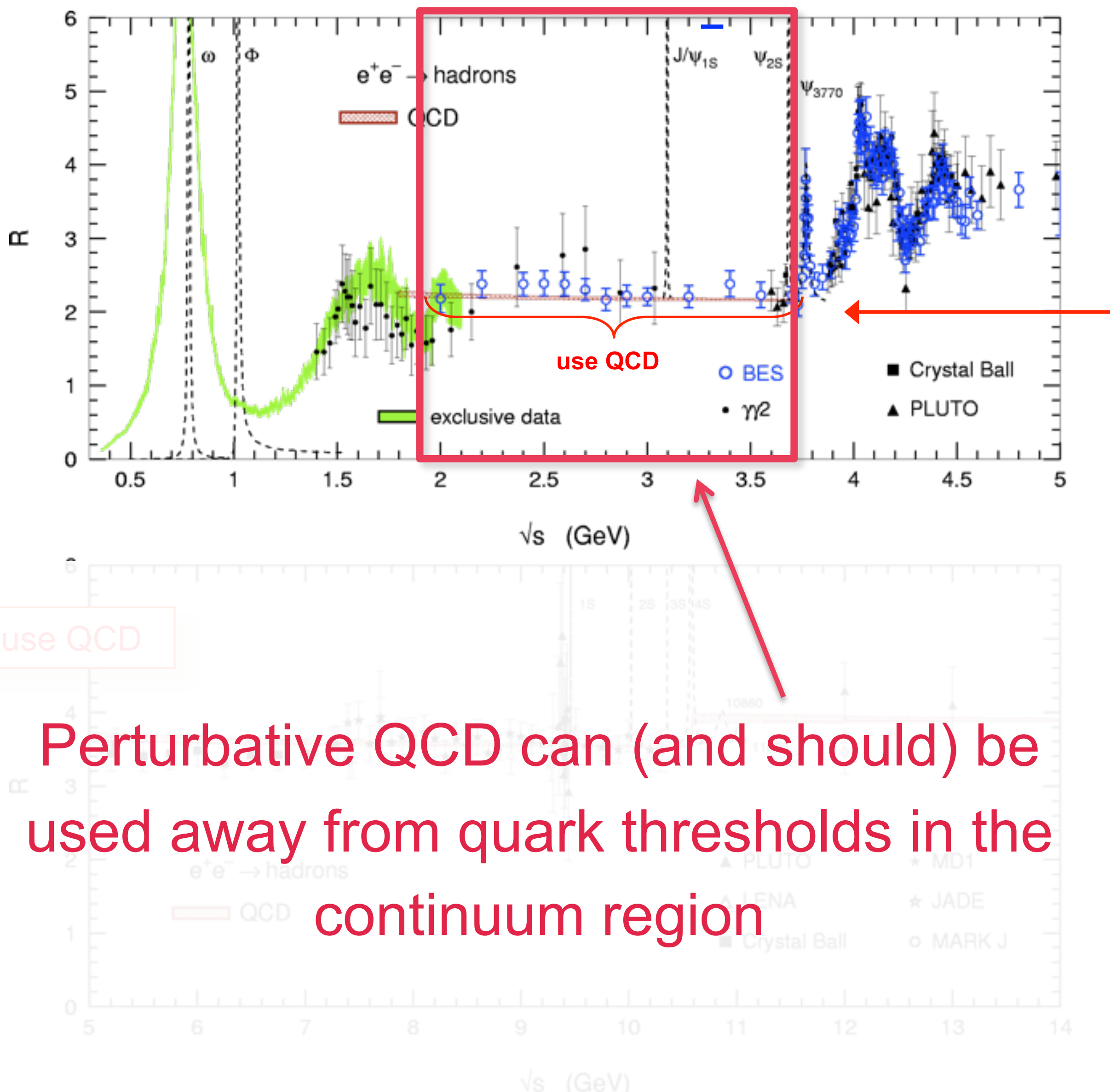


Due to the strongly decaying integration kernel, 73% of dispersion integral stems from $\pi^+\pi^-$ channel, which must be obtained from experiment

$$a_\mu^{\mathrm{had,LO}} = \frac{\alpha^2}{3\pi^2}\int_{4m_\pi^2}^{\infty} ds \frac{K(s)}{s} R(s)$$



At low energy, the inclusive hadronic cross section is obtained by summing up to 26 exclusively measured final states, and by estimating unmeasured modes using isospin symmetry

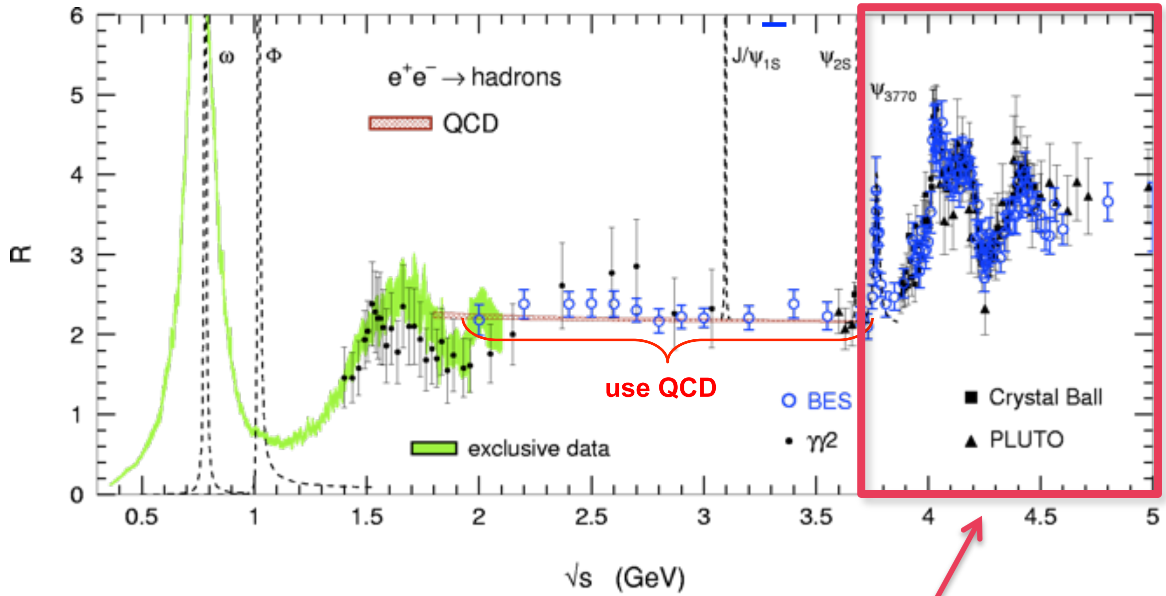


$$a_\mu^{\mathrm{had,LO}} = \frac{\alpha^2}{3\pi^2} \int_{4m_\pi^2}^{\infty} ds \frac{K(s)}{s} R(s)$$

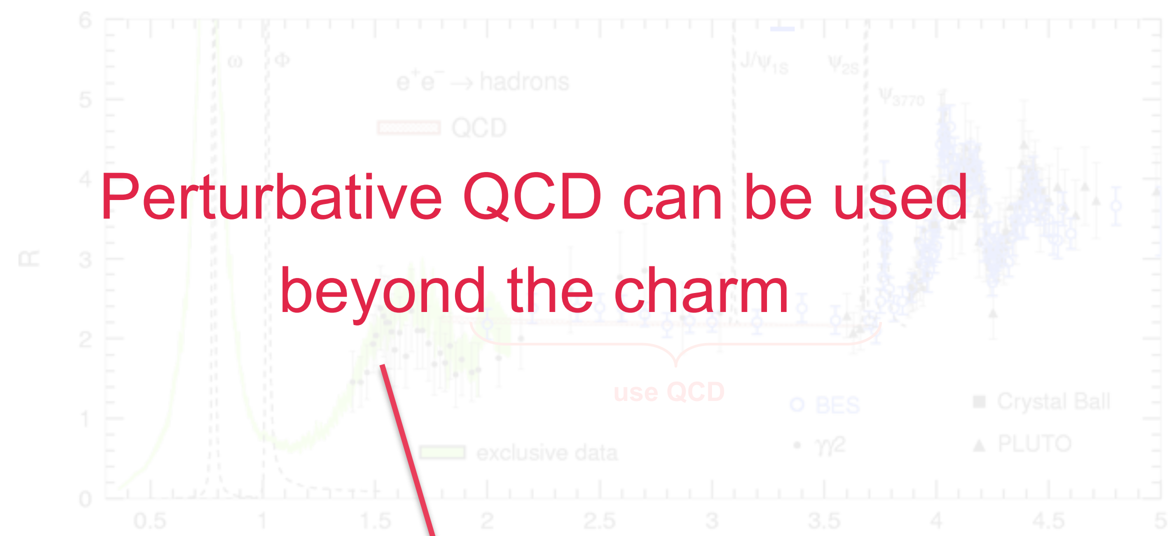
Agreement between Data (BES) and pQCD (within *correlated* systematic errors)

