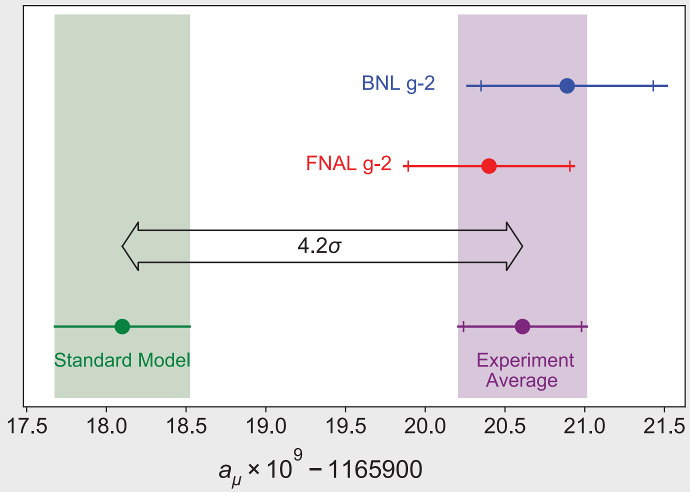
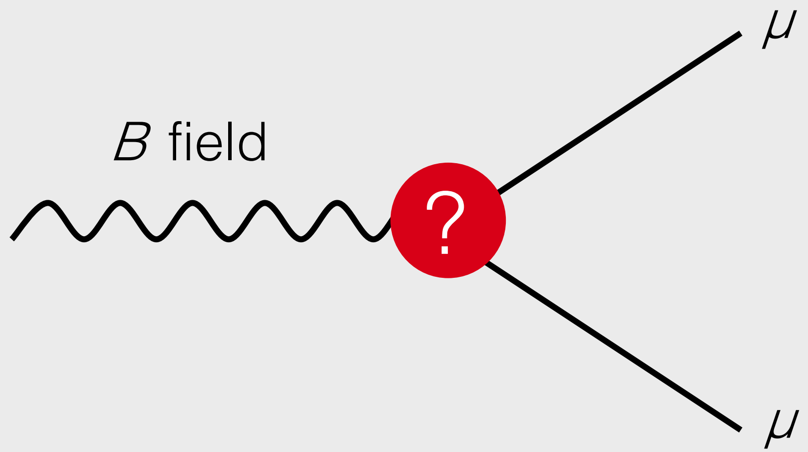
Ultimate precision Standard Model tests: the muon magnetic anomaly — taming its hadronic contributions





Andreas Hoecker (CERN)

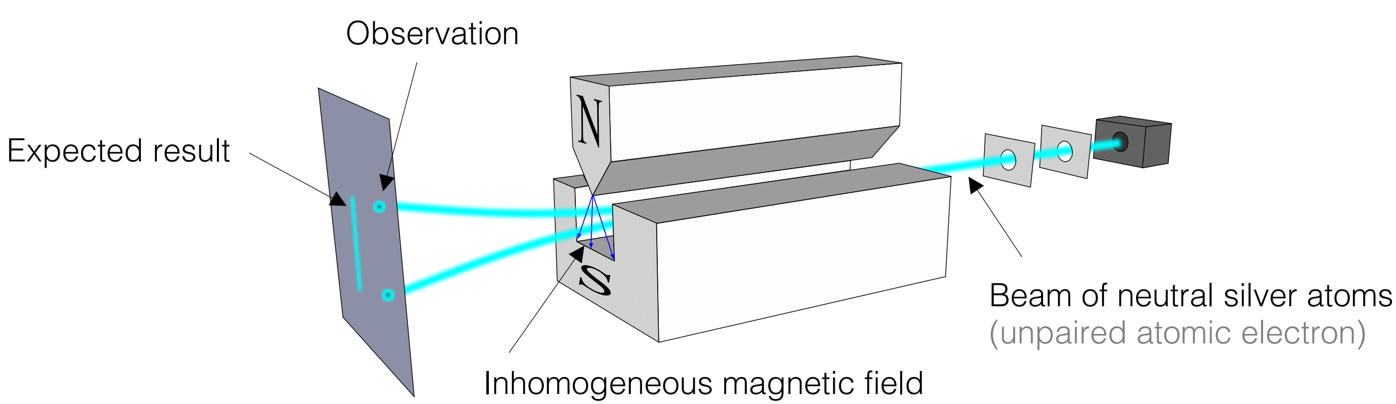
Seminar, Geneva University, May 25th, 2021

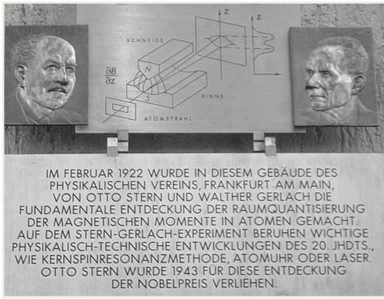
Work in collaboration with M. Davier, B. Malaescu, and Z. Zhang (DHMZ): Eur. Phys. J. C 80, 241 (2020)

Magnetic moment

The magnetic dipole moment of a particle can be observed from its motion in a magnetic field

Intrinsic magnetic moment discovered in Stern-Gerlach experiment, 1922:

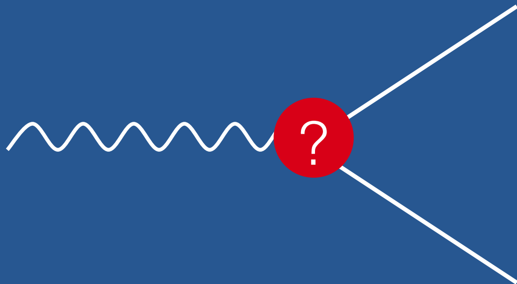




A commemorative plaque at the Frankfurt physics institute

$\rightarrow$ atoms have intrinsic and quantised angular momentum

Uhlenbeck & Goudsmit postulated in 1925 that electrons have spin angular momentum with magnetic dipole moment: $e/2m_e$ (Bohr magneton)



Electron g factor

Paul Dirac

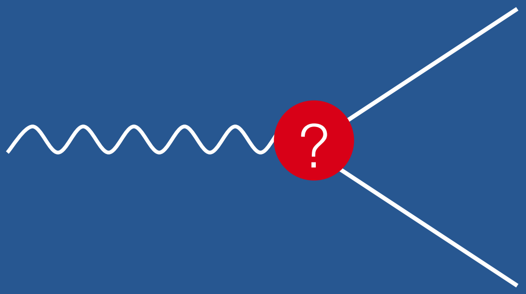
Dirac's relativistic theory of the electron (1928) naturally accounted for quantized particle spin, and described elementary spin-1/2 particles

In the classical limit, one finds the Pauli equation with magnetic moment:

$$\vec{\mu} = -g_e \frac{e}{2m_e} \vec{S} \ ,$$

with $|\boldsymbol{g_e}| = \mathbf{2}$ the gyromagnetic factor
(and radius $R_e = 0$, ie, elementary !)

Here $g_p/g_e = 2.8$ hinted that proton is not elementary



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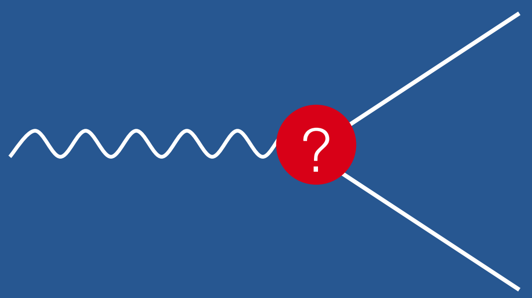
Today, everyone knows that the proton is composite,
and the electron is a point-like particle …

But – is it really ?

→ Precise g_e measurement is key !



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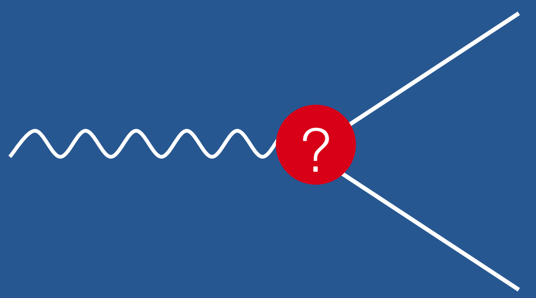
Dirac's prediction was confirmed to 0.1% by Kinsler & Houston in 1934 through studying the Zeeman effect in neon [Phys. Rev. 46, 533 (1934)]

A deviation from $g_e = 2$ was established by Nafe, Nels & Rabi only in 1947 by comparing the hyperfine structure of hydrogen and deuterium spectra [Phys. Rev. 71, 914 (1947)]

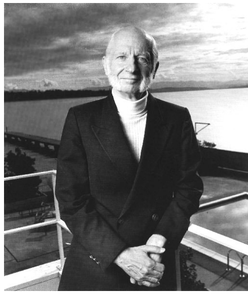
A first precision measurement of $g_e = 2.00344 \pm 0.00012$ (*wrong: 2.00232…!*) was made by Kusch & Foley in 1947 using Rabi's atomic beam magnetic resonance technique [Phys. Rev. 72, 1256 (1947)]



Electron g factor

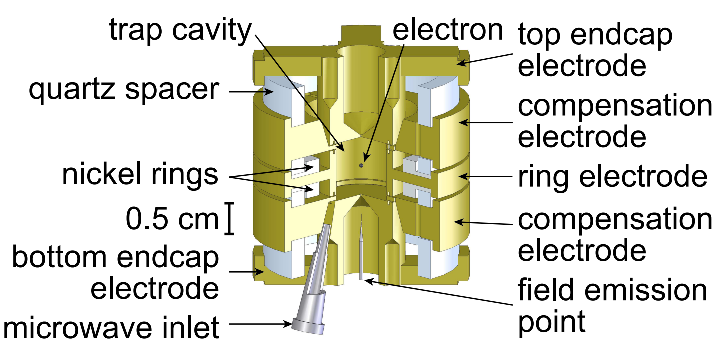
Ever since experimentalists & theorists are racing for g_e precision to test QED

A series of (Nobel prize winning) experiments was performed using single electron capture in a cylindrical Penning trap and measuring the spin (ω_s) to cyclotron ($\omega_c = eB/m_e$) frequency ratio, giving: $g_e/2 = \omega_s/\omega_c$



Hans G. Dehmelt
Nobel 1989

The most precise measurement from 2008 exploits spectroscopy with fully resolved lowest cyclotron and spin levels of a single electron quantum cyclotron in a cold (0.1 K) cylindrical Penning trap cavity immersed in 5.4 T B field



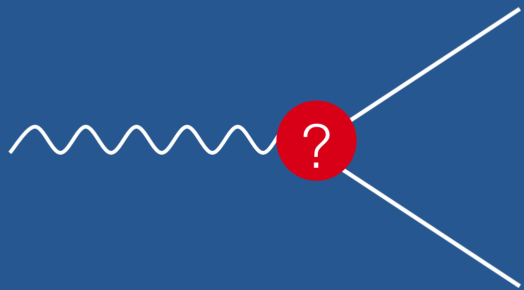
Cylindrical Penning trap cavity used to confine a single electron and inhibit spontaneous emission

Anomalous magnetic moment:

$$a_e = \frac{g_e - 2}{2} = 1\,159\,652\,180.73(28) \cdot 10^{-12}$$

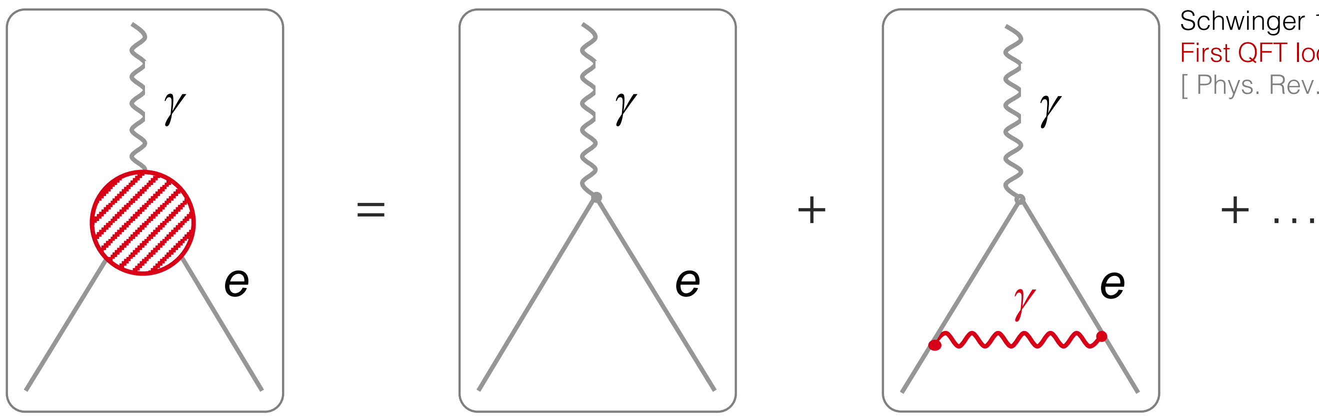
(24 ppb precision, 1.8σ below 1987 value)

[Hanneke, Fogwell, Gabrielse (Harvard), 0801.1134, 1009.4831]



Quantum fluctuations

Dirac's gyromagnetic factor is lowest order QED graph, but there are quantum corrections…



$(g\text{–}2)_e \neq 0$ (full Standard Model)

$(g\text{–}2)_e = 0$ (Dirac)

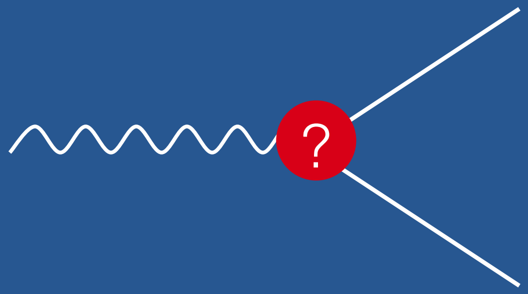
coupling to virtual fields:
$(g\text{–}2)_e \neq 0$ (1st order QED)

Schwinger 1948 (Nobel price 1965)
First QFT loop calculation!
[Phys. Rev. 73, 416 (1948)]

Julian Schwinger

Quantum fluctuations slightly increase gyromagnetic factor, so that:

$$a_e^{\mathrm{QED}} = \frac{\alpha}{2\pi} + \cdots = 0.001\,161\ \ldots$$



Theory and comparison with experiment

In a tour de force, five QED loops have been computed [Kinoshita et al, 1412.8284 (2014)]

$$a_e^{\mathrm{QED}} = \frac{\alpha}{2\pi} - 0.328478444\left(\frac{\alpha}{\pi}\right)^2 + 1.181234\left(\frac{\alpha}{\pi}\right)^3 - 1.912(1)\left(\frac{\alpha}{\pi}\right)^4 - 7.8(3)\left(\frac{\alpha}{\pi}\right)^5$$

adding to these small contributions from hadronic and weak loops, and using the best value α^{-1} = 137.035 999 049 (90), from a measurement of h/m_{Rb} via the recoil velocity of Rubidium atoms when absorbing photon [Bouchendira et al, 1012.3627 (2010), $\alpha^2 = 2R_\infty m_{\mathrm{Rb}} h/(c m_e m_{\mathrm{Rb}})$], one finds:

$$a_e^{\mathrm{SM}} = 1\,159\,652\,181.64\,(4)(76)\cdot 10^{-12}$$

$$a_e^{\mathrm{Exp}} = 1\,159\,652\,180.73(28)\cdot 10^{-12}$$

→ In agreement with experiment

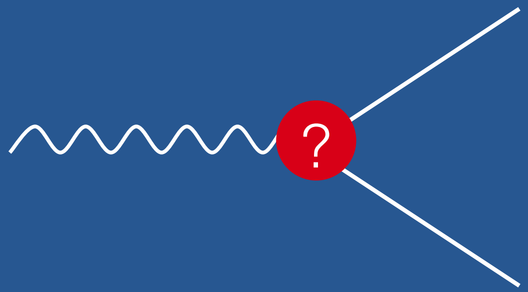
(errors are from loop terms and α, respectively)

Measurement and SM prediction can be used to derive most precise value of α

$$\alpha^{-1}(a_e) = 137.035\,999\,157\,(4)(33)$$

(errors are from theory and experiment, respectively)

(0.25 ppb precision, 3 times better than Rb based value) [Kinoshita et al, 1412.8284 (2014)]



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Recent Update

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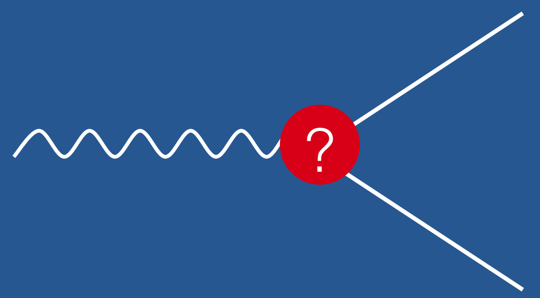

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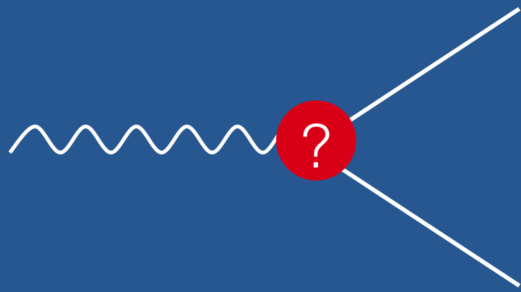
adding to these small contributions from hadronic and weak loops, and using the best value α^{-1} = 137.035 999 049 (90) → 137.035 999 046(27) | 137.035 999 206(10) [> 5σ difference], using the recoil frequency of Cs-133 | Rb-87 atoms in a matter-wave interferometer [Parker et al, Science 360, 191 (2018) | Morel et al. Nature 588, 61 (2020)], one finds:

$$a_e^{\mathrm{SM}} = 1\,159\,652\,181.64\,(76) \cdot 10^{-12} \quad \rightarrow \quad 1\,159\,652\,181.61(23)\ \mid\ \ldots 180.25(10) \cdot 10^{-12}$$

$$a_e^{\mathrm{Exp}} = 1\,159\,652\,180.73(28)\ \cdot 10^{-12}$$

so that the experimental value differs from the SM prediction by –2.4σ | +1.6σ, respectively



Theoretical properties

The anomalous magnetic moment of an elementary particle corresponds to an effective Lagrangian interaction of mass dimension 5. It is finite and calculable

At lowest order in QED, the anomalous magnetic moment is universal:

$$a_e^{\mathrm{QED,LO}} = a_\mu^{\mathrm{QED,LO}} = a_\tau^{\mathrm{QED,LO}} = \frac{\alpha}{2\pi}$$

Differences, ie, lepton mass dependence are introduced at loop level: $(\alpha/\pi)^2$

SM contribution to a_e dominated by mass-independent Feynman diagrams in QED with electrons in internal lines

Lepton mass effects become significant for a_μ !

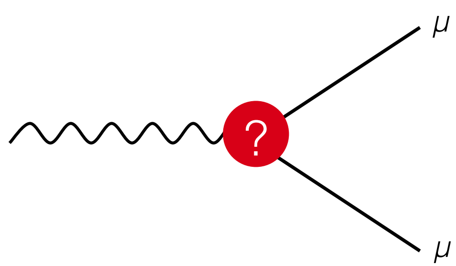
Muon $g-2$

The measurement of the muon $g-2$ is harder as the muon is instable (2.2 μs) $\rightarrow$ why bother ?

$\rightarrow$ All sectors of SM physics contribute measurably to muon $g-2$

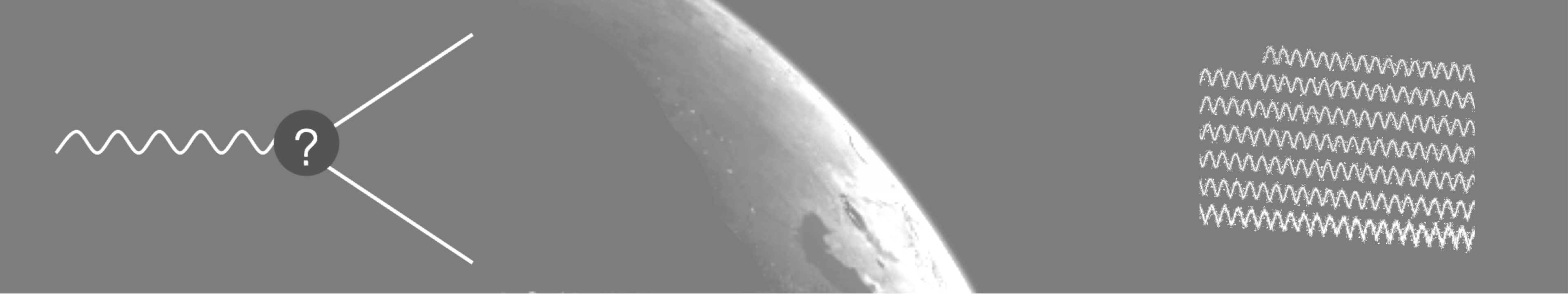
$\rightarrow$ At lowest order where mass effects appear, contributions from heavy virtual "new physics" (NP) particles of mass Λ_{NP} scale as m_ℓ^2

$$a_\ell^{\mathrm{NP}}(\Lambda_{\mathrm{NP}}) \propto \mathcal{O}\left(\frac{m_\ell^2}{\Lambda_{\mathrm{NP}}^2}\right) \quad \longrightarrow \quad \frac{a_\mu^{\mathrm{NP}}}{a_e^{\mathrm{NP}}} \approx \mathcal{O}\left(\frac{m_\mu^2}{m_e^2}\right) \approx 43{,}000$$

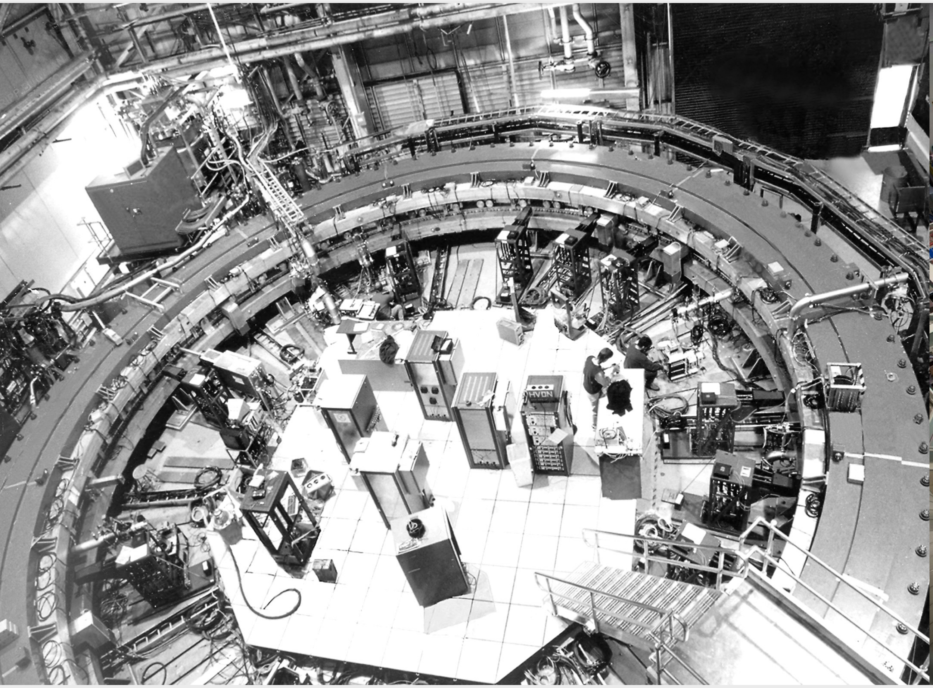


Muon $g-2$ looses "only" about factor 23 (4) in experimental (theoretical) precision,
so a_μ expected to be significantly more sensitive to NP than a_e

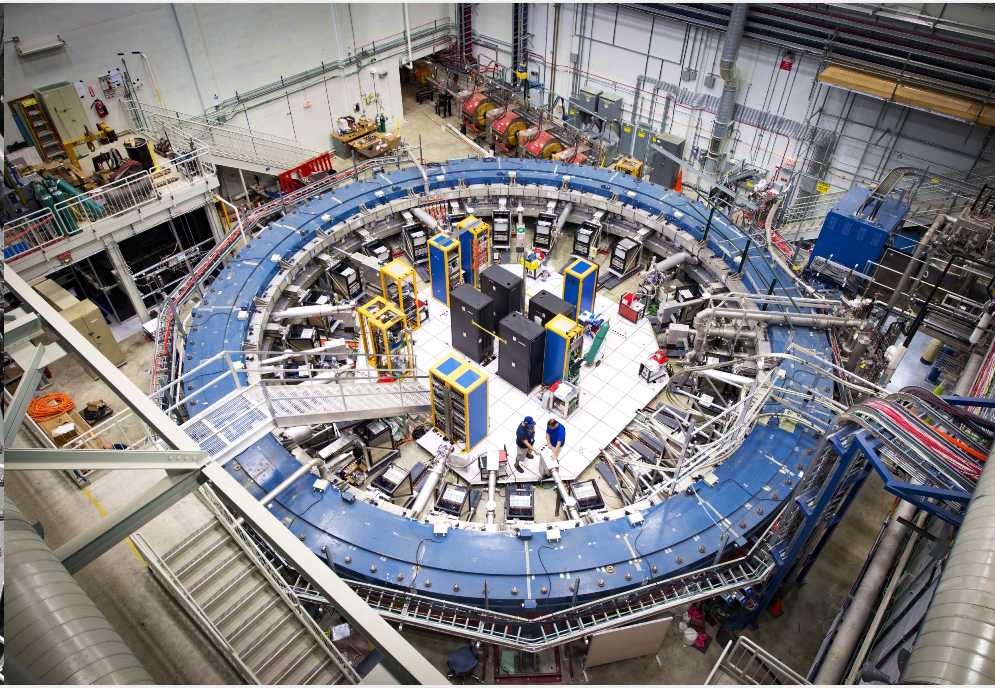
Example: weak + Higgs boson contribution is $1536 \cdot 10^{-12}$ (μ) and $0.030 \cdot 10^{-12}$ (e) $\rightarrow$ ratio of ~ 51,000



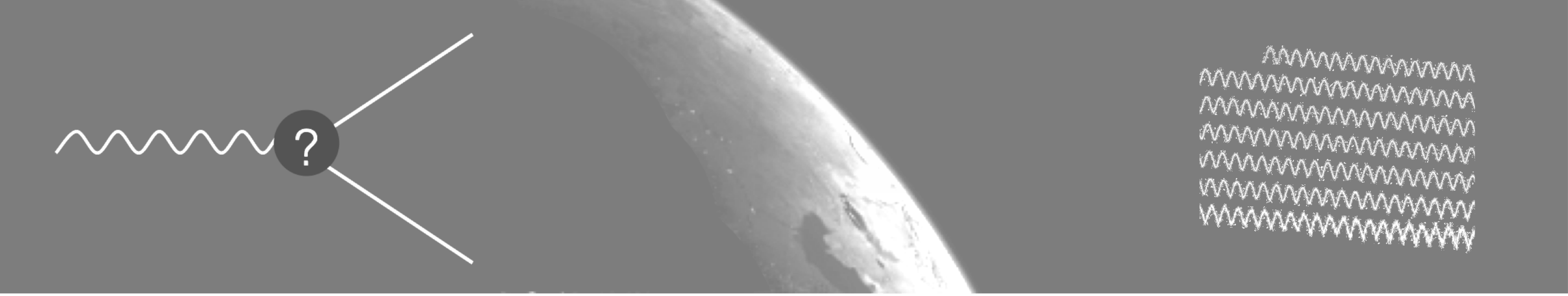
Measuring the muon $g-2$



BNL muon g–2 experiment (E821), 1997–2001 (data taking)



Fermilab muon g–2 experiment, 2018–…



Measuring the muon $g-2$

Analogous approach as for electron: search for discrepancy between the frequencies of cyclotron motion and spin precession

For polarised muons moving in a uniform B field (perp. to muon spin and orbit plane), and focused in an electric quadrupole field, the observed difference between spin precession and cyclotron frequency (= "anomalous frequency"), ignoring μEDM, is:

$$\vec{\omega}_a \equiv \vec{\omega}_s - \vec{\omega}_c = \frac{e}{m_\mu c}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\vec{\beta}\times\vec{E}\right]$$

Motional magnetic field

With electrostatic focusing, no gradient B field focusing needed so that B can be made as uniform as possible !

$\vec{\omega}_a$ Independent of muon momentum

The E field dependence is eliminated at the "magic γ": $\gamma = 29.3 \;\rightarrow\; p_\mu = 3.09$ GeV

The experiment measures $(g_\mu - 2)/2$ directly

[J. Bailey et al., NP B150, 1 (1979)]

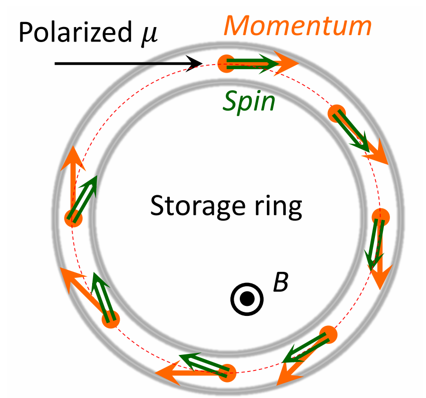
Exploit muon properties in experiment

1. Parity violation polarizes muons in pion decay
   ~97% polarized for forward decays

$$\bar{\nu}_\mu \longleftarrow \pi^- \longrightarrow \mu^-_{\text{polarised}}$$

spin orientation

2. Anomalous frequency proportional to a_μ



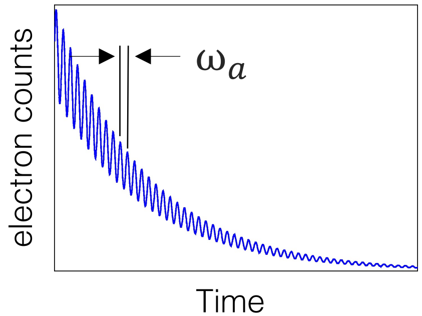
3. Magic γ :

$$\vec{\omega}_a = \frac{e}{m_\mu c}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\vec{\beta}\times\vec{E}\right] \approx \frac{e}{m_\mu c} a_\mu \vec{B}$$

4. Parity violation in muon decay

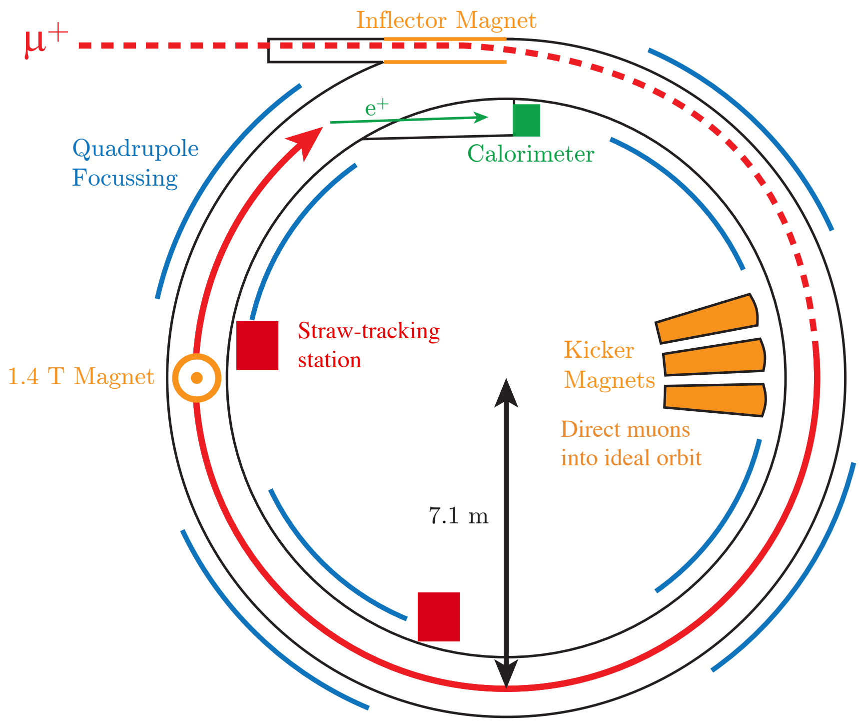
$$\mu^+_{\text{polarised}} \longrightarrow e^+ + \nu_e + \bar{\nu}_\mu$$

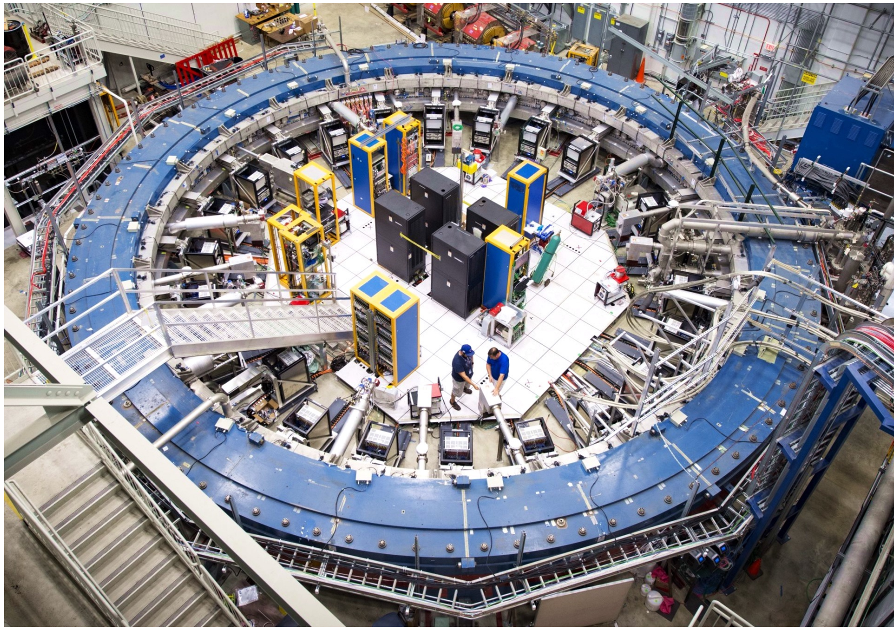
fast positron emitted preferentially in direction of muon spin



Fermilab muon $g-2$ experiment

- 8 GeV proton beam incident on a target produces large number of pions that decay to muons
- 3.1 GeV muons (relativistically enhanced lifetime of 64 μs) is injected into a 45 m circumference ring with 1.4 T vertical magnetic field, which produces cyclotron motion matching the ring radius
- Vertical focusing via electrostatic quadrupole lenses around the ring

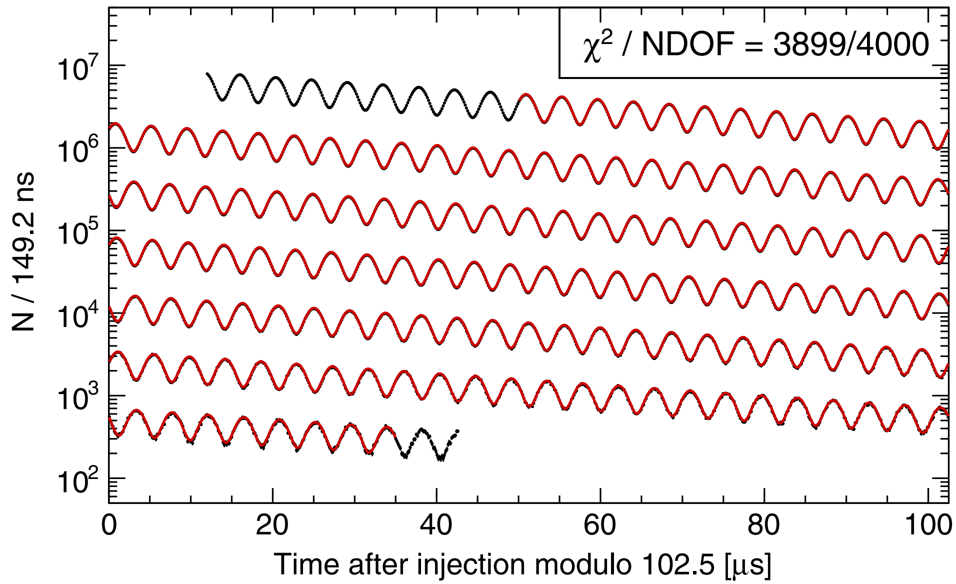




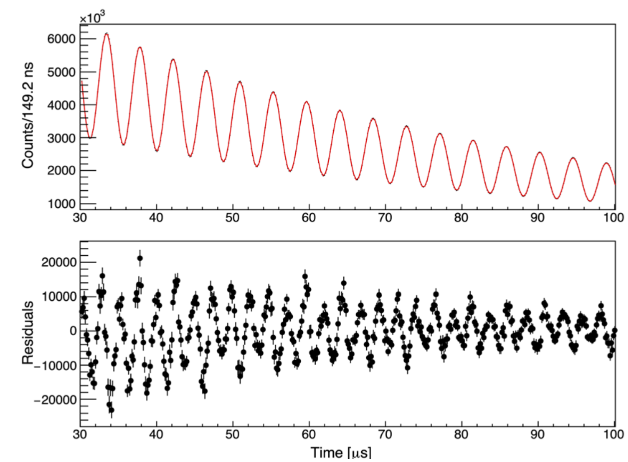
- Decay positrons (correlated with μ spin precession) counted vs. time in calorimeters inside ring ($\rightarrow \omega_a$)
- Muon beam profile measured with tracking stations
- Precise measurement of ω_a and B allows to extract a_μ

Fermilab muon $g-2$ experiment

PRD 103, 072002 (2021)



Observed positron rate in successive 102.5 μs periods



5-paramerter fit not adequate to describe data

Anomalous frequency:

$$\omega_a \approx \frac{e}{m_\mu c} a_\mu B$$

obtained from time-dependent fit to electron counts (for given energy *E*), in simplified form:

$$N(t) = N_0 e^{-t/\gamma\tau}[1 - A \cdot \cos(\omega_a t + \phi)]$$

In blue: fit parameters (full fit has 22 free parameters)

Statistical uncertainty on ω_a: 0.43 ppm

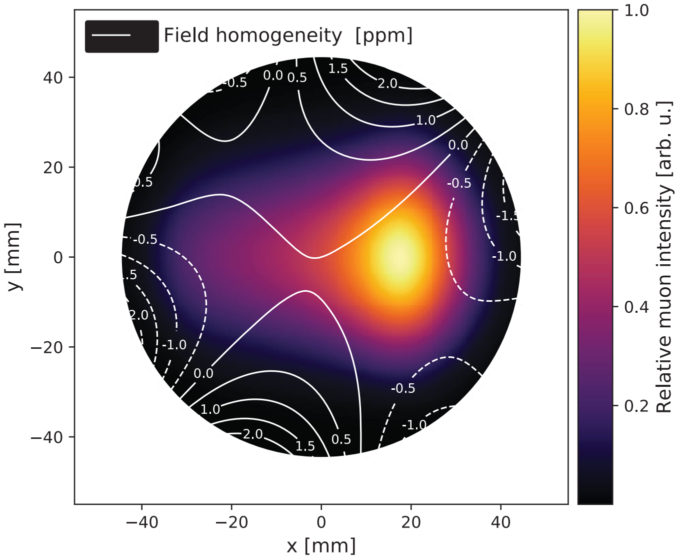
Total systematic uncertainty on ω_a: 0.06 ppm, with largest contributors:

- Pileup (~in-time arrival of two low-E positrons)
- Calorimeter gain changes
- Fit model
- Coherent betatron oscillation

Additional beam-dynamics systematic: 0.09 ppm

B-field is mapped with 17 NMR probes on a trolley pulled through beampipe every 2-3 days

PRL 126, 141801 (2021)



Azimuthal average magnetic field contours overlaid on the time and azimuthally averaged muon distribution

B-field is proportional to free proton precession frequency ω_p ($B \propto \omega_p/\mu_p$) measured by NMR probes, so one can write:

$$a_\mu = \frac{\omega_a}{B}\frac{m_\mu c}{e} = \frac{\omega_a}{\omega_p} \cdot \frac{\mu_p}{\mu_e}\frac{m_\mu}{m_e}\frac{g_e}{2}$$

Precise external data (total uncertainty: 0.03 ppm)

Measured by Fermilab experiment

where was used:

$$B = \frac{\hbar\omega_p}{2\mu_p} \quad \text{and} \quad e = \frac{4m_e\mu_e}{\hbar g_e}$$

The full expression includes corrections for the muon distribution and NMR temperature reference

→ Systematic uncertainty on ω_p is 0.06 ppm

→ Additional systematic from magnetic transients: 0.10 ppm

ω_a and ω_p measured independently in blind analyses → doubly blind experiment!