Perturbative QCD can (and should) be used away from quark thresholds in the continuum region

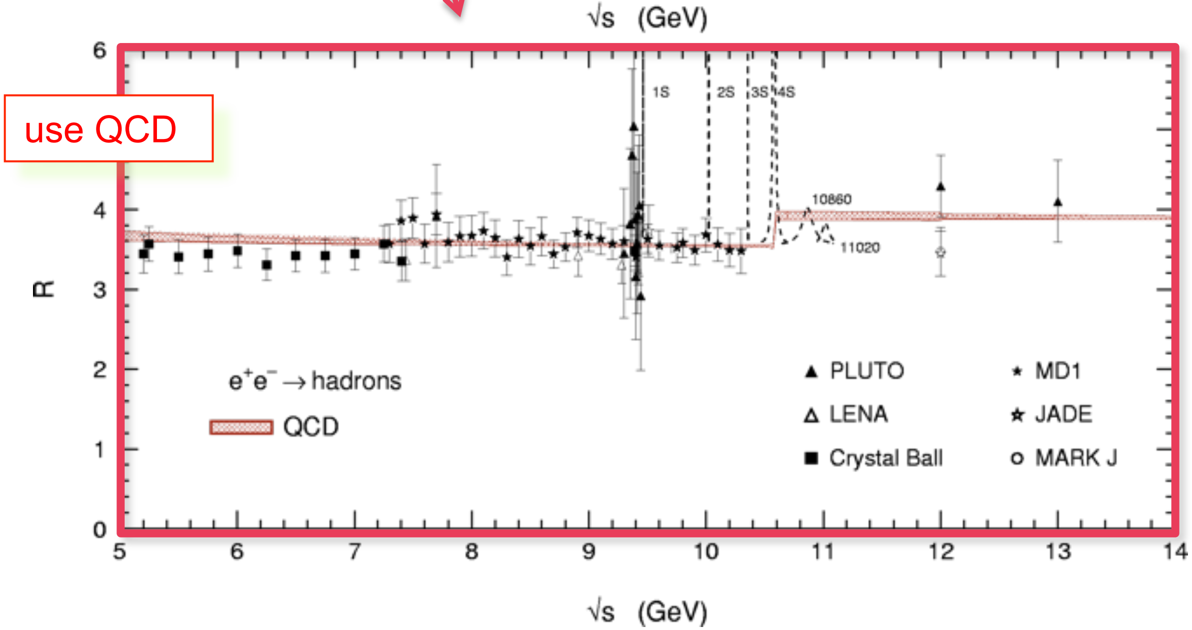
$$a_\mu^{\text{had,LO}} = \frac{\alpha^2}{3\pi^2} \int_{4m_\pi^2}^{\infty} ds \frac{K(s)}{s} R(s)$$



Experimental data must be used in the charm anti-charm resonance region



Perturbative QCD can be used beyond the charm



$$a_\mu^{\text{had,LO}} = \frac{\alpha^2}{3\pi^2} \int_{4m_\pi^2}^{\infty} ds \frac{K(s)}{s} R(s)$$

Hadronic Contribution to Muon $g - 2$

[units in10^{-10}]

$$a_\mu^{\text{SM}} \equiv \left(\frac{g-2}{2}\right)_\mu = a_\mu^{\text{QED}} + a_\mu^{\text{had,LO}} + a_\mu^{\text{had,NLO}} + a_\mu^{\text{weak}}$$

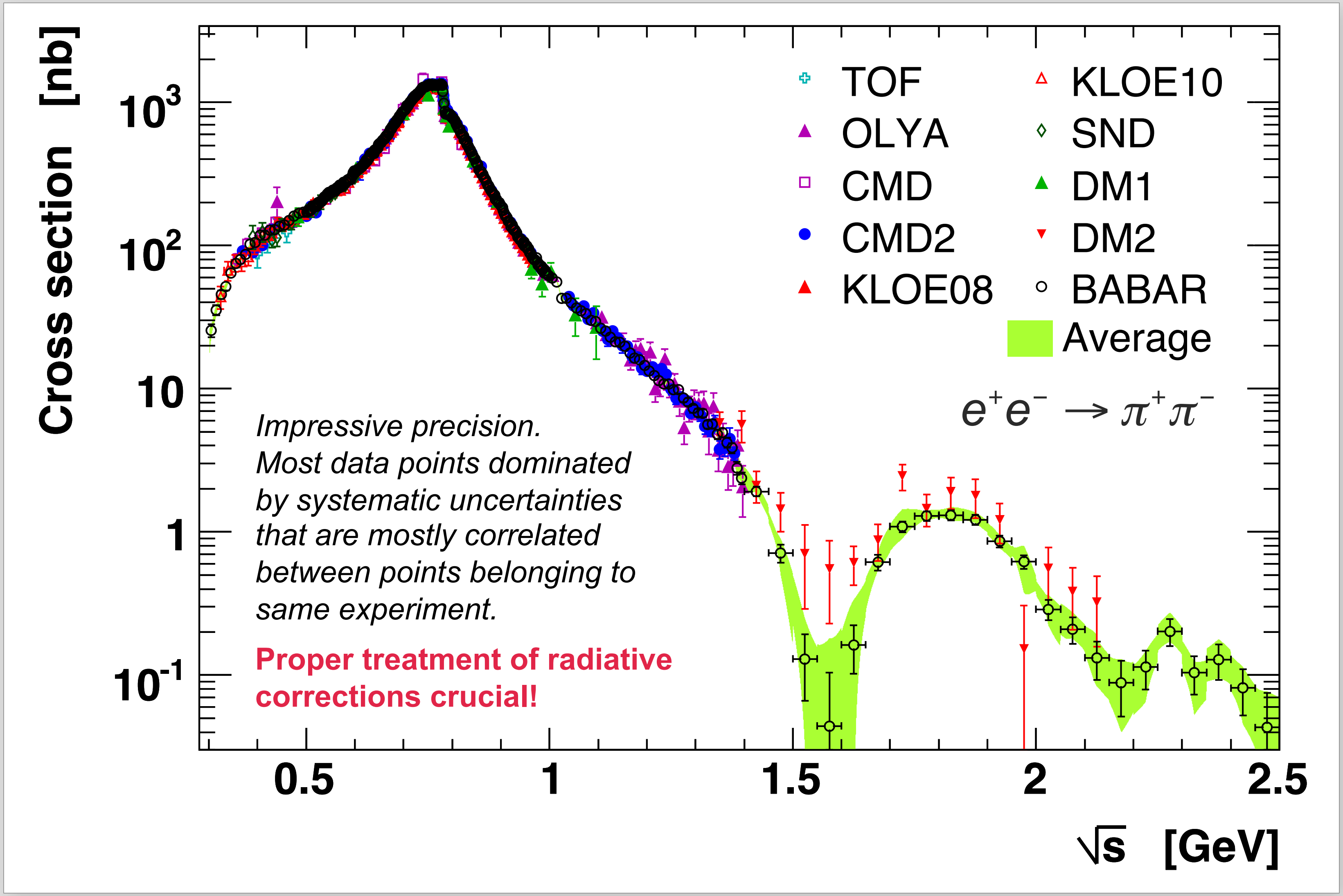
$\sigma^{\text{Exp}} = 6.3$ | $\sigma^{\text{SM}}_{\text{QED}} \simeq 0.02$ | $\sigma^{\text{SM}}_{\text{had,LO}} \approx 4$ | $\sigma^{\text{SM}}_{\text{had,NLO}} \simeq \sigma^{SM}_{\text{had,LBLS}} \approx 3$ | $\sigma^{\text{SM}}_{\text{weak}} \approx 0.2$

→ SM error on a_μ dominated by hadronic part, ie, by **experimental data** !