Result, and comparison with earlier experiments

Fermilab g–2 Result (Run 1): [PRL 126, 141801 (2021)]

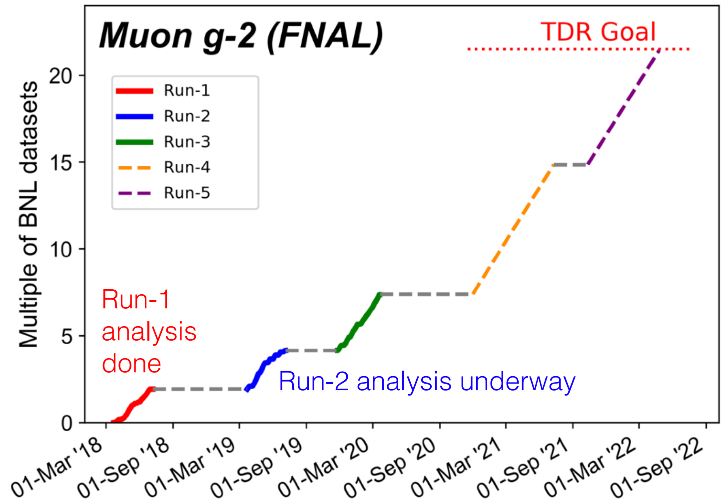
$$a_\mu = 11\,659\,204.0\,(5.1)(1.8)\,\cdot 10^{-10}$$

(0.46 ppm precision, assumes CPT invariance)

Combination with BNL E821 result:

[PRD 73, 072003 (2006) with updated reference values]

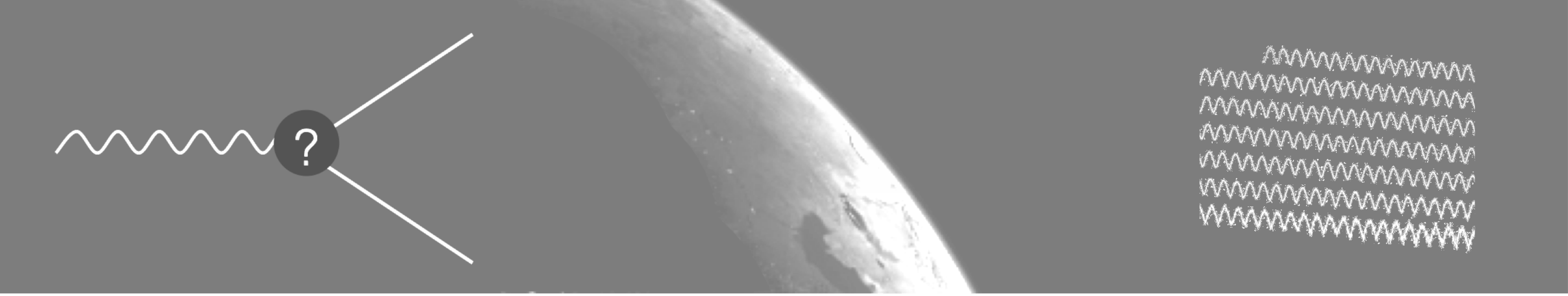
$$a_\mu = 11\,659\,206.1\,(4.1)\,\cdot 10^{-10}$$ (0.35 ppm)



Evolution of the experimental sensitivity:

[See, eg, Miller, de Rafael, Roberts, hep-ph/0703049]

Experiment	Beam	Measurement	$\delta a_\mu / a_\mu$	Required theor. terms	
Columbia-Nevis ('57)	μ^+	g = 2.00 (σ = 0.10)		$g = 2$	Muon behaves like heavy electron
Columbia-Nevis ('60)	μ^+	0.001 13 (+16)(−12)	12 %	$\alpha/2\pi$	
CERN 1 (SC, 1961)	μ^+	0.001 145 (22)	1.9 %	$\alpha/2\pi$	
CERN 1 (SC, 1962)	μ^+	0.001 162 (5)	0.43 %	$(\alpha/\pi)^2$	
CERN 2 (PS, 1968)	μ^+	0.001 166 16 (31)	266 ppm	$(\alpha/\pi)^3$	
CERN 3 (PS, 1979)	$\mu^\pm$	0.001 165 923 0 (84)	7.2 ppm	$(\alpha/\pi)^3$ + had (60 ppm)	Electrostatic focusing, magic γ
BNL E821 (1997–2001)	$\mu^\pm$	0.001 165 920 91 (63)	0.54 ppm	$(\alpha/\pi)^4$ + had + weak + ?	
Fermilab 2020	μ^+	0.001 165 920 40 (54)	0.46 ppm	same	

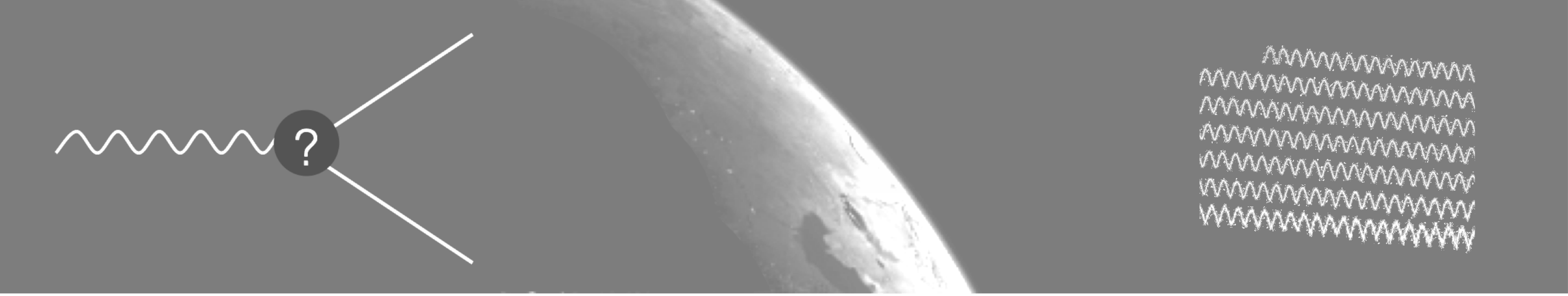


Confronting Experiment with Theory

The Standard Model prediction of a_μ is decomposed in its main contributions:

$$a_\mu^{\mathrm{SM}} = \frac{g_\mu - 2}{2} = a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{EW}} + a_\mu^{\mathrm{Had}}$$

of which the hadronic contribution has the largest uncertainty



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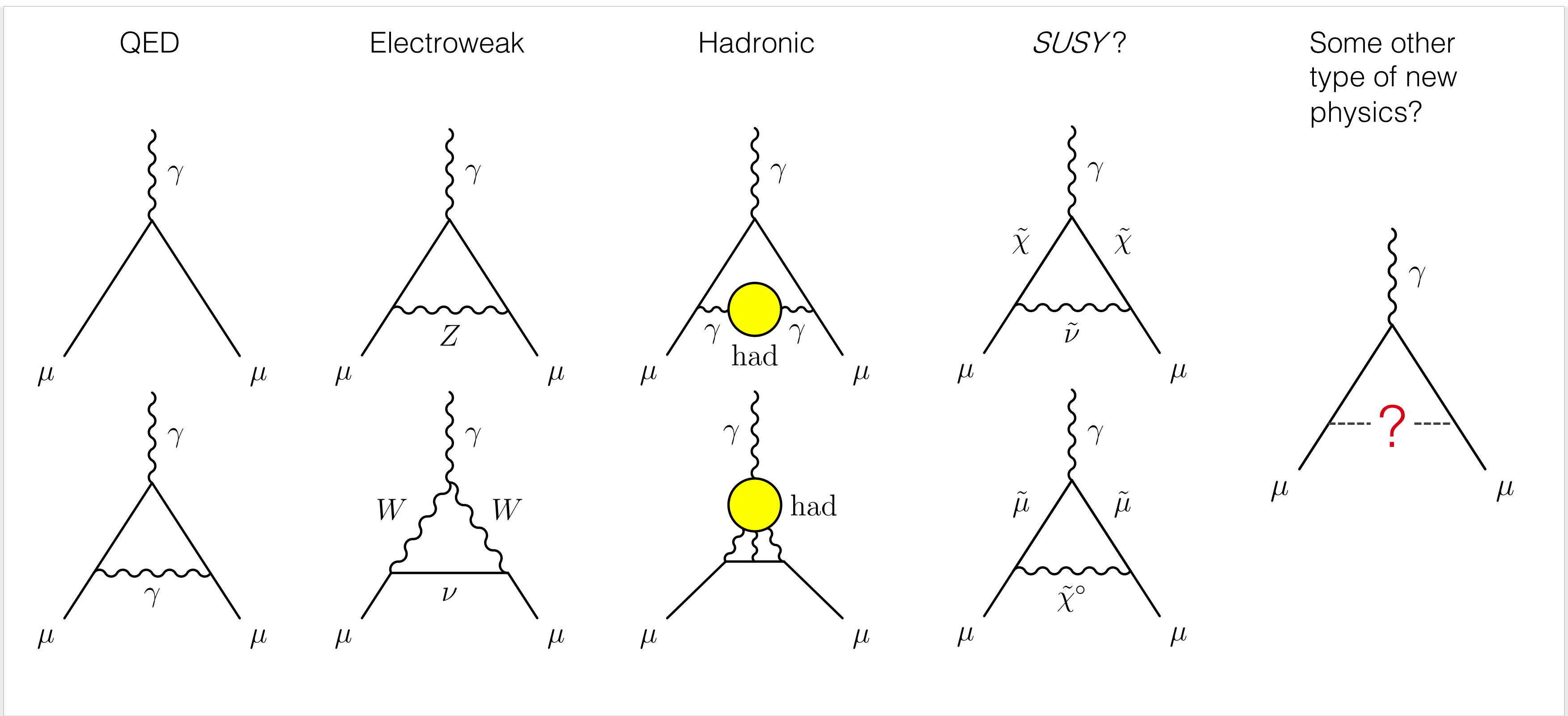
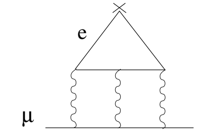


QED contribution

Known to 5 loops, good convergence, diagrams with internal electron loops enhanced:

$$a_\mu^{\mathrm{QED}} = \frac{\alpha}{2\pi} + 0.765\,857\,425(17)\left(\frac{\alpha}{\pi}\right)^2 + 24.050\,509\,96(32)\left(\frac{\alpha}{\pi}\right)^3 + 130.880(6)\left(\frac{\alpha}{\pi}\right)^4 + 753.3(1.0)\left(\frac{\alpha}{\pi}\right)^5$$

[Schwinger term]



3-loop light-by-light scattering with electron loop

[5-loop: Aoyama, Hayakawa, Kinoshita, Nio, 1205.5370 (2012)]

Using α = 137.035 999 049 (90) from Rubidium recoil measurement, gives:

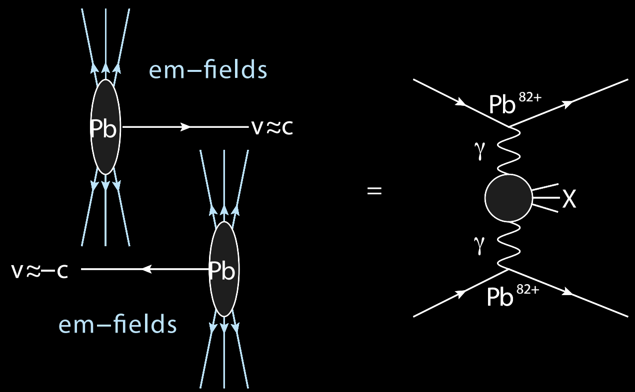
$$a_\mu^{\mathrm{QED}} = 11\,658\,471.895(0.008) \cdot 10^{-10}$$

with negligible uncertainty compared to experimental error of $6.3 \cdot 10^{-10}$

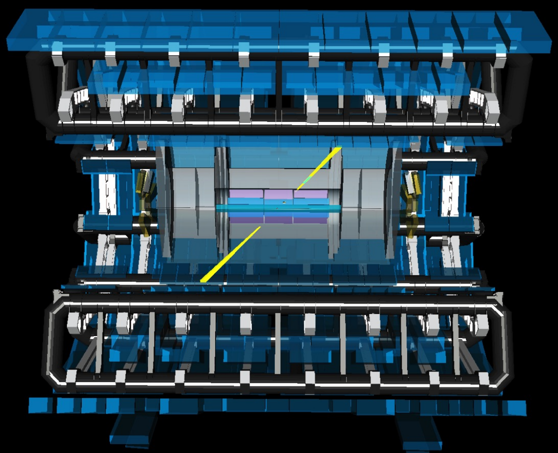
Observation of $\gamma\gamma \rightarrow \gamma\gamma$ light-by-light scattering (LBLS) by ATLAS in 5.02 TeV ultraperipheral Pb+Pb collisions

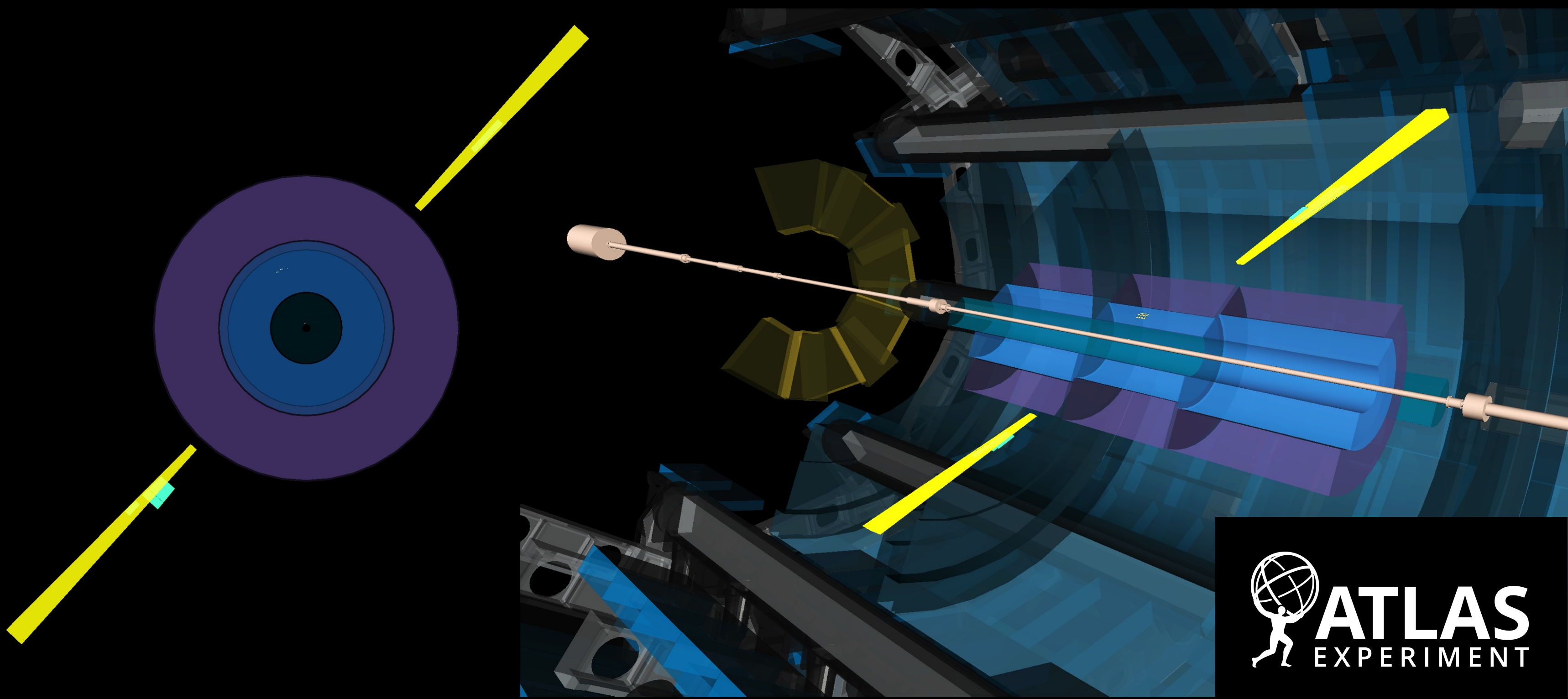
Picture shows LBLS candidate: two E_T = 4.9 GeV back-to-back photons with no additional activity

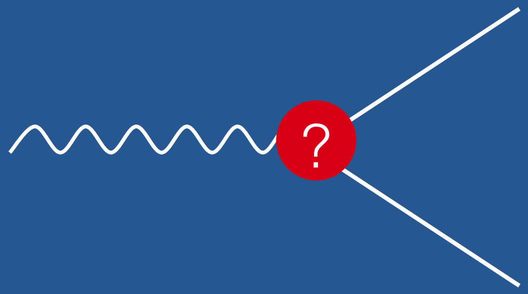
[PRL 123 (2019) 052001]



Field strength of up to 10^{25} V/m reached





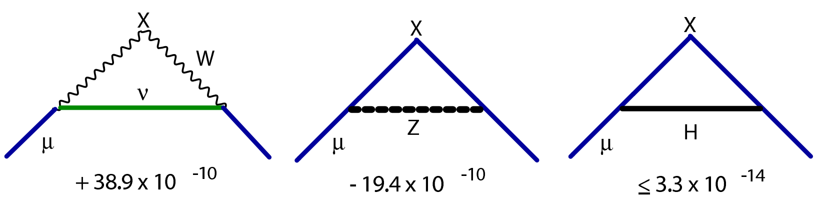


Electroweak contribution

EW contribution involving *W*, *Z* or Higgs is suppressed at least by a factor: $\frac{\alpha}{\pi}\frac{m_\mu^2}{m_W^2} \approx 4 \cdot 10^{-9}$

The first loop gives: [Jackiw, Weinberg and others 1972]

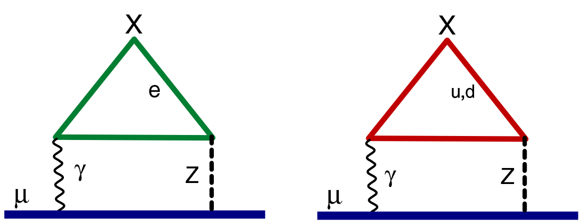
$$a_\mu^{\mathrm{EW,1-loop}} = \frac{G_F m_\mu^2}{8\sqrt{2}\pi^2}\left[\frac{5}{3} + \frac{1}{3}(1 - 4\sin^2\theta_W)^2 + \mathcal{O}\left(\frac{m_\mu^2}{m_W^2}\right) + \mathcal{O}\left(\frac{m_\mu^2}{m_H^2}\right)\right] = 19.48 \cdot 10^{-10}$$