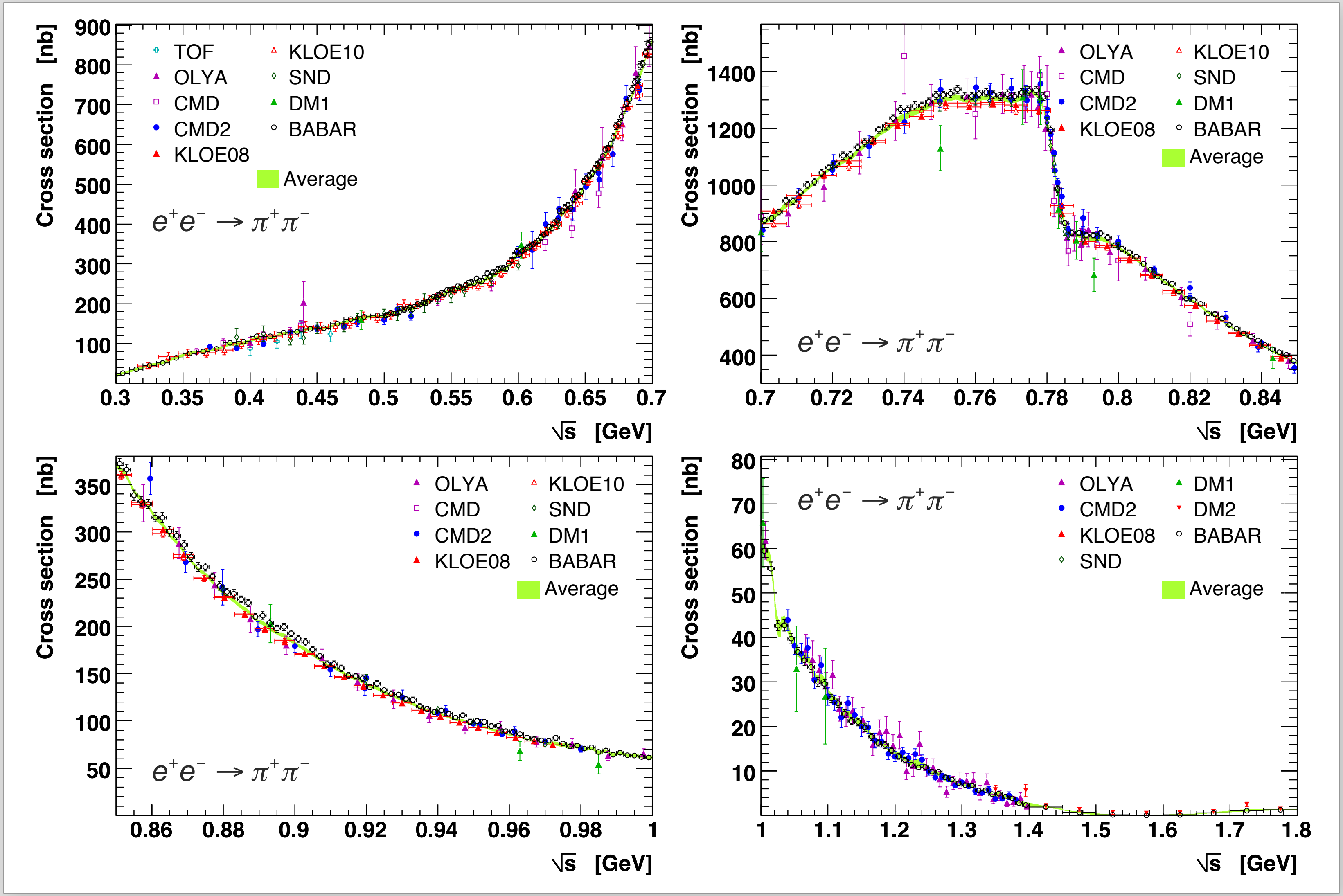
Huge 20-years effort by many experimentalists and phenomenologists to reduce error on lowest-order hadronic part:

- Improved e^+e^- cross section data from Novisibirsk (Russia)
- More use of perturbative QCD
- Technique of "radiative return" allows to use cross section data from Φ and *B* factories
- Isospin symmetry allows us to also use τ hadronic spectral functions

$e^+e^- \to \pi^+\pi^-$ Cross Section



$e^+e^- \to 4\pi$ Cross Sections



Adding all (28) Contributions Together

Hadronic LO contribution: $a_\mu^{\text{had,LO}}[e^+e^-] = (695.5 \pm 4.0_{\text{exp}} \pm 0.7_{\text{QCD}}) \times 10^{-10}$

Davier et *al.* arXiv:0908.4300

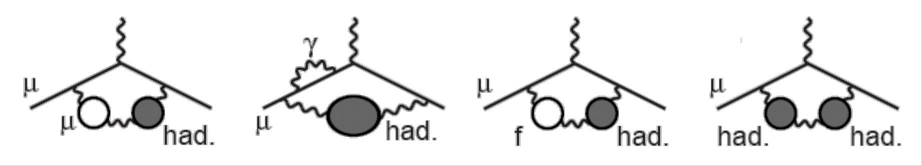
Hadronic NLO contributions:

Vacuum polarization (1-loop) + additional photon or VP insertion

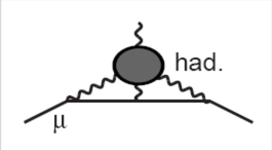
- Computed akin to LO part via dispersion integral with modified kernel function

$$a_\mu^{\text{had,NLO}} = -9.8(0.1) \times 10^{-10}$$

HLMNT 2010 (and others)



Light-by-light scattering



- Dispersion relation approach not possible (4-point function)
- No first-principle calculation yet (*e.g.*, on the lattice)
- Model calculations using short dist. quark loops, π^0, $\eta^{(\prime)}$, … pole insertions and $\pi^\pm$ loops in the large-N_C limit

$$a_\mu^{\text{had,LBL}} = +10.5(2.6) \times 10^{-10}$$

Prades-deRafael-Vainshtein (and others)

Pre-Tau-2010 Results for Muon $g-2$

$$a_\mu^{SM}[e^+e^-] = (11\,659\,183.4 \pm 4.1_{\text{had,LO}} \pm 2.6_{\text{NLO}} \pm 0.2_{\text{QED+weak}}) \times 10^{-10}$$

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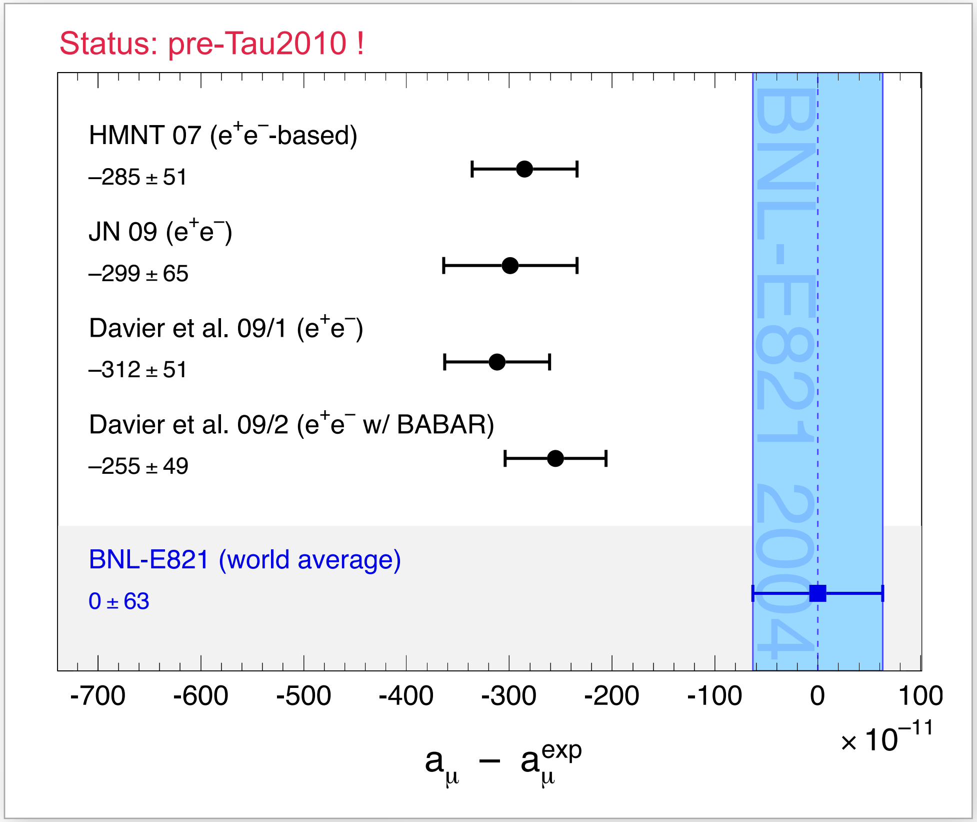


BNL E821 (2004):
$a_\mu^{\text{exp}} = (11\,659\,208.9 \pm 6.3)\ 10^{-10}$

Observed Difference with Experiment:

$$\boldsymbol{a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (25.5 \pm 8.0)} \times 10^{-10}$$

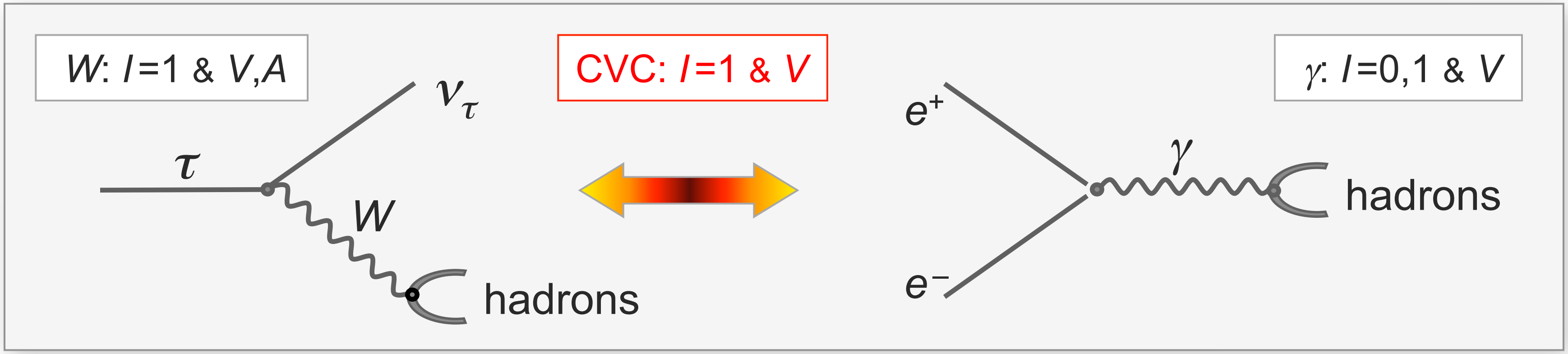
➡ 3.2 ”standard deviations“

Amount of discrepancy in ballpark of SUSY with mass scale of several 100 GeV !

$$\Delta a_\mu^{\text{SUSY}} \approx 13 \cdot 10^{-10}\ \text{sgn}(\mu) \left(\frac{100\ \text{GeV}}{m_{\text{SUSY}}}\right)^2 \tan\beta$$

Tau Hadronic Spectral Functions

Can exploit precise Tau data to increase precision on a_μ



Hadronic physics factorizes in Spectral Functions :

Isospin symmetry connects I =1 e^+e^- cross section (neutral) to τ vector spectral functions (charged):

$$\sigma^{(I=1)}\left[e^+e^- \to \pi^+\pi^-\right] = \frac{4\pi\alpha^2}{s}\upsilon\left[\tau^- \to \pi^-\pi^0\nu_\tau\right]$$

$$\upsilon\left[\tau^- \to \pi^-\pi^0\nu_\tau\right] \propto \frac{\mathrm{BR}\left[\tau^- \to \pi^-\pi^0\nu_\tau\right]}{\mathrm{BR}\left[\tau^- \to e^-\bar{\nu}_e\nu_\tau\right]} \, \frac{1}{N_{\pi\pi^0}}\frac{dN_{\pi\pi^0}}{ds} \, \frac{m_\tau^2}{\left(1-s/m_\tau^2\right)^2\left(1+s/m_\tau^2\right)} \, \frac{R_{\mathrm{IB}}(s)}{S_{\mathrm{EW}}}$$

Branching fractions — Mass spectrum — Kinematic factor (PS) — Isospin correction

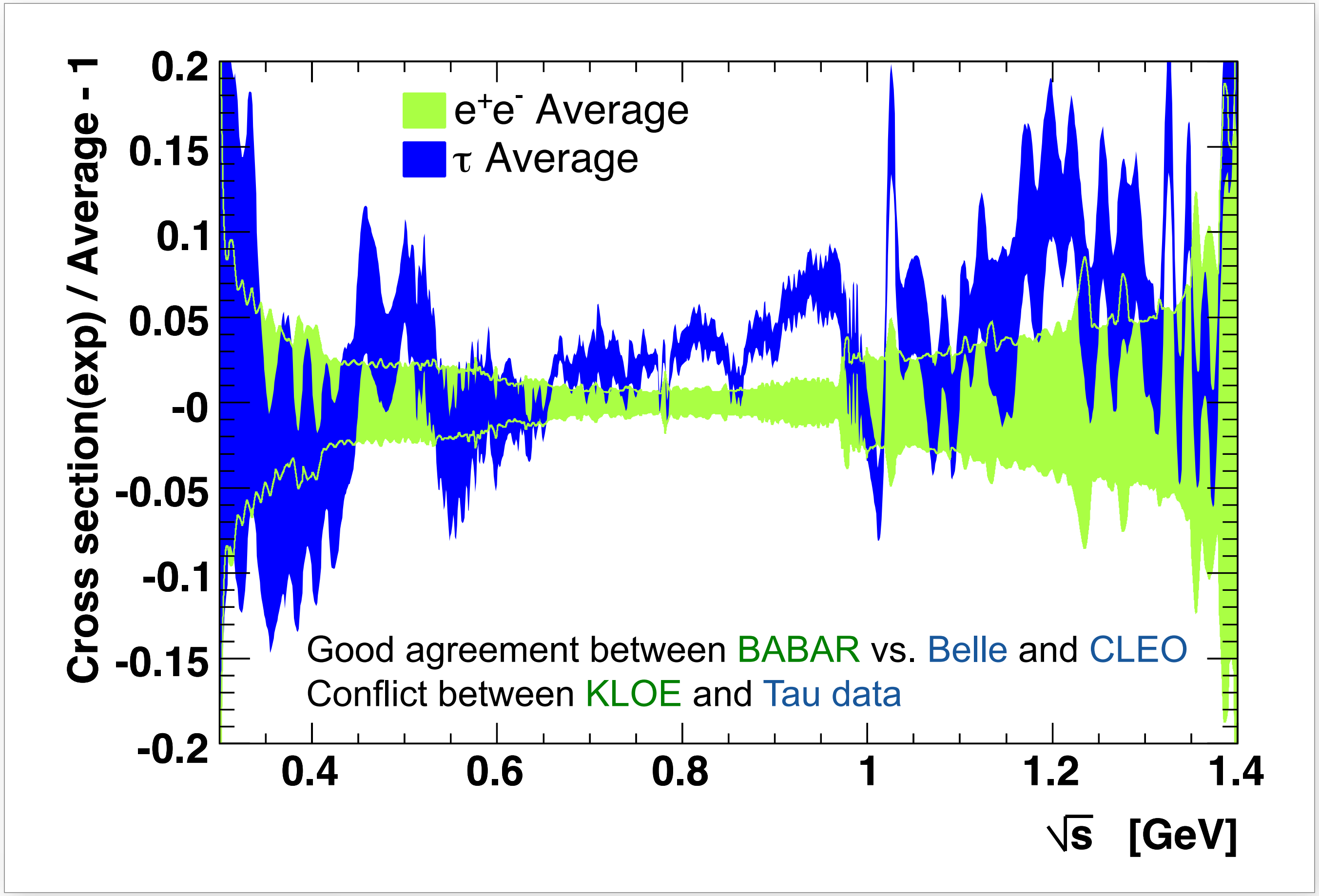
Can exploit precise Tau data to increase precision on a_μ

- In practice, used for 2π and 4π channels with isospin rotation
- Tau spectral functions measured by ALEPH, Belle, CLEO, OPAL
- Excellent precision of tau data. Branching ratio (ie, spectral function normalisation) for $\tau \to \pi\pi^0\nu$ known to 0.4%.
- Invariant mass spectrum requires unfolding using detector simulation, which is however under good control
- Main experimental challenge: abundance and shape modeling of feed-through from other tau final states
- Main theoretical challenge: **isospin breaking**
  Radiative corrections, charged vs. neutral mass splitting and electromagnetic decays: (–3.2 ± 0.4)% correction to $a_\mu^{\rm had}$