1-loop diagrams (some cancellation between *W*/*Z* graphs)

Two-loop contribution surprisingly large due to large ln(m_Z/m_μ): [Czarnecki, Krause, Marciano, 1995, and others]

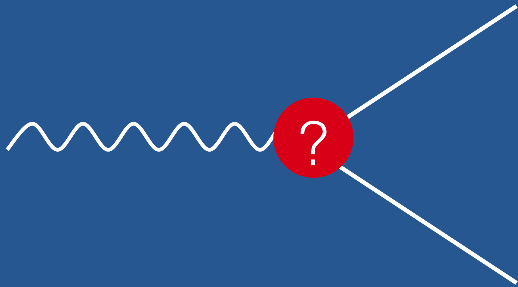
$$a_\mu^{\mathrm{EW,2-loop}} = -4.12(0.10) \cdot 10^{-10}$$



2-loop diagrams (+ Higgs exchange)

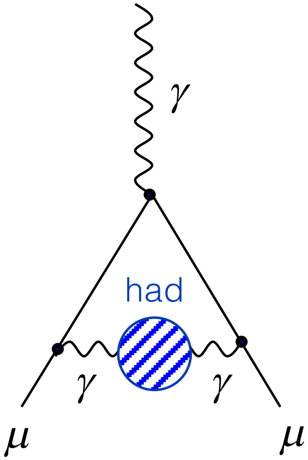
$$\Rightarrow a_\mu^{\mathrm{EW,1+2-loop}} = 15.36(0.10) \cdot 10^{-10}$$

Three-loop leading logarithms are found to be small (~10^{-12}) [Degrassi, Giudice, hep-ph/9803384, and others]



Hadronic contribution

The dominant hadronic contribution and uncertainty stems from the lowest order contribution, $a_\mu^{\mathrm{Had,LO}}$, which cannot be calculated from perturbative QCD as it is in the nonperturbative regime



Tools to approach low-energy QCD:

1. Lattice QCD (encouraging results, but precision is challenging)
2. Effective QFT with hadrons such as chiral perturbation theory (limited validity range)
3. Hadronic models (hard to estimate robust uncertainties)
4. Dispersion relations and experimental data

Digression: Running of $\alpha_{QED}(m_Z)$

Photon vacuum polarisation function $\Pi_\gamma(q^2)$

$$i\int d^4x\, e^{iqx}\langle 0|TJ^\mu_{em}(x)\left(J^\nu_{em}(0)\right)^\dagger|0\rangle = -\left(g^{\mu\nu}q^2 - q^\mu q^\nu\right)\Pi_\gamma(q^2)$$

Only vacuum polarisation "screens" electron charge

$$\alpha(s) = \frac{\alpha(0)}{1-\Delta\alpha(s)}$$

with: $\Delta\alpha(s) = -4\pi\alpha\,\mathrm{Re}\left[\Pi_\gamma(s) - \Pi_\gamma(0)\right]$

split into leptonic and hadronic contribution

Leptonic $\Delta\alpha_{lep}(s)$ calculable in QED (known to 3-loops). However, quark loops are modified by long-distance hadronic physics, cannot be calculated with perturbative QCD

Born: $\sigma^{(0)}(s) = \sigma(s)\left(\alpha/\alpha(s)\right)^2$

Way out: Optical Theorem (unitarity)

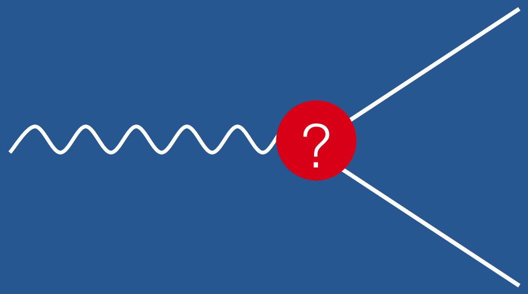
$$12\pi\,\mathrm{Im}\Pi_\gamma(s) = \frac{\sigma^{(0)}[e^+e^- \to \text{hadrons}]}{\sigma^{(0)}[e^+e^- \to \mu^+\mu^-]} \equiv R(s)$$

and the subtracted dispersion relation of $\Pi_\gamma(q^2)$ (analyticity)

$$\Pi_\gamma(s) - \Pi_\gamma(0) = \frac{s}{\pi}\int_0^\infty ds' \frac{\mathrm{Im}\Pi_\gamma(s')}{s'(s'-s)-i\varepsilon} \quad\Longrightarrow\quad \Delta\alpha_{had}(s) = -\frac{\alpha s}{3\pi}\mathrm{Re}\int_0^\infty ds' \frac{R(s')}{s'(s'-s)-i\varepsilon}$$

Precise knowledge $\alpha(m_Z)$ important ingredient to global electroweak fit

$\Delta\alpha_{had}(s)$ uncertainty contributes 1.8 MeV to m_W SM prediction (total error of SM: 8 MeV), but dominant uncertainty to $\sin^2\theta_{eff}$ (SM)

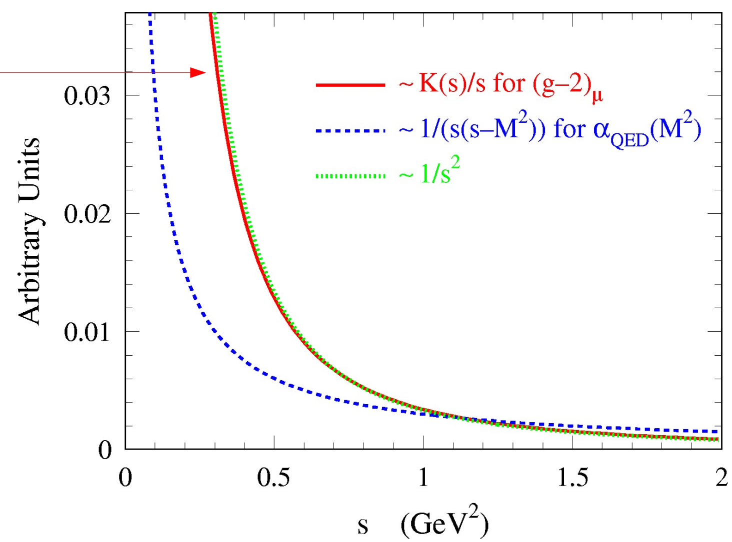


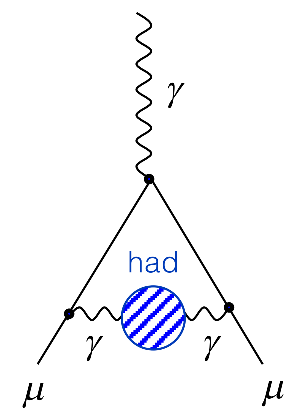
Hadronic contribution

Akin to $\Delta\alpha_{\text{had}}(s)$, the lowest-order hadronic contribution to a_μ can be obtained from a dispersion relation: [Bouchiat, Michel, 1961]

[Brodsky, de Rafael, 1968]

$$a_\mu^{\text{Had,LO}} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_{m_\pi^2}^{\infty} ds \frac{K(s)}{s} R(s)$$





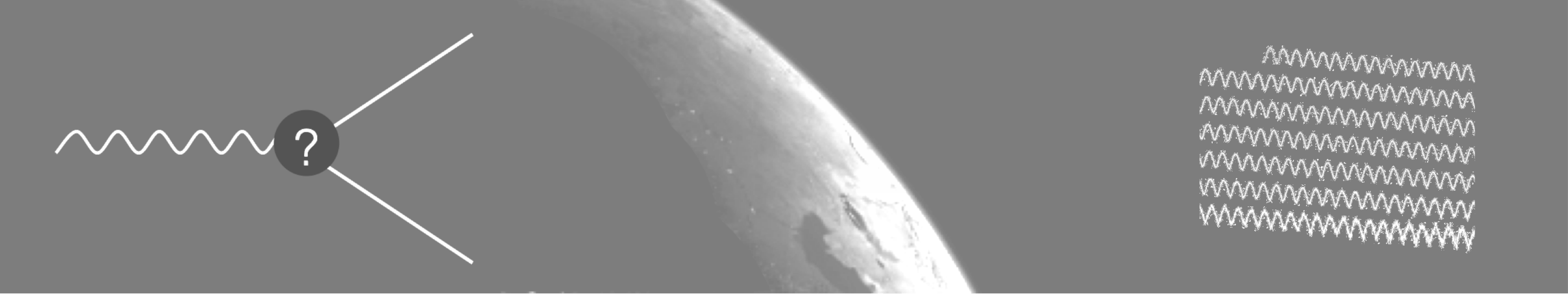
Integration kernel steeply falls with s, putting emphasis on the low-mass $R(s)$, dominated by low-multiplicity exclusive hadronic states, such as $e^+e^- \to \pi^+\pi^-$

Most recent DHMZ evaluation (2019):

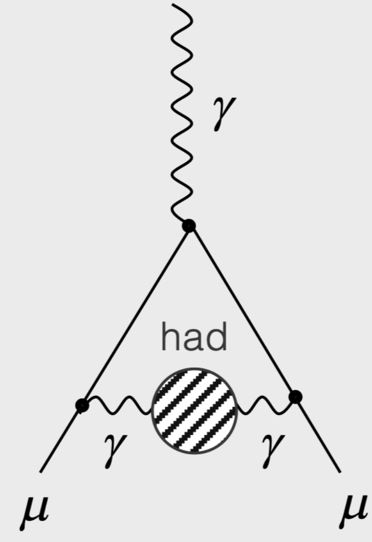
$$\Longrightarrow a_\mu^{\text{Had,LO}} = 694.0(4.0) \cdot 10^{-10}$$

[DHMZ,1908.00921 (2019), EPJ C 80, 241 (2020)]

$$\sigma(a_\mu^{\text{SM}})[10^{-10}] = 4.4 = 0.0^{\text{QED}} \oplus 0.1^{\text{EW}} \oplus 4.0^{\text{Had,LO}} \oplus 0.1^{\text{Had,N(N)LO}} \oplus 1.8^{\text{Had,LBL}}$$

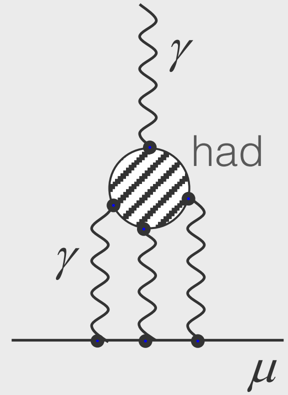


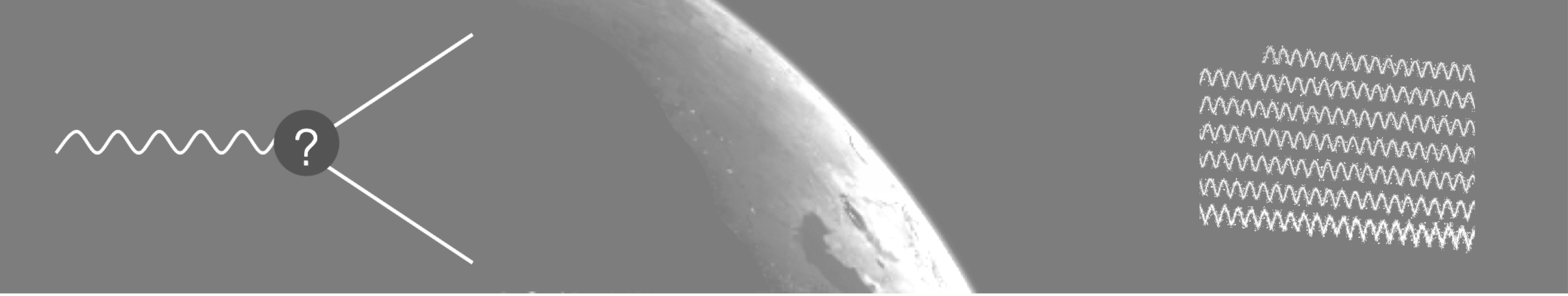
The hadronic contribution to the muon g–2



All hadronic contributions (LO, NLO, NNLO), except for light-by-light scattering (LBLS), can be obtained via dispersion relations using a mix of experimental data and perturbative QCD

The LBLS contribution is a four-point function that is currently estimated using meson models



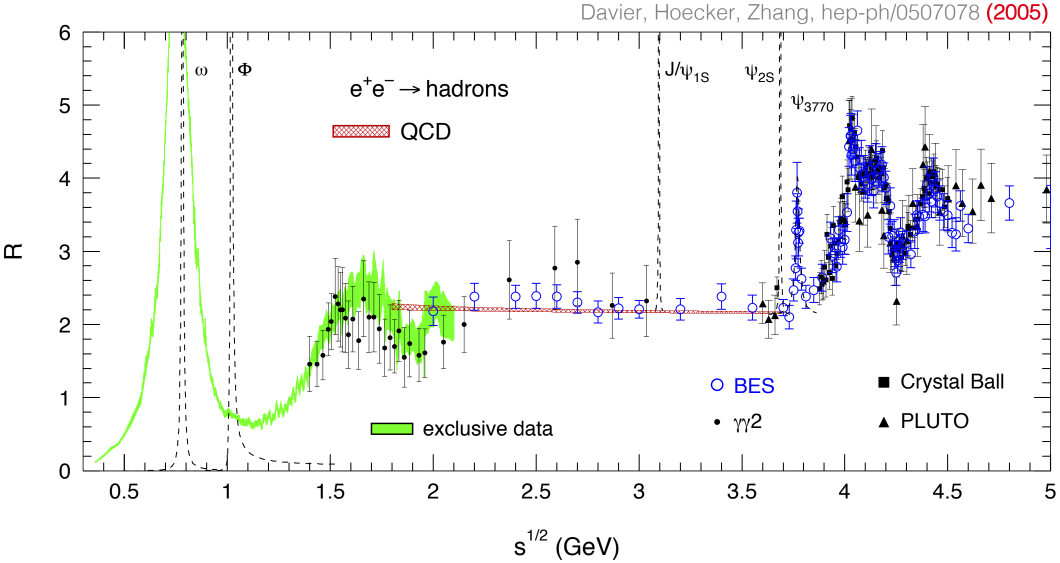


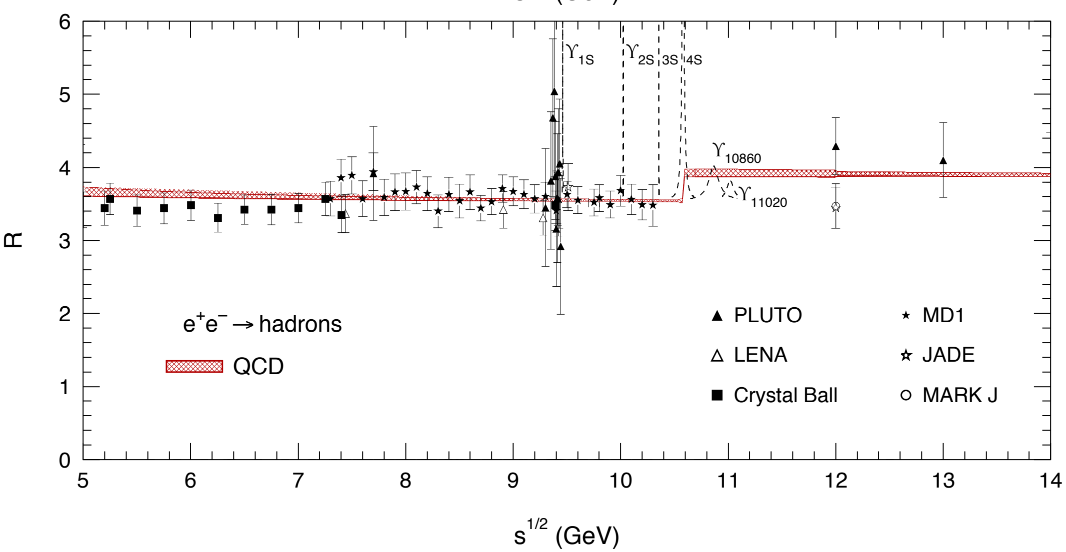
The hadronic contribution to the muon g–2

In the following, all a_μ numbers are given in units of 10^{-10}

The dispersion relation is solved using a mix of $e^+e^- \to$ had data and QCD, depending on s

Davier, Hoecker, Zhang, hep-ph/0507078 (2005)





$$a_\mu^{\mathrm{Had,LO}} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_{m_\pi^2}^{\infty} ds \frac{K(s)}{s} R(s)$$

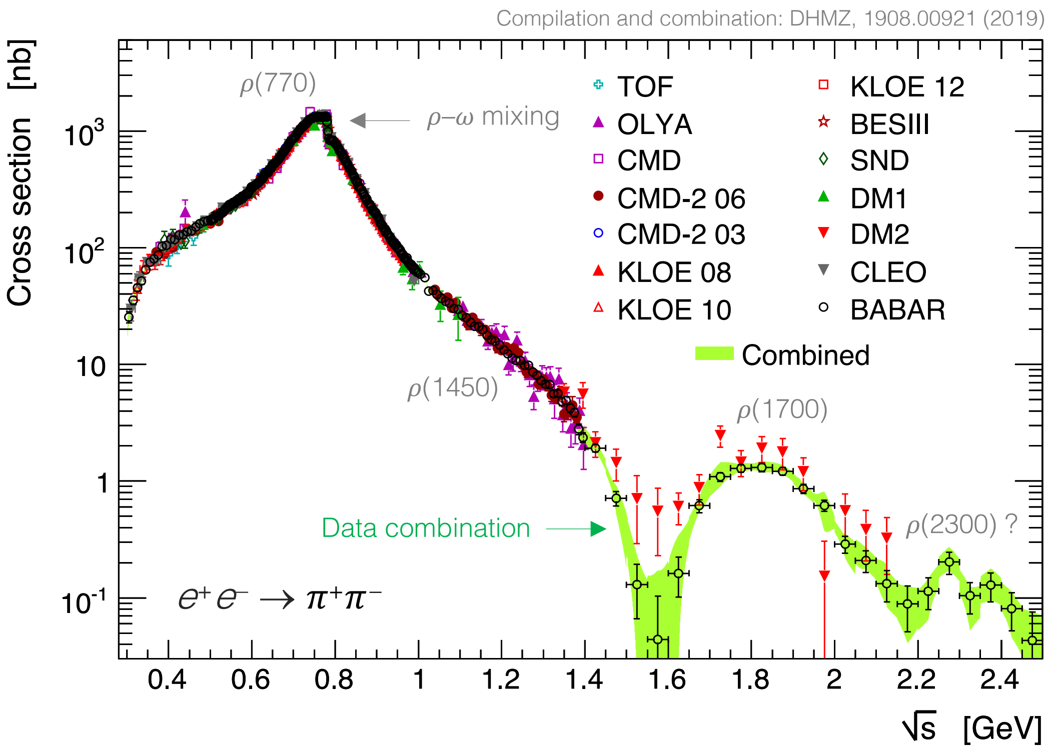
- **[$\pi^0\gamma$ – 1.8 GeV]**: sum of 34 exclusive channels; very few unmeasured channels are estimated using isospin symmetry
- **[1.8 – 3.7 GeV]**: agreement between data and QCD for *uds* continuum → more precise QCD NNLO used; J/ψ & ψ(2S) resonances from Breit-Wigners
- **[3.7 – 5.0 GeV]**: open charm pair production: use of data
- **[5.0 GeV – ∞]**: NNNLO QCD (assuming global quark-hadron duality to hold across *bb* threshold)

The dominant $e^+e^- \to \pi^+\pi^-$ contribution

$e^+e^- \to \pi^+\pi^-$ contributes 73% to $a_\mu^{\mathrm{Had,LO}}$ and 58% to total uncertainty-squared