$\tau \rightarrow \pi\pi^0\nu$ Spectral Functions

Comparing tau to e^+e^- data:

DHMZ, Tau 2010



→ see talk by B. Malaescu for more detailed comparisons between all experiments

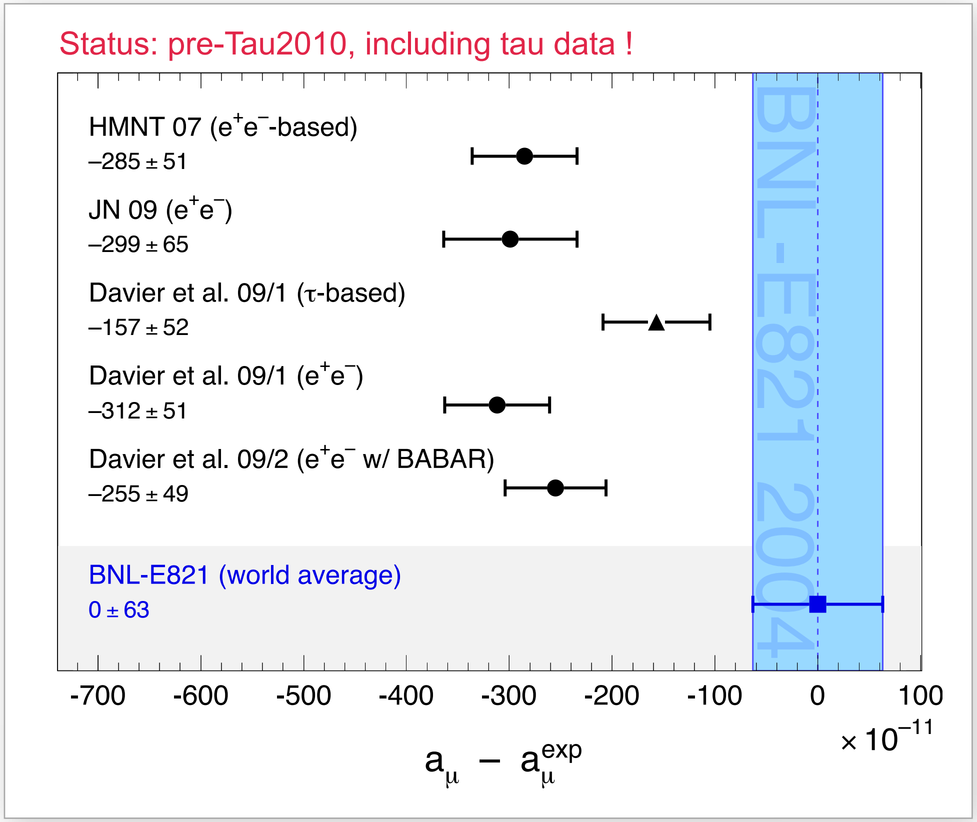
Pre-Tau-2010 Results for Muon $g-2$

$$a_\mu^{SM}[e^+e^-] = (11\,659\,183.4 \pm 4.1_{\text{had,LO}} \pm 2.6_{\text{NLO}} \pm 0.2_{\text{QED+weak}}) \times 10^{-10}$$

$$a_\mu^{SM}[\tau\text{-based}] = (11\,659\,193.2 \pm 4.0_{\text{had,LO}} \pm 2.1_{\text{IB}} \pm 2.6_{\text{NLO}} \pm 0.2_{\text{QED+weak}}) \times 10^{-10}$$

Davier *et al.* arXiv: 0908.4300

Davier *et al.* arXiv: 0906.5443



Observed Difference with Experiment:

$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (15.7 \pm 8.1) \times 10^{-10}$$

➡ 1.9 "standard deviations" for τ data

Discrepancy between e^+e^- and tau data significantly reduced with new data.

In particular BABAR and Belle show excellent agreement !

Pre-Tau-2010 Results for Muon $g-2$

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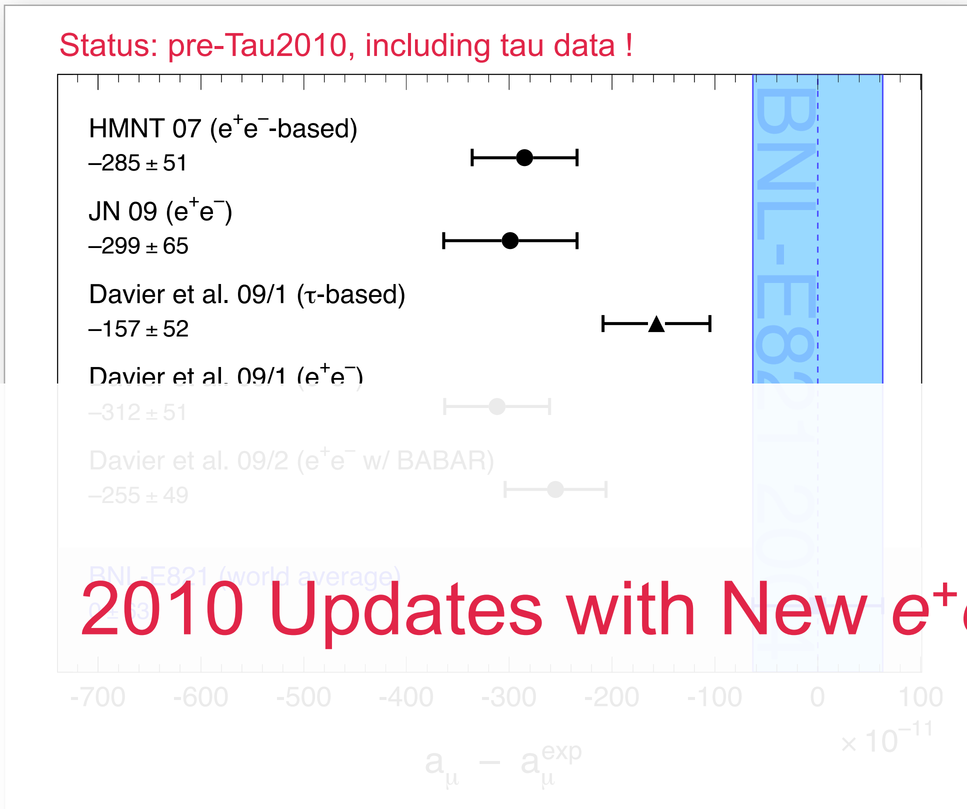
Davier et *al.* arXiv: 0906.5443



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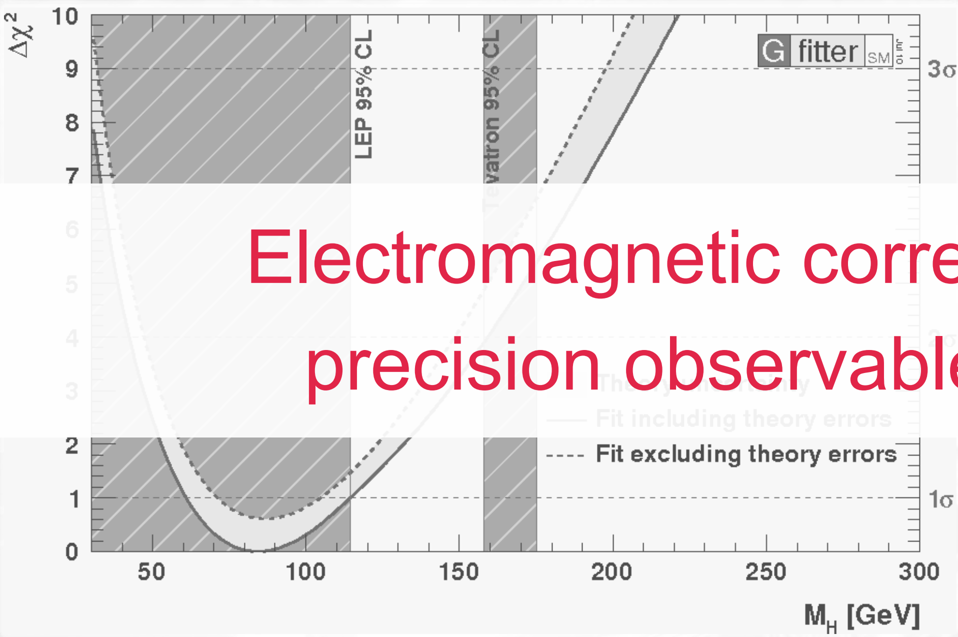
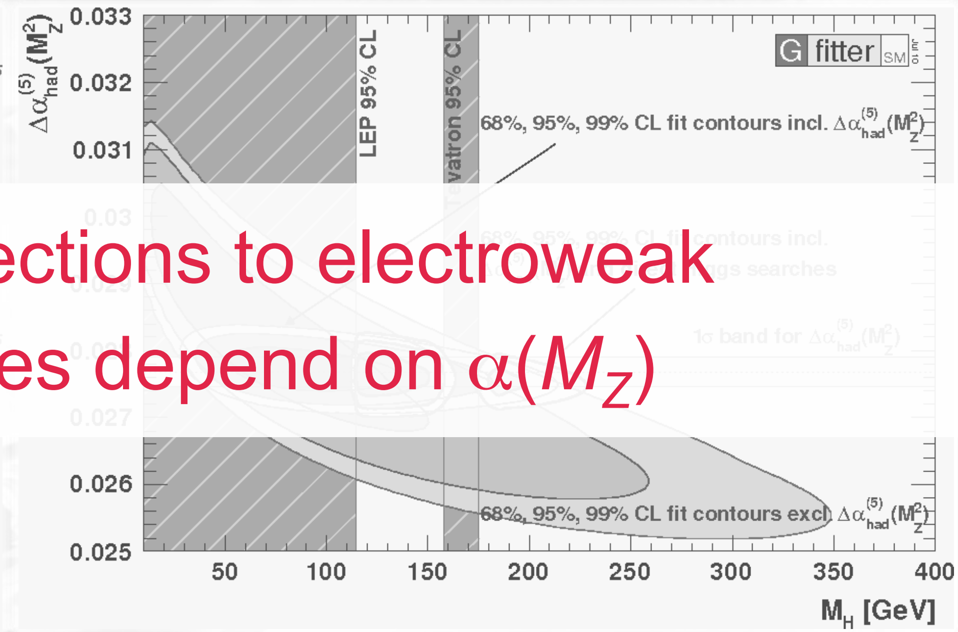
2010 Updates with New e^+e^- Data Later Today !

There is also the Running of α_{QED} !





Electromagnetic corrections to electroweak precision observables depend on $\alpha(M_Z)$

The Running α_{QED} at M_Z

Same principle as for *g* – 2: energy-dependent vacuum polarisation effects screen the bare electromagnetic coupling. Leptonic contributions computed via QED, hadronic contributions obtained from dispersion relation:

$$\alpha(s) = \frac{\alpha(0)}{1-\Delta\alpha(s)} \quad \text{with:} \quad \Delta\alpha(s) = \Delta\alpha_{\text{lep}}(s) + \Delta\alpha_{\text{had}}(s) = -4\pi\alpha\,\mathrm{Re}\left[\prod_\gamma(s) - \prod_\gamma(0)\right]$$

$$\Delta\alpha_{\text{lep}}^{\text{3-loop}}(M_Z^2) = 0.031497686$$

Steinhauser, hep-ph/9803313 (1998)

$$\Delta\alpha_{\text{had}}(M_Z^2) = -\frac{\alpha M_Z^2}{3\pi}\int_{m_{\pi^0}^2}^{\infty} \boxed{\frac{R(s)}{s\left(s-M_Z^2\right)-i\varepsilon}}\,ds = 0.02768(22)_{\text{had (5)}} - 0.000073(2)_{\text{top}}$$

Integration kernel more "democratic" than for *g* – 2
(influence of tau data less pronounced)

The Running α_{QED} at M_Z

Same principle as for *g* – 2: energy-dependent vacuum polarisation effects screen the bare electromagnetic coupling. Leptonic contributions computed via QED, hadronic contributions obtained from dispersion relation:

$$\alpha(s) = \frac{\alpha(0)}{1-\Delta\alpha(s)} \qquad \text{with:} \quad \Delta\alpha(s) = \Delta\alpha_{\text{lep}}(s) + \Delta\alpha_{\text{had}}(s) = -4\pi\alpha\,\mathrm{Re}\left[\Pi_\gamma(s) - \Pi_\gamma(0)\right]$$

$$\Delta\alpha_{\text{lep}}^{\text{3-loop}}(M_Z^2) = 0.031497686$$

Steinhauser, hep-ph/9803313 (1998)

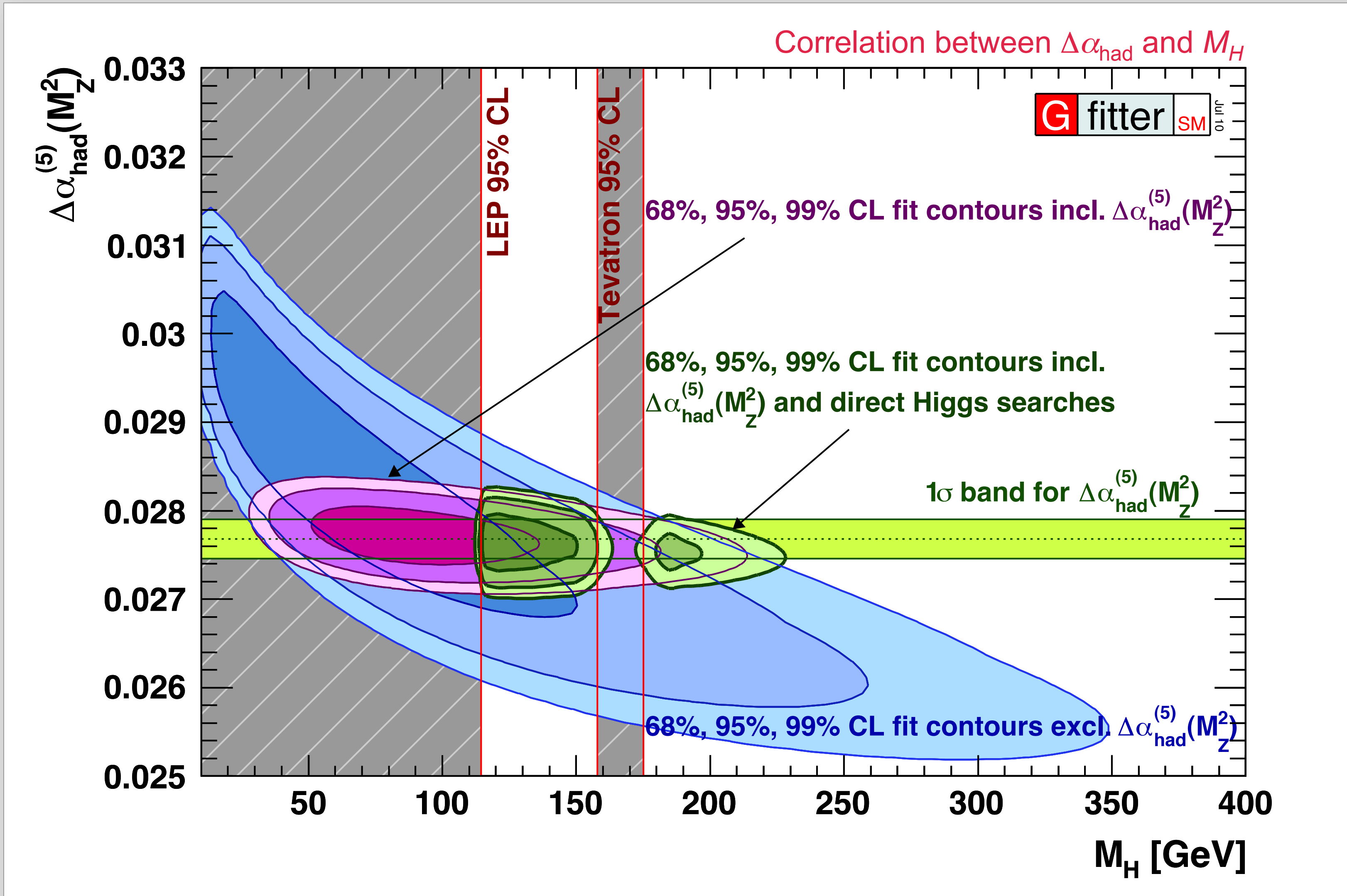
$$\Delta\alpha_{\text{had}}(M_Z^2) = -\frac{\alpha M_Z^2}{3\pi} \int_{m_{\pi^0}^2}^{\infty} \frac{R(s)}{s\left(s-M_Z^2\right) - i\varepsilon}\, ds = 0.02768(22)_{\text{had (5)}} - \ 0.000073(2)_{\text{top}}$$