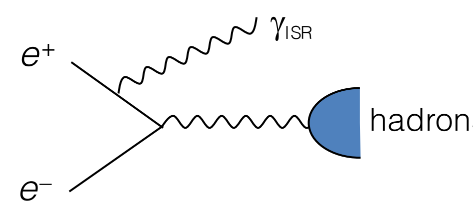
Relative to uncertainty[2] due to quadratic addition (neglecting inter-channel correlations here)

Many of the efforts of the last twenty years focused on that channel.
Most measurements dominated by systematic uncertainties



Three types of input data:

- **Energy scans**: CMD-2 ($\delta_{\mathrm{syst}} \sim 0.8\%$), SND ($\delta_{\mathrm{syst}} \sim 1.5\%$), + DM1, DM2, OLYA, TOF
- **ISR-based measurements**: BABAR ($\delta_{\mathrm{syst}} \sim 0.5\%$), BES-III ($\delta_{\mathrm{syst}} \sim 0.9\%$), KLOE ($\delta_{\mathrm{syst}} \sim 0.8$–$1.4\%$)



- **Hadronic tau decay data** via isospin symmetry (CVC): ALEPH, OPAL, CLEO, Belle ($\delta_{\mathrm{syst\text{-}combined}} \sim 0.7\%$),

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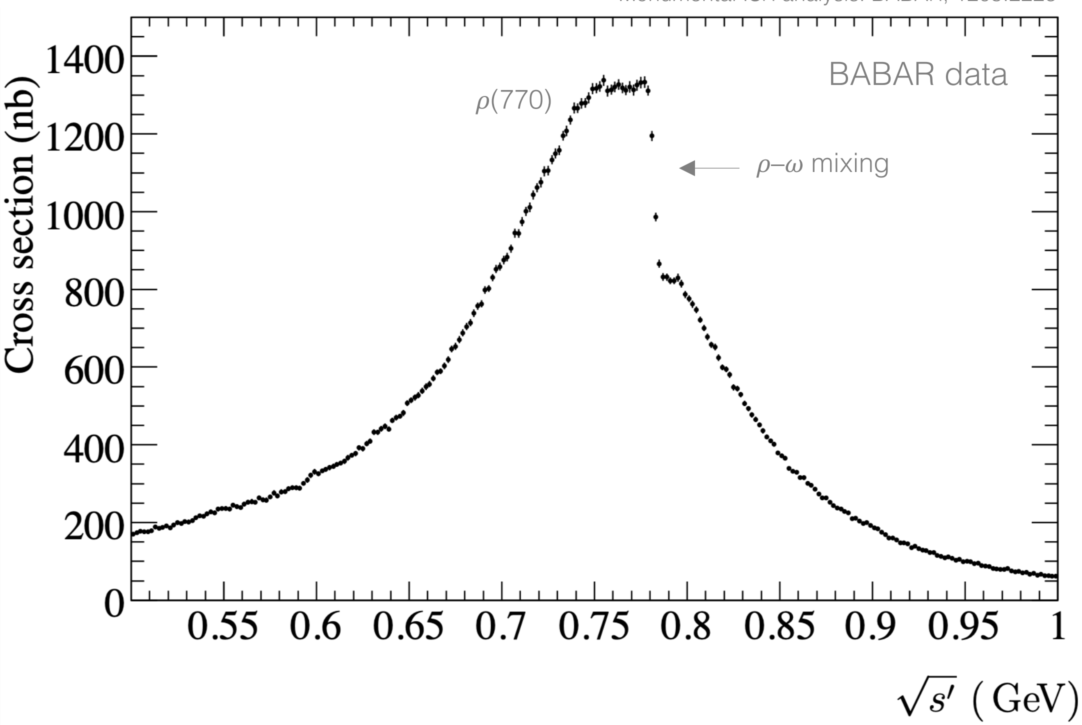
Monumental ISR analysis: BABAR, 1205.2228



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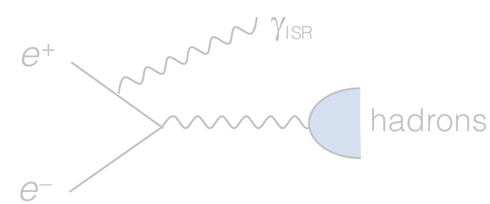

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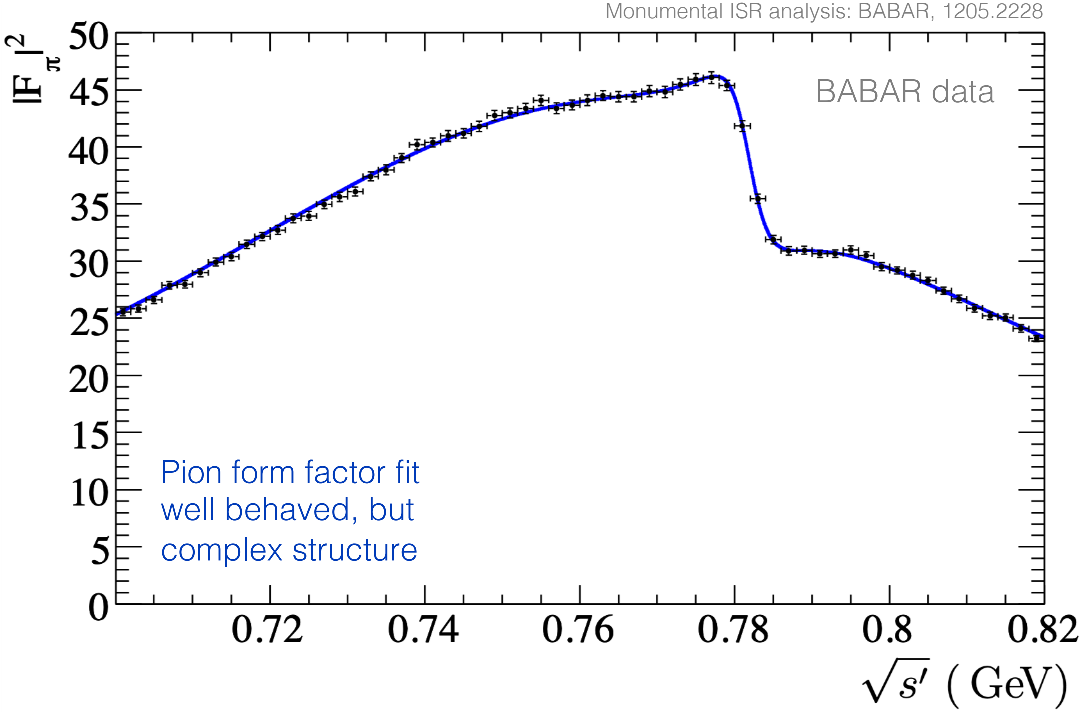

Three types of input data:

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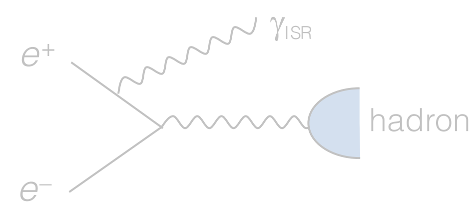

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Relative to uncertainty² due to quadratic addition (neglecting inter-channel correlations here)

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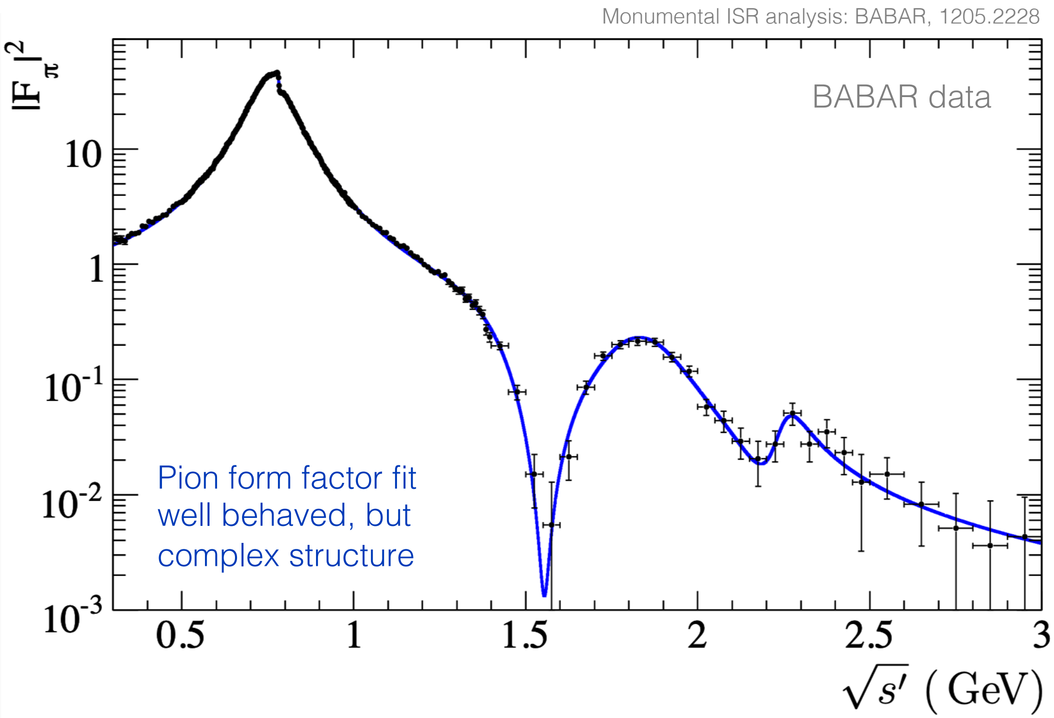
Most measurements dominated by systematic uncertainties



Three types of input data:

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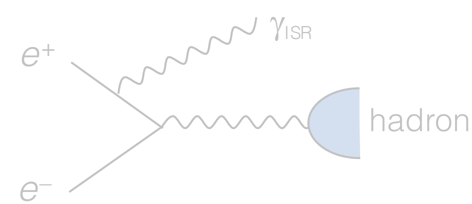

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Many of the efforts of the last twenty years focused on that channel.
Most measurements dominated by systematic uncertainties

Dominant systematic uncertainties / challenges:
(in parentheses uncertainties for best measurements)

- **Energy scan measurements** (ex. CMD-2 / 0.8%): detection efficiency, radiative corrections (0.4%), beam energy (0.3%), …
- **ISR-based measurements** (ex. BABAR / >0.5%): pion identification (0.3%), $\mu^+\mu^-$ reference (0.4%), …
- **Tau data** (ALEPH, 0.3% on normalisation): π^0 and photon reconstruction (0.2%), hadronic interactions (0.2%), …

Huge amount of precision data, but — with a close look — one notices issues…

Three types of input data:

- **Energy scans**: CMD-2 ($\delta_{\mathrm{syst}} \sim 0.8\%$), SND ($\delta_{\mathrm{syst}} \sim 1.5\%$), + DM1, DM2, OLYA, TOF
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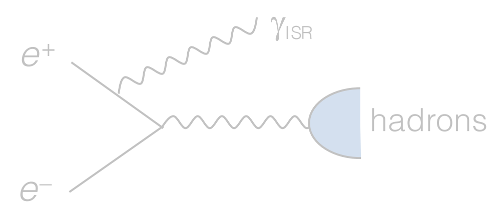


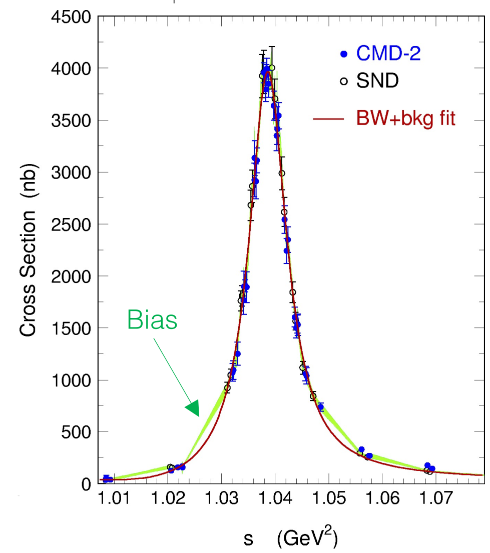
- **Hadronic tau decay data** via isospin symmetry (CVC): ALEPH, OPAL, CLEO, Belle ($\delta_{\mathrm{syst\text{-}combined}} \sim 0.7\%$),

Digression: Combining data points for integration

The integration of data points belonging to different experiments, with different within-experiment, inter-experiment and inter-channel correlated systematic uncertainties, and with different data densities requires a careful treatment

It is thereby mandatory to test the accurateness of the integration procedure in terms of central value and uncertainty using representative models with known truth

Trapezoidal rule is biased

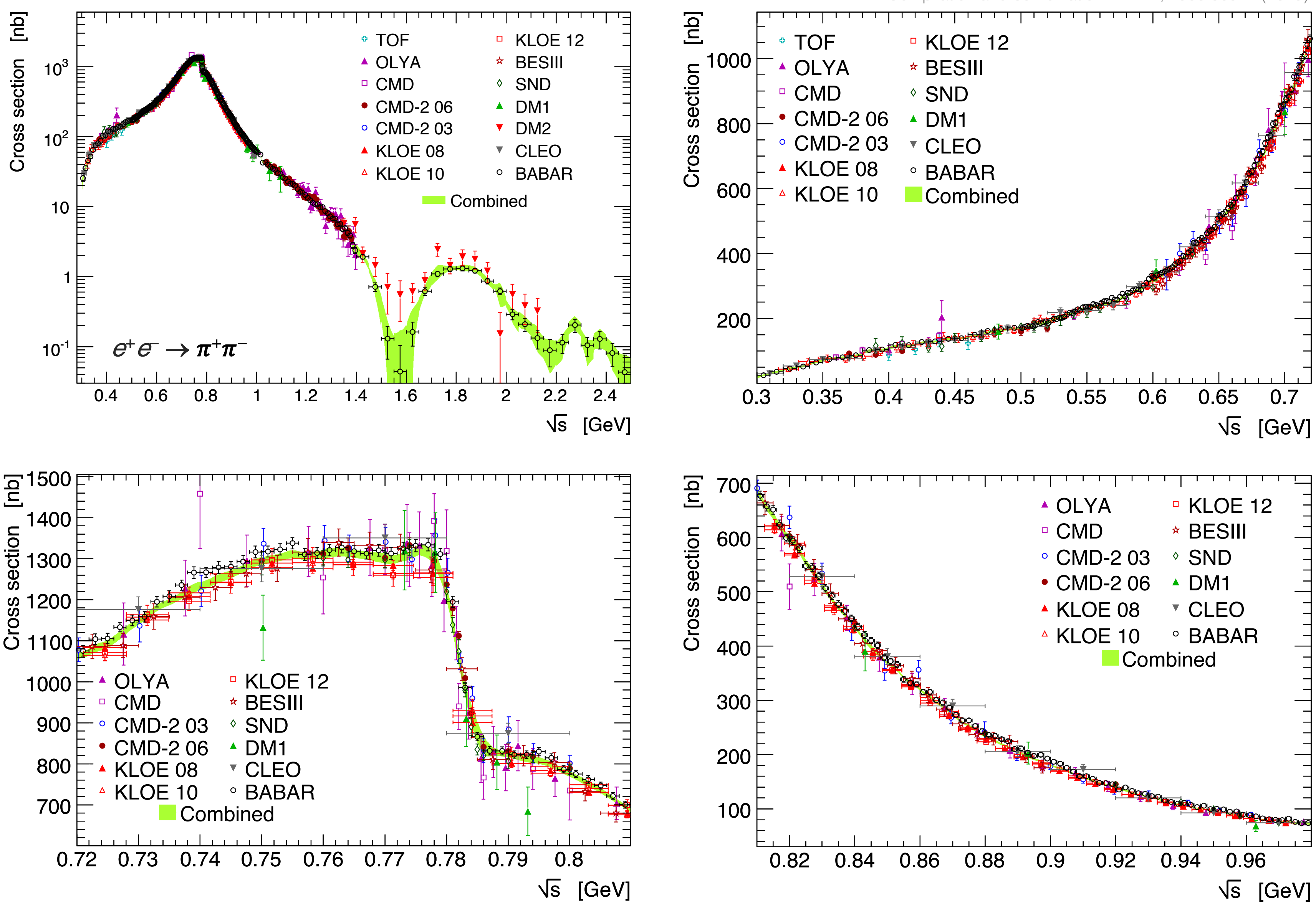


DHMZ approach for a given channel:

- Quadratic interpolation of the data points for each experiment
- Local weighted average between interpolations performed in infinitesimal bins (1 MeV); local PDG error rescaling in case of incompatibility
- Full covariance matrices: correlations between data points of an experiment (systematic errors), between experiments and channels
- Error propagation (up to dispersion integrals) using pseudo experiments
- Possible bias tested in 2π channel using a form factor model (closure test): negligible for quadratic interpolation, but not for linear model (trapezoidal rule)

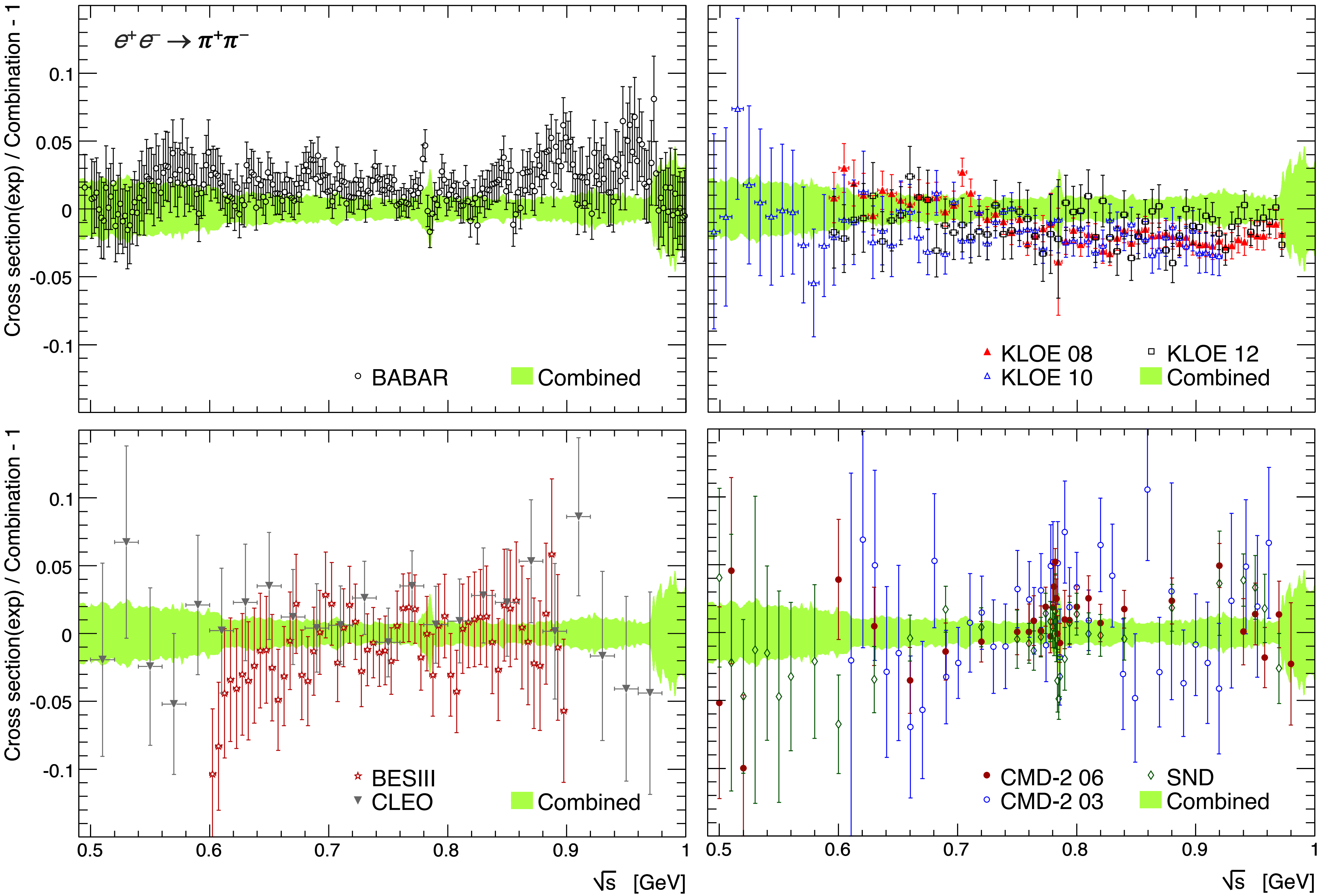
The dominant $e^+e^- \to \pi^+\pi^-$ contribution