Result: $\alpha^{-1}(M_Z^2) = \ 128.937 \pm 0.030$

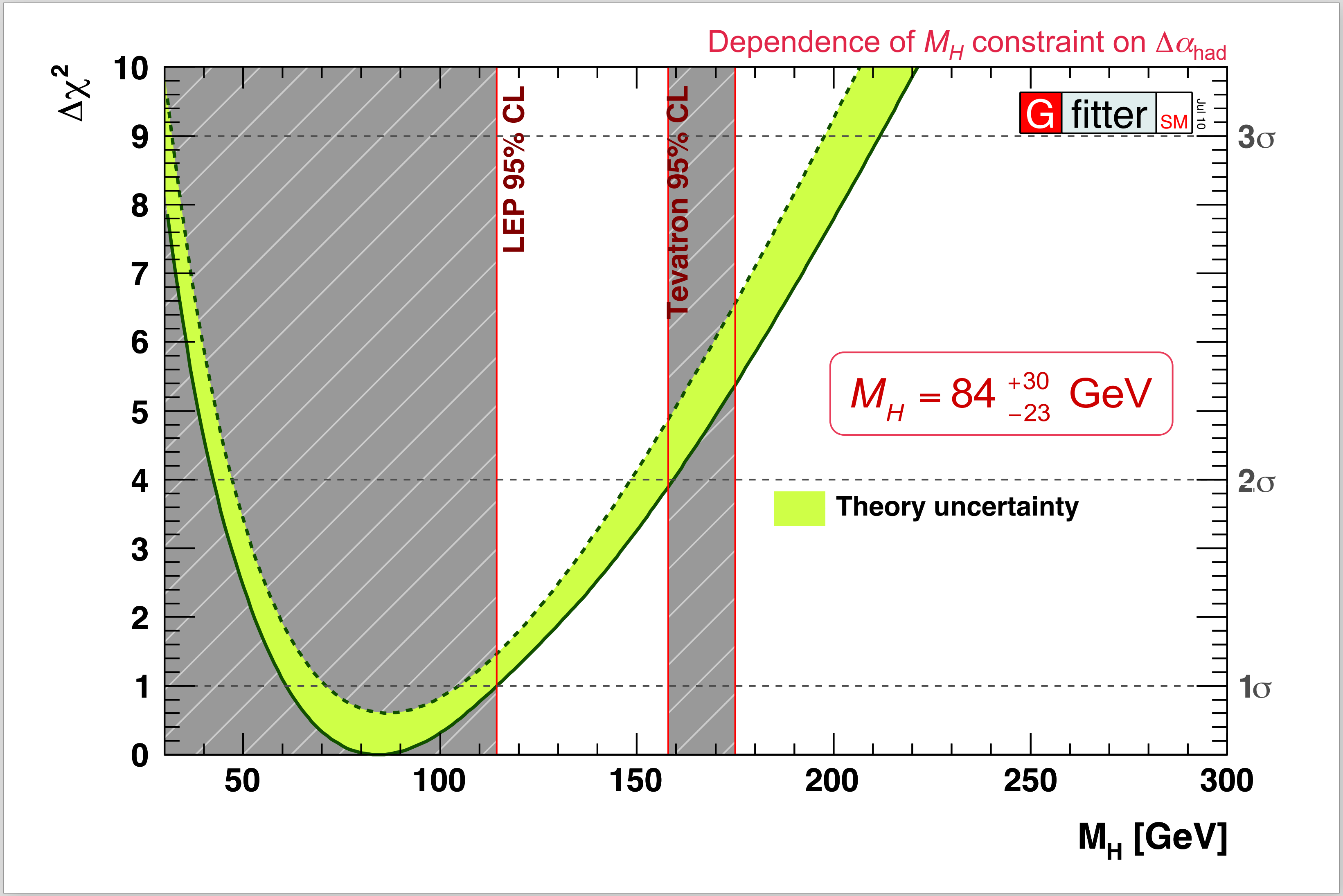
HLMNT arXiv:hep-ph/0611102 (2006)

The current precision suffices for the global electroweak fit and the constraint of the Higgs boson mass, but the central value has an impact !

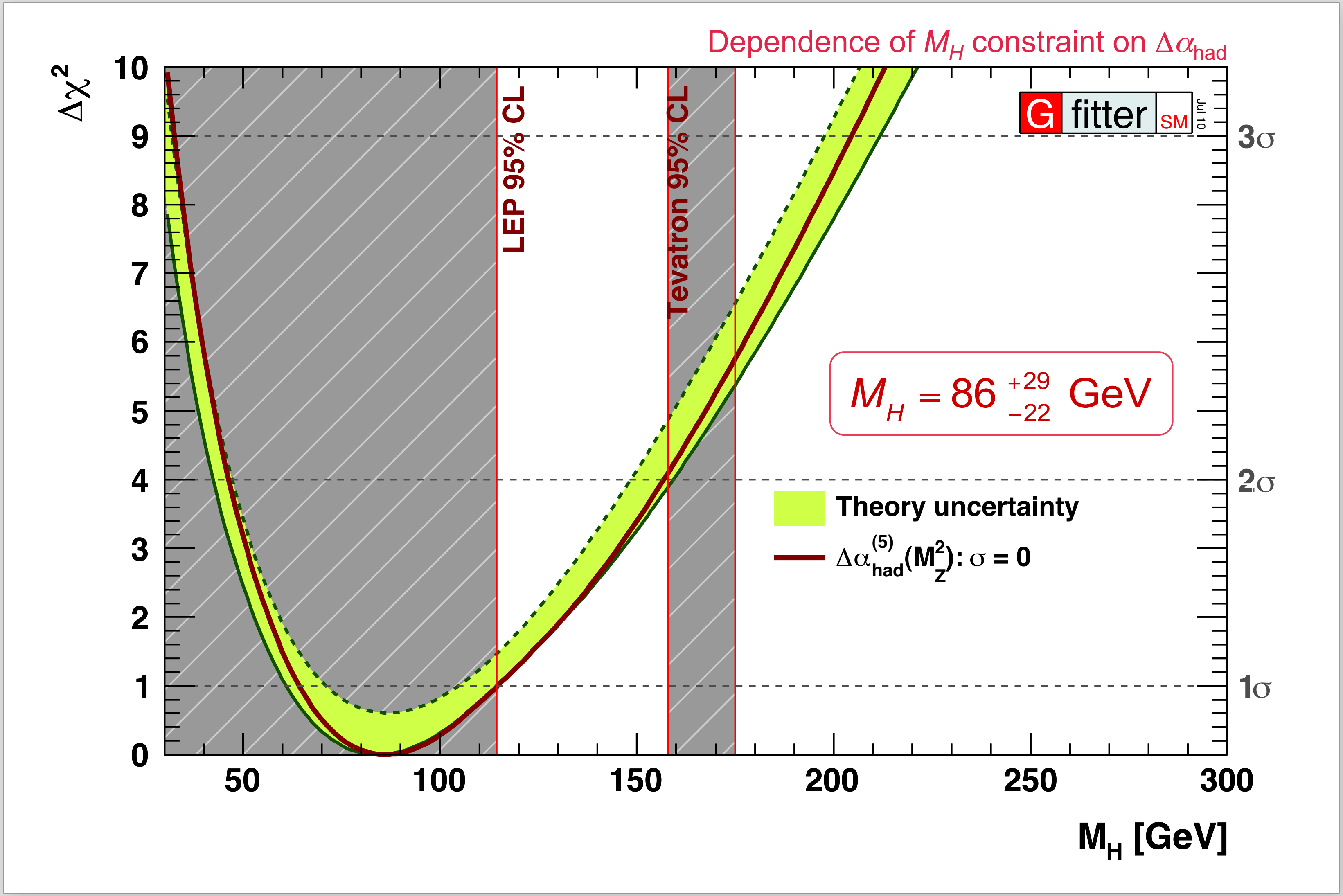
Global Electroweak Fit



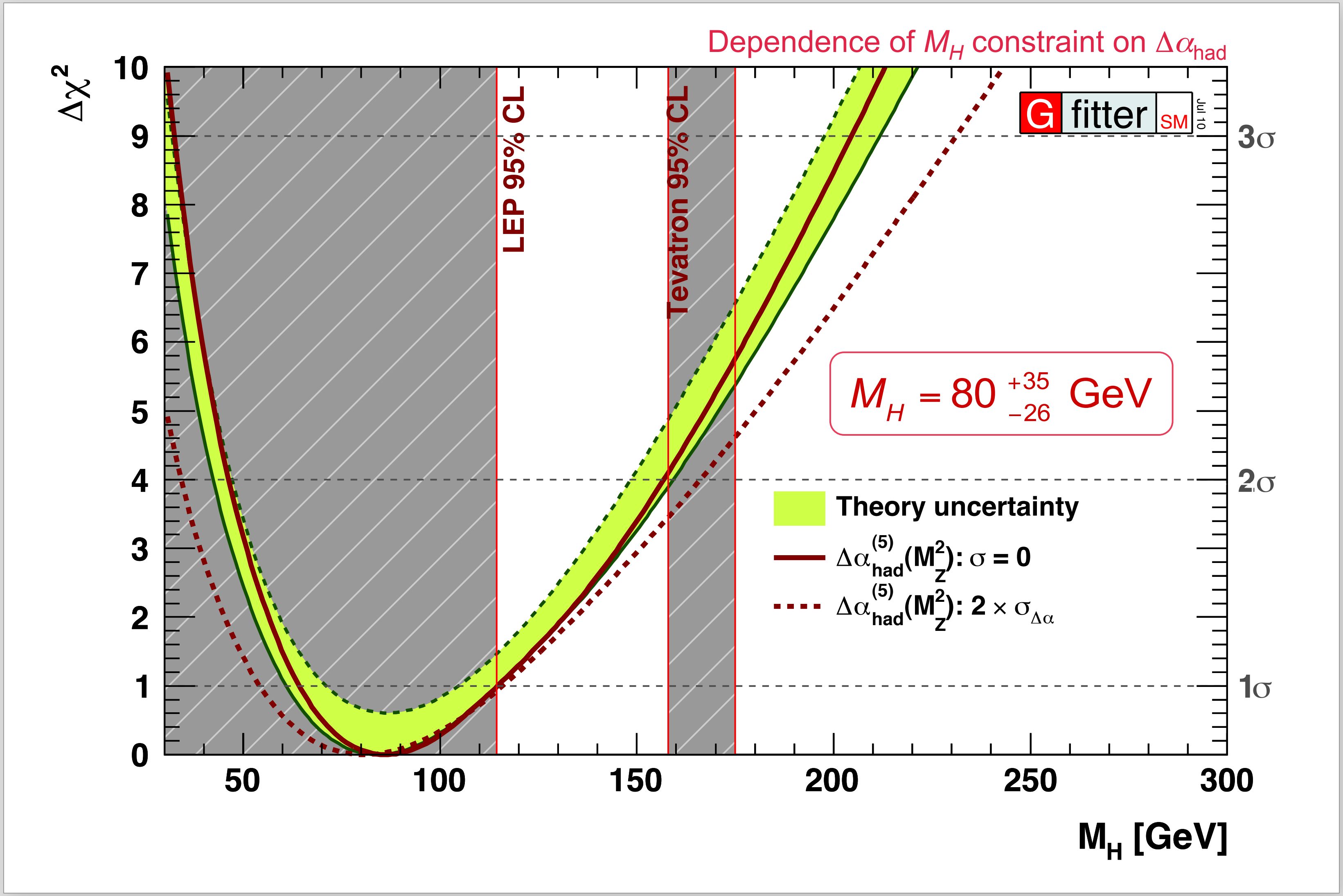
Global Electroweak Fit



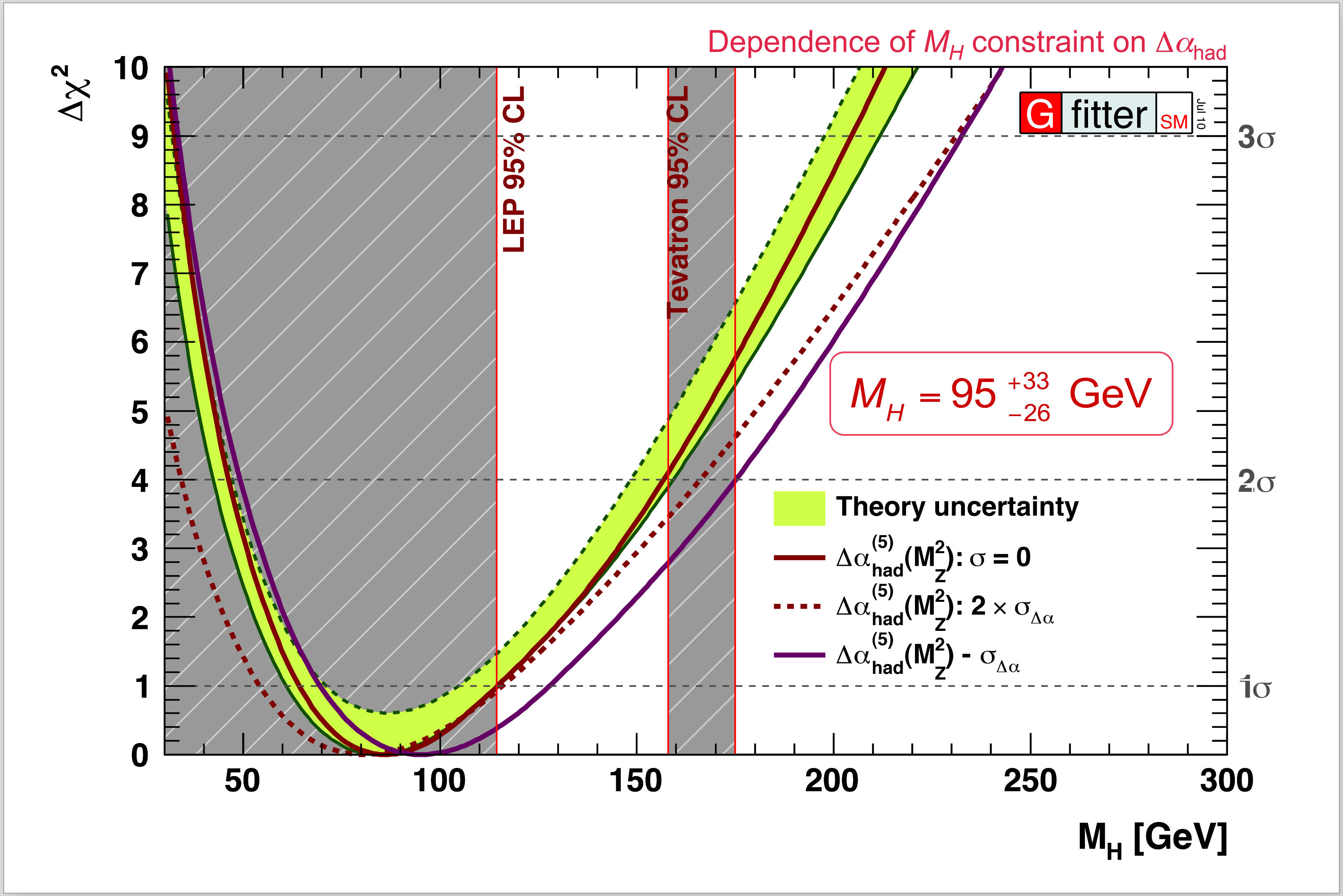
Global Electroweak Fit

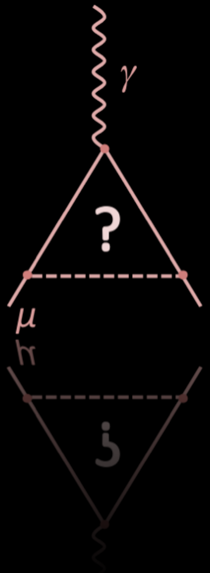


Global Electroweak Fit



Global Electroweak Fit





Will hear about many important results and developments at this session:

- ISR simulation [Henryk Czyz]
- KLOE and BABAR $e^+e^- \to \pi^+\pi^-$ results using ISR technique [Graziano Venanzoni, Bogdan Malaescu]
- Hadronic cross section measurements at Novisibirsk [Boris Shwartz]
- CVC tests in rare modes [Simon Eidelman]
- New muon $g - 2$ and $\alpha(M_Z)$ results [Thomas Teubner, AH]
- Future muon $g - 2$ experimental projects [Lee Roberts, Tsutomu Mibe]