Compilation and combination: DHMZ, 1908.00921 (2019)



The dominant $e^+e^- \to \pi^+\pi^-$ contribution — close comparison

Compilation and combination: DHMZ, 1908.00921 (2019)



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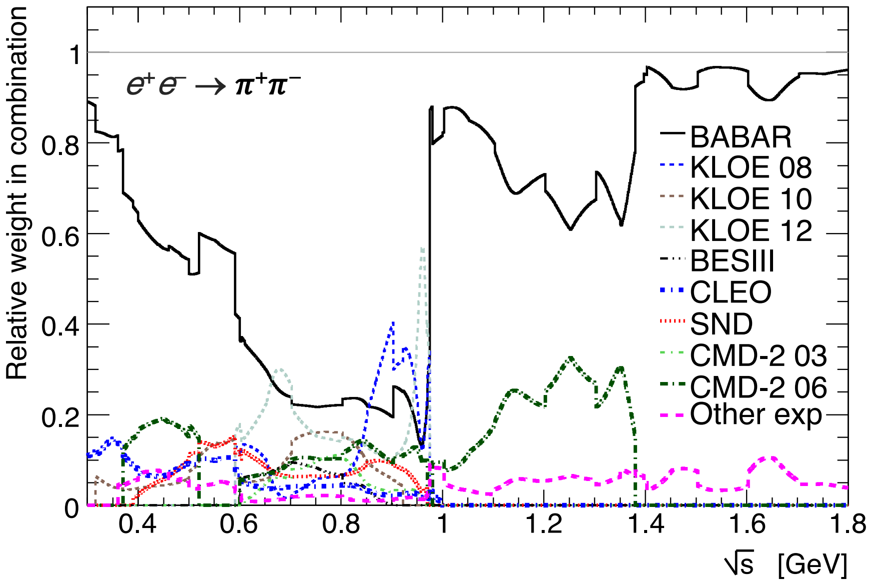
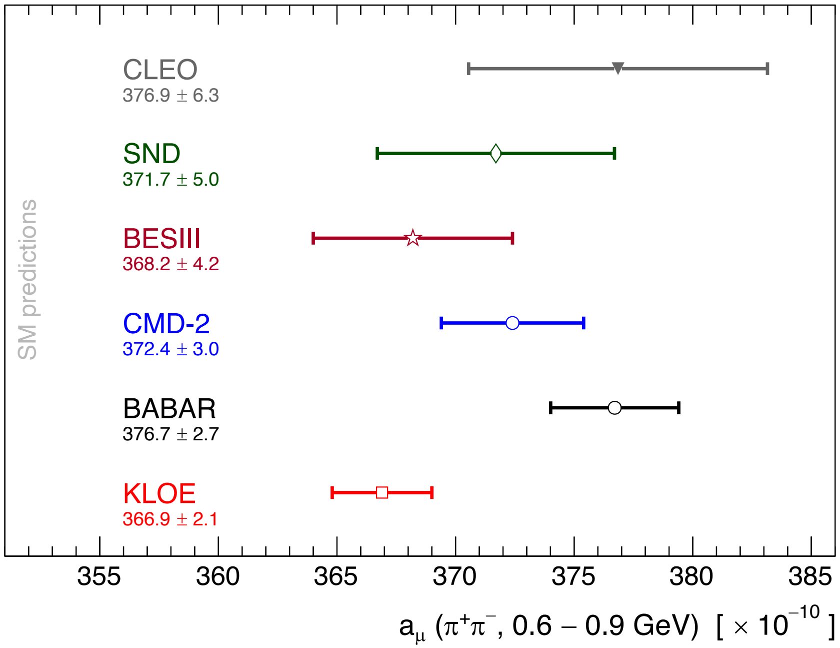
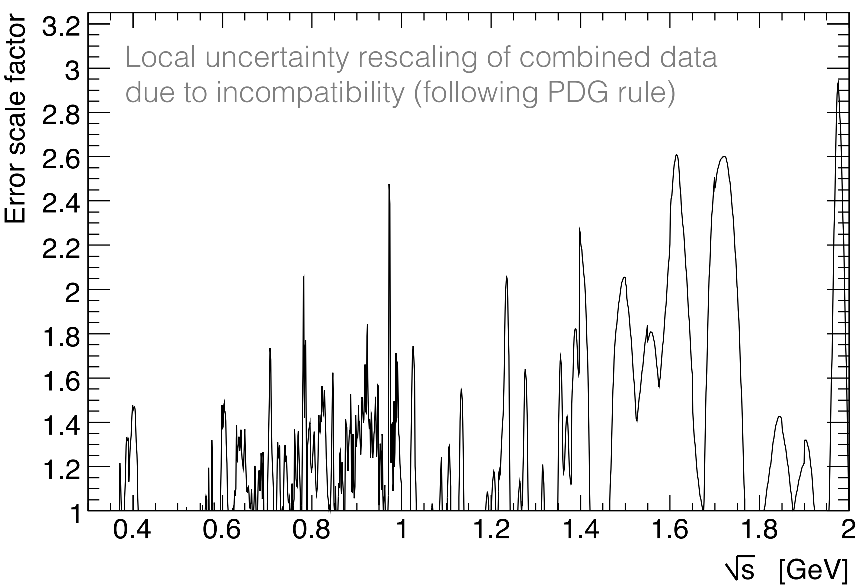






Above: comparison of results for $a_\mu^{\text{Had,LO}}[\pi^+\pi^-]$ in the interval 0.6 – 0.9 GeV for various experiments

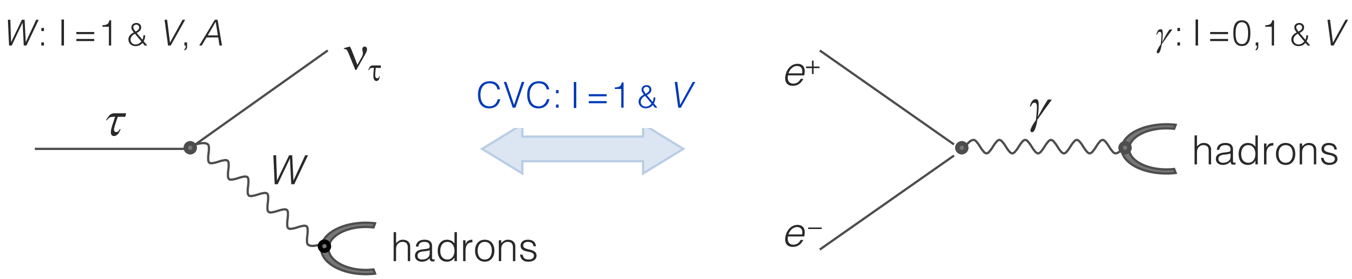
(for KLOE, the result of their public combination is shown)

The dominant $e^+e^- \to \pi^+\pi^-$ contribution — impact on a_μ

Full combination:	$a_\mu^{\mathrm{Had,LO}}[\pi^+\pi^-]$	=	507.0 ± 1.9	Local uncertainty rescaling does not fully capture global BABAR-KLOE discrepancy
Removing BABAR:		=	505.1 ± 2.1	
Removing KLOE:		=	510.6 ± 2.2	
Full combination incl. *global* error rescaling:		=	**507.9 ± 0.8 ± 3.2**	

[DHMZ 1908.00921 numbers]

It is possible to also use precise $\tau \to \pi^+\pi^0\nu$ data via isospin symmetry (CVC):



But it requires to correct for isospin breaking effects. Last estimate: 516.2 ± 2.9 ± 2.0_{IB} (3.5) [2.1σ above e^+e^-]

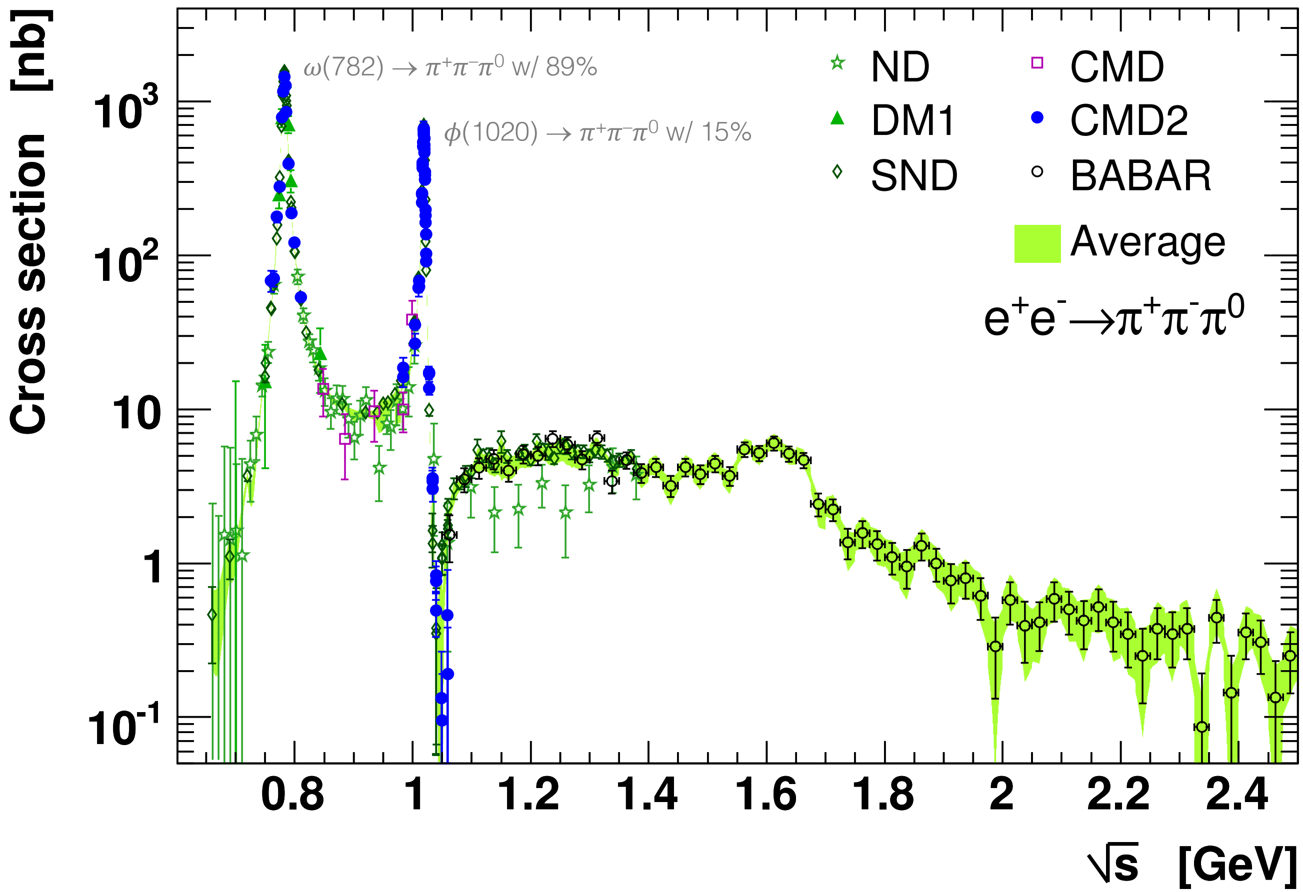
Some IB effects still under debate (eg, γ–ρ mixing). While the use of tau data helped significantly in the 1990-ies when the quality of the e^+e^- data was insufficient, with the much improved e^+e^- precision we consider the tau data less appealing for the a_μ estimate

The $e^+e^- \to \pi^+\pi^-\pi^0$ contribution

$e^+e^- \to \pi^+\pi^-\pi^0$ contributes with 6.6% to $a_\mu^{\mathrm{Had,LO}}$ and 13% to its uncertainty-squared

Good agreement among precision data (no BABAR data yet below 1.04 GeV)

Compilation and combination: DHMZ, Davier 1612.02743 (2016)



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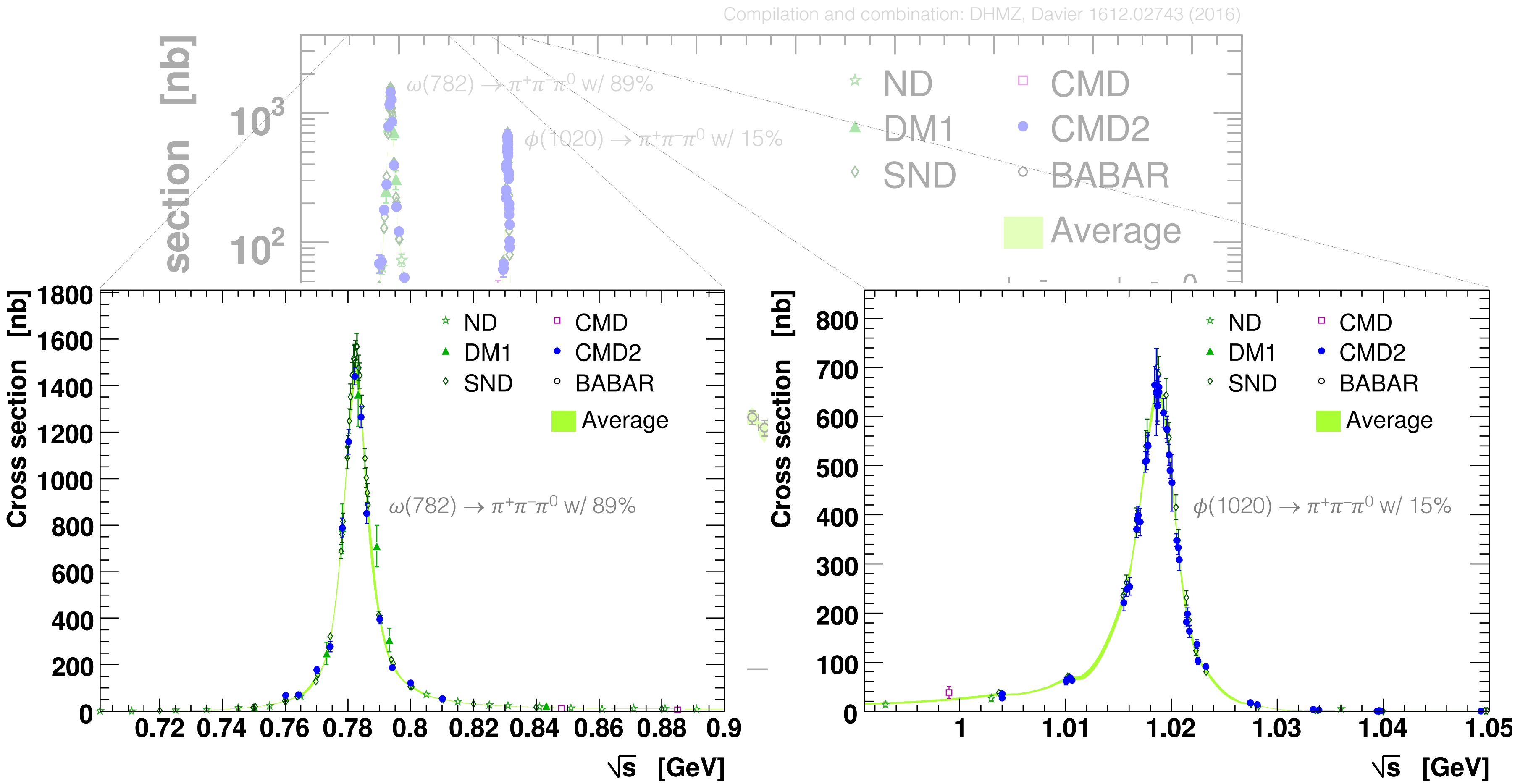
$e^+e^- \to \pi^+\pi^-\pi^0$ contributes with 6.6% to $a_\mu^{\mathrm{Had,LO}}$ and 13% to its uncertainty-squared

Good agreement among precision data (no BABAR data yet below 1.04 GeV)



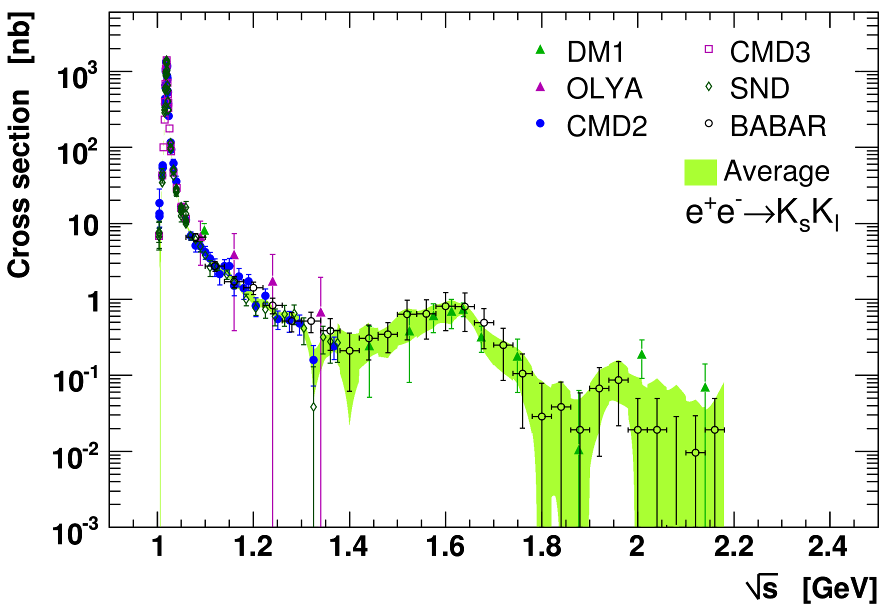
The $e^+e^- \to \phi(1020) \to K_S K_L$, K^+K^- contributions

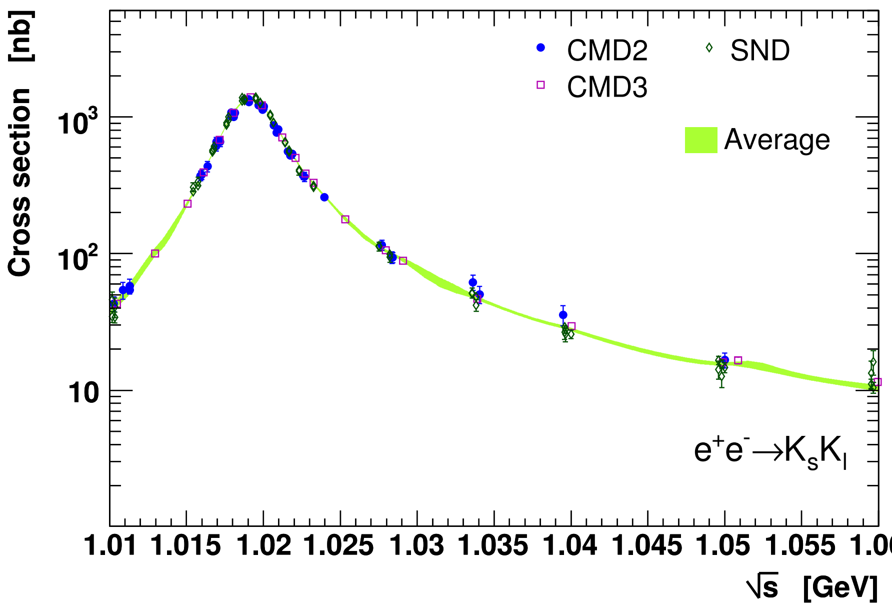
$e^+e^- \to K_S K_L$, K^+K^- contribute to 5.1% to $a_\mu^{\mathrm{Had,LO}}$ and 1.6% to its uncertainty-squared

Good consistency in $K_S K_L$ final state, new data from CMD-3 and BABAR

BABAR reconstructed K_L directly via their nuclear interactions in the electromagnetic calorimeter

Compilation and combination: DHMZ, Davier 1612.02743 (2016)





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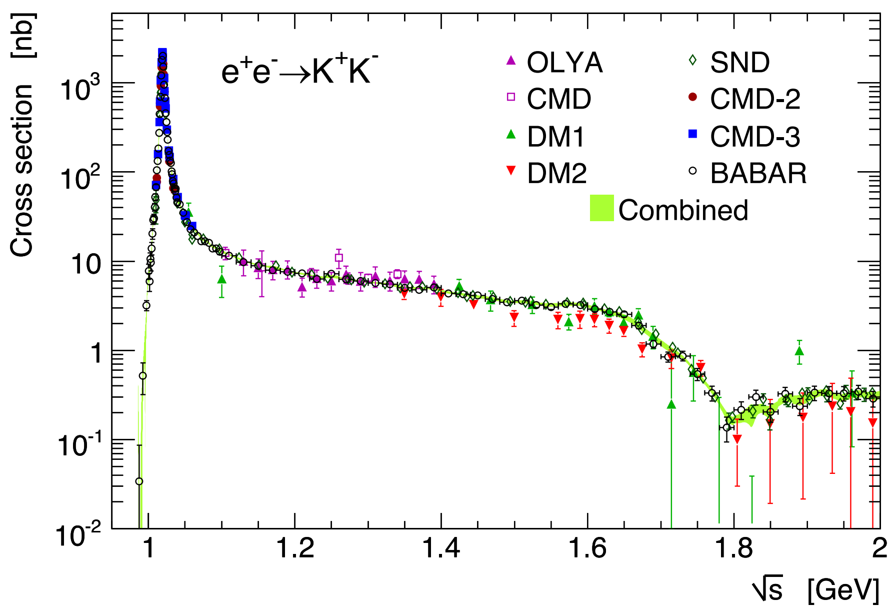
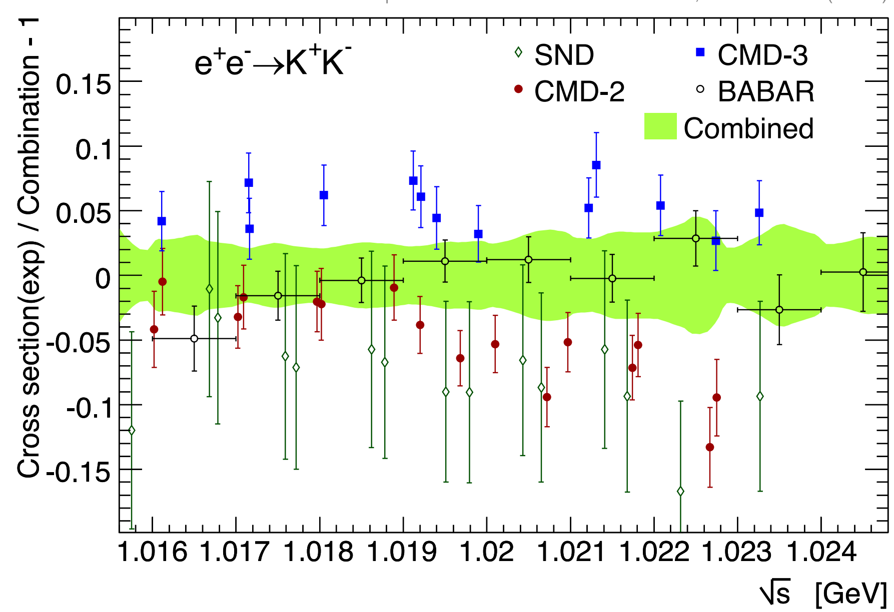
$e^+e^- \to K_S K_L$, K^+K^- contribute to 5.1% to $a_\mu^{\mathrm{Had,LO}}$ and 1.6% to its uncertainty-squared

Problems in K^+K^- channel, discrepancy between BABAR and CMD-2/3/SND (VEPP-2000)

K^+K^- final state with low kaons at threshold hard to reconstruct for energy-scan experiments. Easier in BABAR due to ISR boost

Compilation and combination: DHMZ, 1908.00921 (2019)





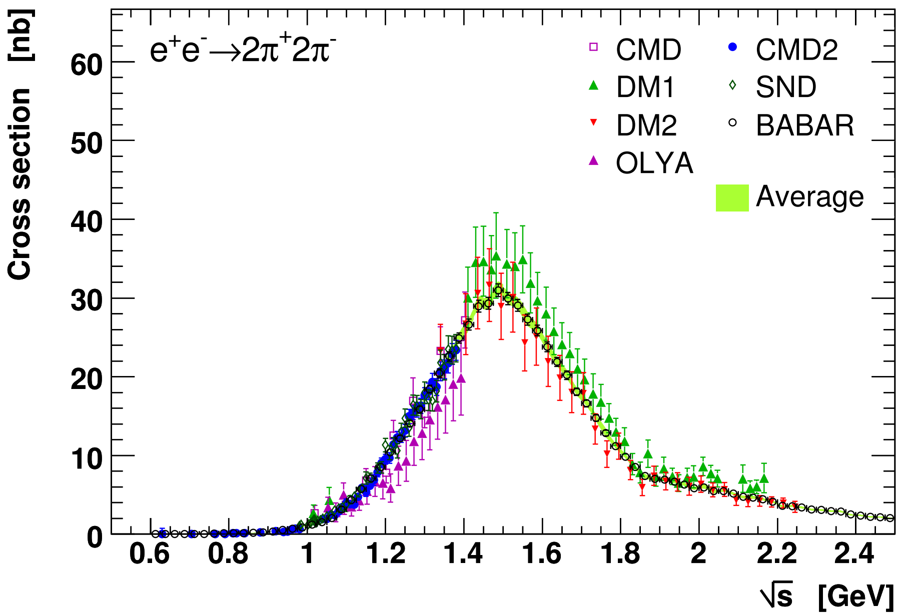
BABAR (σ_{syst} = 0.7%) higher by 5.1% compared to CMD-2 (σ_{syst} = 2.2%) and by 9.6% compared to SND (σ_{syst} = 7.1%).
New SND results agree with BABAR and supersede previous ones. New CMD-3 results are 5.5% *higher* than BABAR.
11% difference between CMD-2 and CMD-3 results, compared to 2.2% systematic uncertainties quoted
→ apply global error rescaling as for $\pi^+\pi^-$

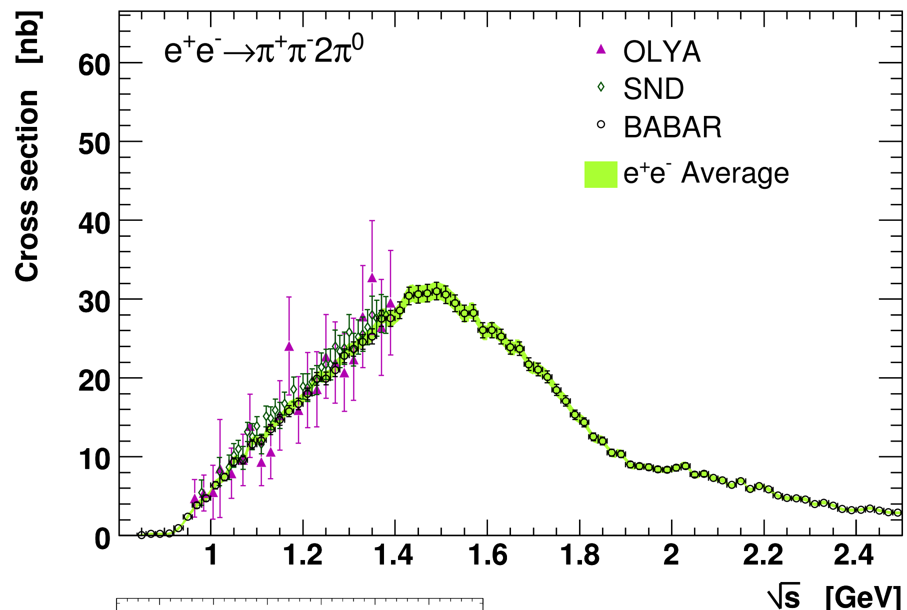
The $e^+e^- \to \pi^+\pi^-\pi^+\pi^-$, $\pi^+\pi^-\pi^0\pi^0$ contributions

The four pion channels contribute with 4.6% to $a_\mu^{\mathrm{Had,LO}}$ and 2.4% to its uncertainty-squared

$\pi^+\pi^-\pi^+\pi^-$ channel pretty well known since long, but $\pi^+\pi^-\pi^0\pi^0$ challenging. Discrepancies in earlier data, but recent precise (~3.1% systematic) measurement from BABAR much improving

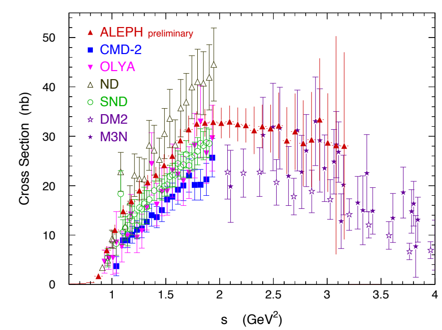
Compilation and combination: DHMZ,,1612.02743 (2016)





Situation before BABAR data; then also tau data were used

$a_\mu^{\mathrm{Had,LO}}$ increased by 1.4 (absolute) with BABAR & uncertainty < halved

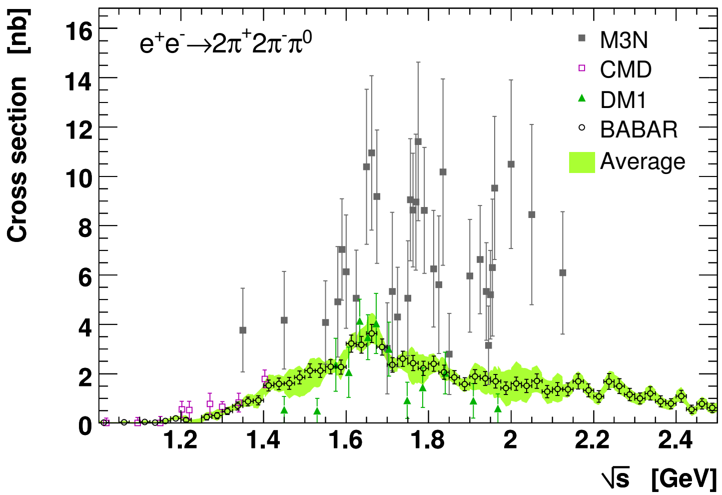


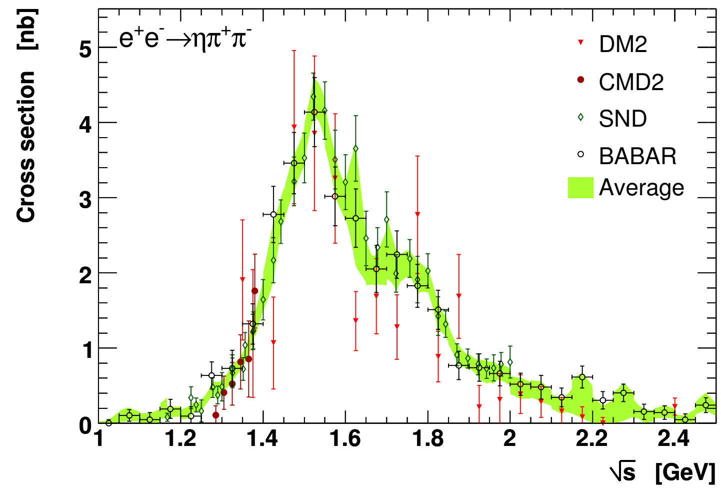
Tau data from ALEPH significantly above BABAR

The $e^+e^- \to \geq 5\pi$ contributions

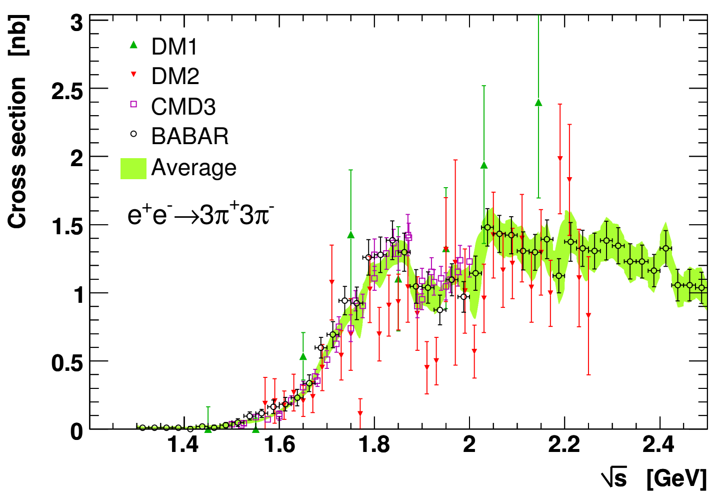
$\geq 5\pi$ channels (incl. $\eta\pi\pi$) contribute with 0.7% to $a_\mu^{\mathrm{Had,LO}}$ and 0.2% to its uncertainty-squared

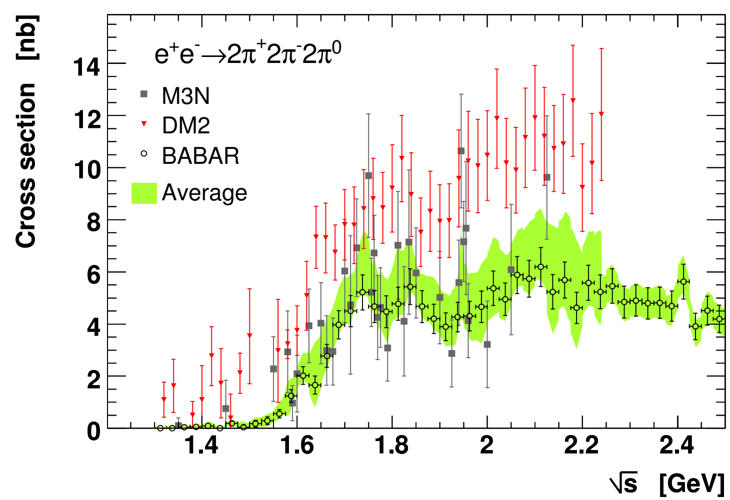
Also here large improvement from BABAR ISR data, problems in older datasets





Compilation and combination: DHMZ, 1612.02743 (2016)

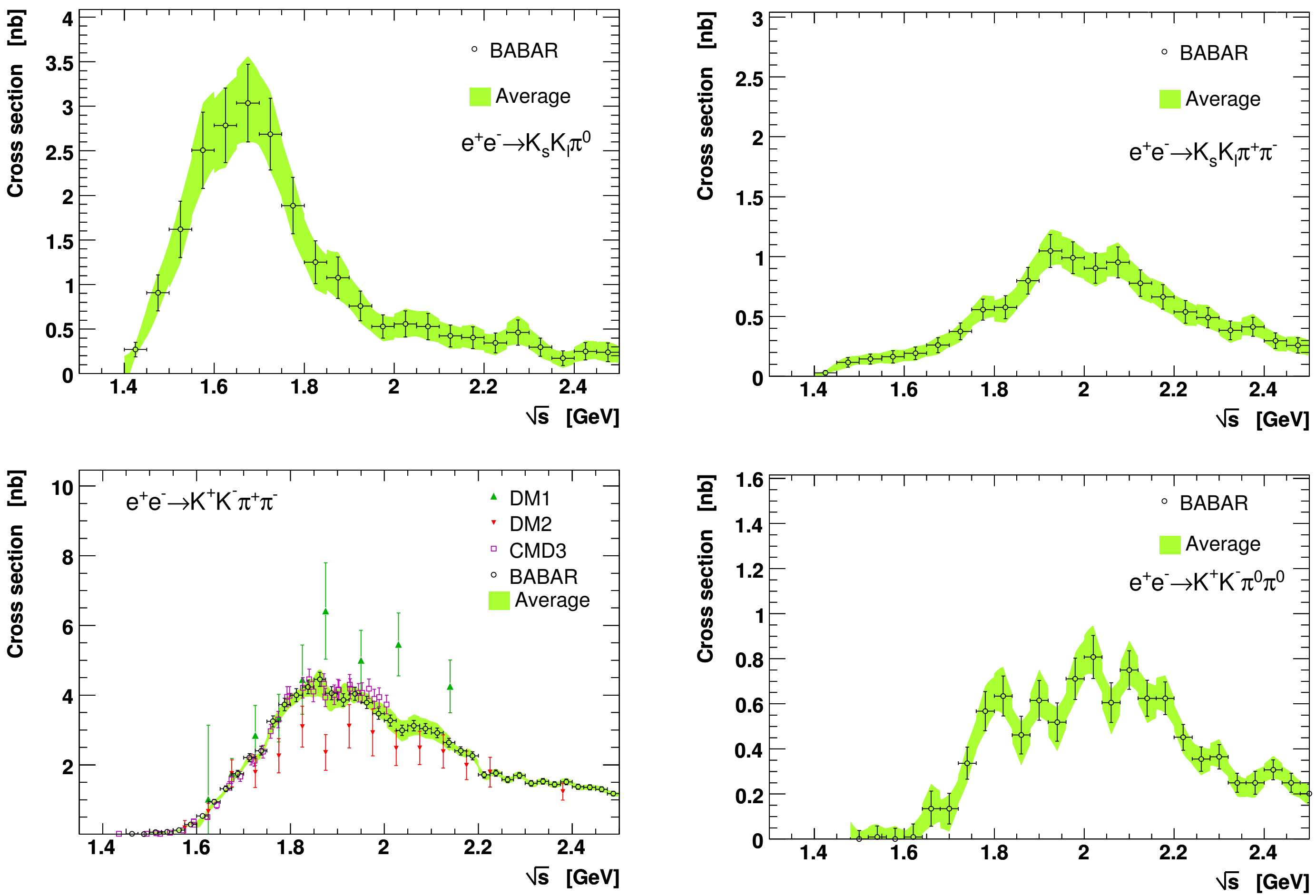




The $e^+e^- \to KK\pi(\pi\pi)$ contributions (many charge combinations)

Past analyses suffered from missing final states that were estimated by symmetry arguments

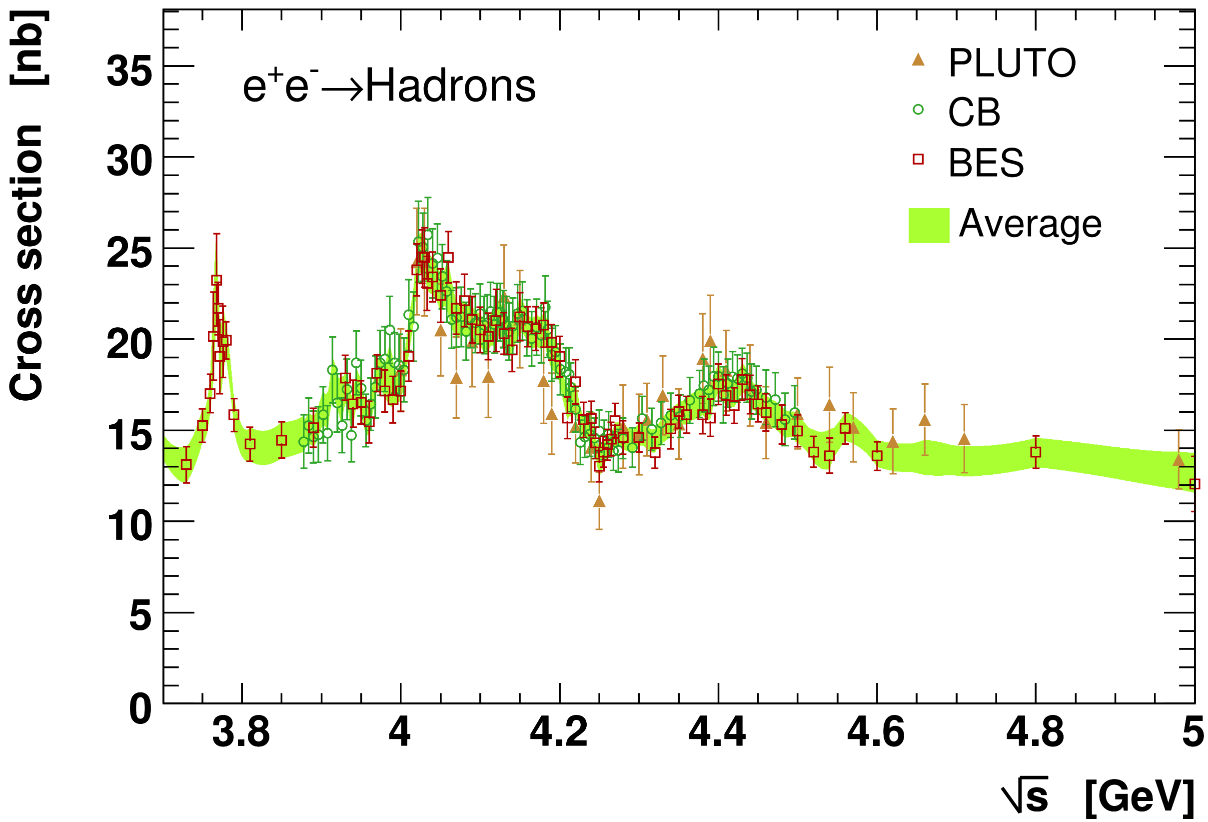
Systematic measurement of exclusive processes by BABAR completes the $KK\pi$ and (almost) all $KK\pi\pi$ final states. Their sum contributes 0.4% to $a_\mu^{\mathrm{Had,LO}}$ and 0.1% to uncertainty-squared



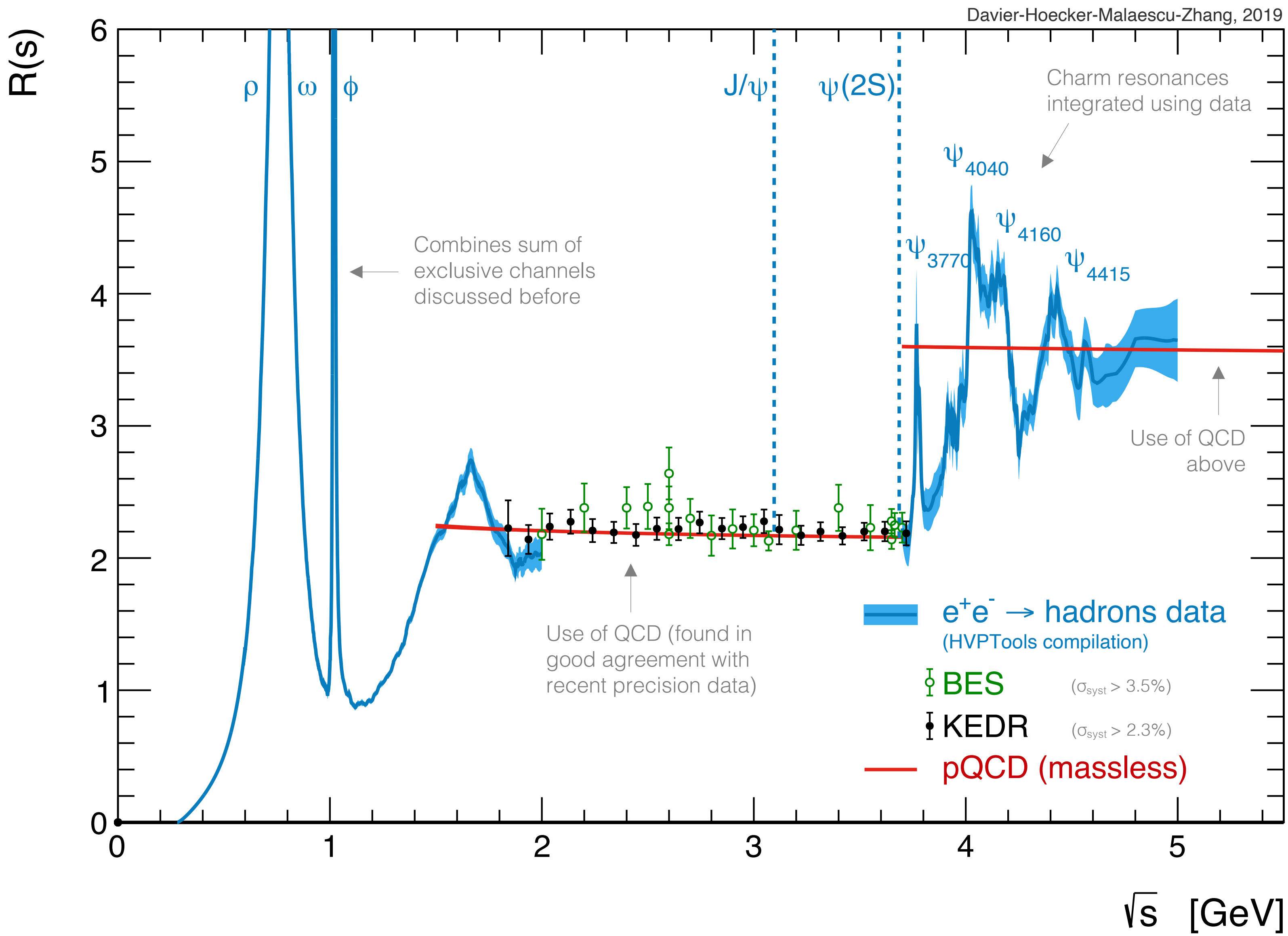
Selection of measurements by BABAR and others

3.7–5.0 GeV region contributes with 1.1% to $a_\mu^{\mathrm{Had,LO}}$ and 0.6% to its uncertainty-squared

Good agreement between measurements. Precision dominated by BES (σ_{syst} ~ 3.5%)



The big picture



Full compilation in numbers

DHMZ, 1908.00921 (2019)

Legend: First error statistical, second channel-specific systematic, third common systematic (correlated)
For R_{QCD}, uncertainties are due to: α_S, NNNLO truncation, resummation (FOPT vs. CIPT), quark masses

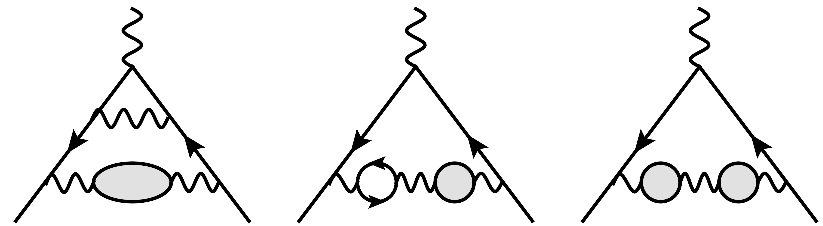
Channel	$a_\mu^{\text{had,LO}}$ $[10^{-10}]$
$\pi^0\gamma$	$4.41 \pm 0.06 \pm 0.04 \pm 0.07$
$\eta\gamma$	$0.65 \pm 0.02 \pm 0.01 \pm 0.01$
$\pi^+\pi^-$	$507.85 \pm 0.83 \pm 3.23 \pm 0.55$
$\pi^+\pi^-\pi^0$	$46.21 \pm 0.40 \pm 1.10 \pm 0.86$
$2\pi^+2\pi^-$	$13.68 \pm 0.03 \pm 0.27 \pm 0.14$
$\pi^+\pi^-2\pi^0$	$18.03 \pm 0.06 \pm 0.48 \pm 0.26$
$2\pi^+2\pi^-\pi^0$ (η excl.)	$0.69 \pm 0.04 \pm 0.06 \pm 0.03$
$\pi^+\pi^-3\pi^0$ (η excl.)	$0.49 \pm 0.03 \pm 0.09 \pm 0.00$
$3\pi^+3\pi^-$	$0.11 \pm 0.00 \pm 0.01 \pm 0.00$
$2\pi^+2\pi^-2\pi^0$ (η excl.)	$0.71 \pm 0.06 \pm 0.07 \pm 0.14$
$\pi^+\pi^-4\pi^0$ (η excl., isospin)	$0.08 \pm 0.01 \pm 0.08 \pm 0.00$
$\eta\pi^+\pi^-$	$1.19 \pm 0.02 \pm 0.04 \pm 0.02$
$\eta\omega$	$0.35 \pm 0.01 \pm 0.02 \pm 0.01$
$\eta\pi^+\pi^-\pi^0$(non-ω,ϕ)	$0.34 \pm 0.03 \pm 0.03 \pm 0.04$
$\eta 2\pi^+2\pi^-$	$0.02 \pm 0.01 \pm 0.00 \pm 0.00$
$\omega\eta\pi^0$	$0.06 \pm 0.01 \pm 0.01 \pm 0.00$
$\omega\pi^0$ ($\omega \to \pi^0\gamma$)	$0.94 \pm 0.01 \pm 0.03 \pm 0.00$
$\omega 2\pi$ ($\omega \to \pi^0\gamma$)	$0.07 \pm 0.00 \pm 0.00 \pm 0.00$
ω (non-$3\pi, \pi\gamma, \eta\gamma$)	$0.04 \pm 0.00 \pm 0.00 \pm 0.00$

$$a_\mu^{\text{Had,LO}} = (694.0 \pm 4.0) \cdot 10^{-10}$$

Channel	$a_\mu^{\text{had,LO}}$ $[10^{-10}]$
K^+K^-	$23.08 \pm 0.20 \pm 0.33 \pm 0.21$
$K_S K_L$	$12.82 \pm 0.06 \pm 0.18 \pm 0.15$
ϕ (non-$K\overline{K}, 3\pi, \pi\gamma, \eta\gamma$)	$0.05 \pm 0.00 \pm 0.00 \pm 0.00$
$K\overline{K}\pi$	$2.45 \pm 0.05 \pm 0.10 \pm 0.06$
$K\overline{K}2\pi$	$0.85 \pm 0.02 \pm 0.05 \pm 0.01$
$K\overline{K}\omega$	$0.00 \pm 0.00 \pm 0.00 \pm 0.00$
$\eta\phi$	$0.33 \pm 0.01 \pm 0.01 \pm 0.00$
$\eta K\overline{K}$ (non-ϕ)	$0.01 \pm 0.01 \pm 0.01 \pm 0.00$
$\omega 3\pi$ ($\omega \to \pi^0\gamma$)	$0.06 \pm 0.01 \pm 0.01 \pm 0.01$
7π ($3\pi^+3\pi^-\pi^0$ + estimate)	$0.02 \pm 0.00 \pm 0.01 \pm 0.00$
J/ψ (BW integral)	6.20 ± 0.11
$\psi(2S)$ (BW integral)	1.56 ± 0.05
R data $[3.7 - 5.0]$ GeV	$7.29 \pm 0.05 \pm 0.30 \pm 0.00$
R_{QCD} $[1.8 - 3.7\text{ GeV}]_{uds}$	$33.45 \pm 0.28 \pm 0.65_{\text{dual}}$
R_{QCD} $[5.0 - 9.3\text{ GeV}]_{udsc}$	6.86 ± 0.04
R_{QCD} $[9.3 - 12.0\text{ GeV}]_{udscb}$	1.20 ± 0.01
R_{QCD} $[12.0 - 40.0\text{ GeV}]_{udscb}$	1.64 ± 0.00
R_{QCD} $[> 40.0\text{ GeV}]_{udscb}$	0.16 ± 0.00
R_{QCD} $[> 40.0\text{ GeV}]_{t}$	0.00 ± 0.00
Sum	$694.0 \pm 1.0 \pm 3.5 \pm 1.6 \pm 0.1_\psi \pm 0.7_{\text{QCD}}$

NLO two-point correlation contributions to $a_\mu^{\mathrm{Had,NLO}}$ can be computed akin to the LO part via (a sum of) dispersion relations

$$a_\mu^{\mathrm{Had,NLO}(i)} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_{m_\pi^2}^{\infty} ds \, \frac{K^{(i)}(s)}{s} R(s)$$



Each diagram corresponds to specific kernel function $K^{(i)}$

$$\Longrightarrow a_\mu^{\mathrm{Had,NLO}} = (-9.87 \pm 0.09) \cdot 10^{-10}$$

Kurz et al, 1511.08222 (2015)

Kurz et al, 1511.08222 (2015)

NNLO two-point function corrections have also been computed: $a_\mu^{\mathrm{Had,NNLO}} = (1.24 \pm 0.01) \cdot 10^{-10}$

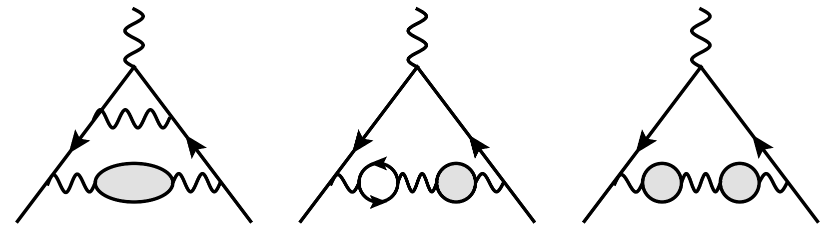
Higher order hadronic terms

NLO two-point correlation contributions to $a_\mu^{\text{Had,NLO}}$ can be computed akin to the LO part via (a sum of) dispersion relations

$$a_\mu^{\text{Had,NLO}(i)} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_{m_\pi^2}^{\infty} ds \, \frac{K^{(i)}(s)}{s} R(s)$$

$$\Longrightarrow a_\mu^{\text{Had,NLO}} = (-9.87 \pm 0.09) \cdot 10^{-10}$$

Kurz et al, 1511.08222 (2015)



Each diagram corresponds to specific kernel function $K^{(i)}$

Kurz et al, 1511.08222 (2015)

NNLO two-point function corrections have also been computed: $a_\mu^{\text{Had,NNLO}} = (1.24 \pm 0.01) \cdot 10^{-10}$

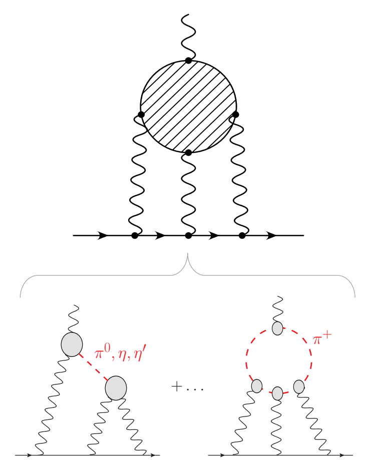
The four-point hadronic LBL scattering contribution, however, cannot be obtained this way and models are used instead

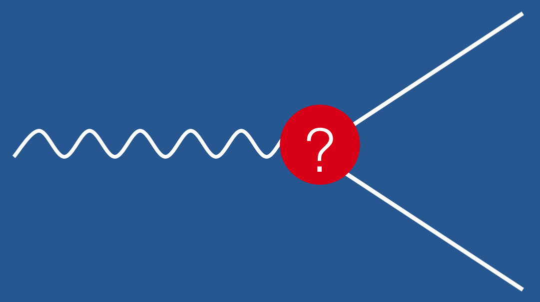
Calculation uses hadronic models with π^0, $\eta^{(\prime)}$, … pole insertions and $\pi^\pm$ loops in the large-N_C limit, Lattice QCD offers promising alternative

$$\Longrightarrow a_\mu^{\text{Had,LBL}} = (9.2 \pm 1.8) \cdot 10^{-10}$$

Muon g–2 Theory White Paper, 2006.04822 (2020)

Educated guess (other groups find smaller / larger uncertainty)





Muon $g-2$ summary

Summing all contributions [$\cdot\ 10^{-10}$]:

$$a_\mu^{\mathrm{QED}} = 11\,658\,471.895 \pm 0.008$$

$$a_\mu^{\mathrm{EW}} = 15.36 \pm 0.10$$

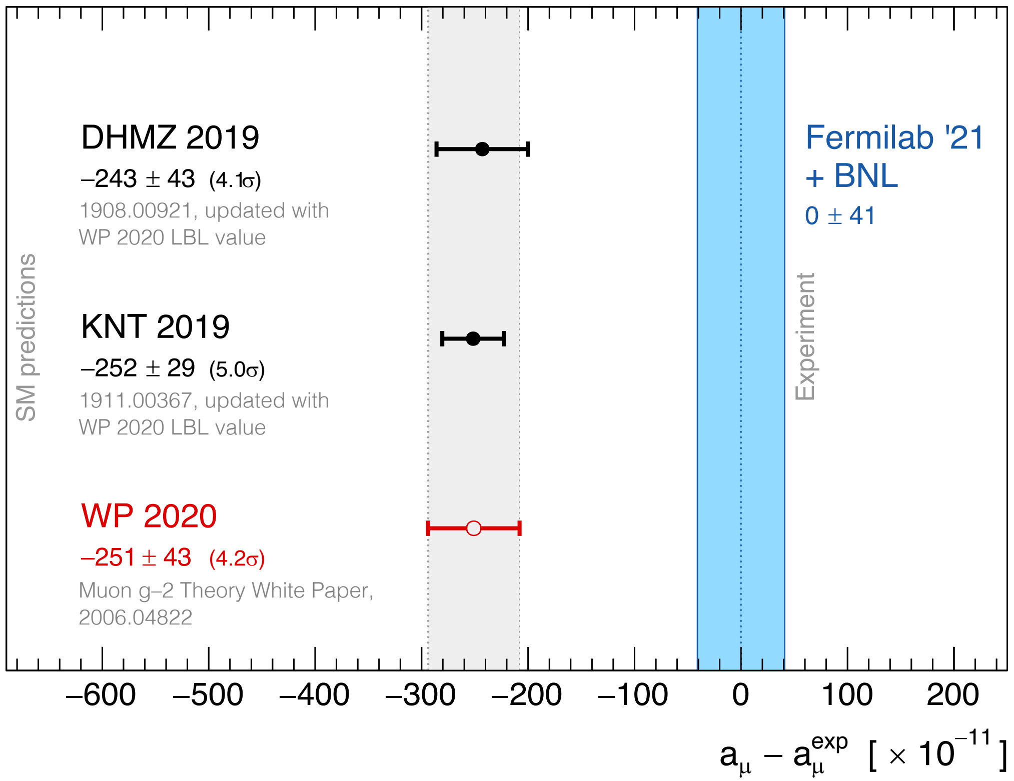
$$a_\mu^{\mathrm{Had,LO}} = 694.0 \pm 4.0$$

$$a_\mu^{\mathrm{Had,NLO}} = -9.87 \pm 0.09$$

$$a_\mu^{\mathrm{Had,NNLO}} = 1.24 \pm 0.01$$

$$a_\mu^{\mathrm{Had,LBL}} = 9.2 \pm 1.8$$

$$a_\mu^{\mathrm{SM}} = (11\,659\,181.8 \pm 4.3) \cdot 10^{-10}$$



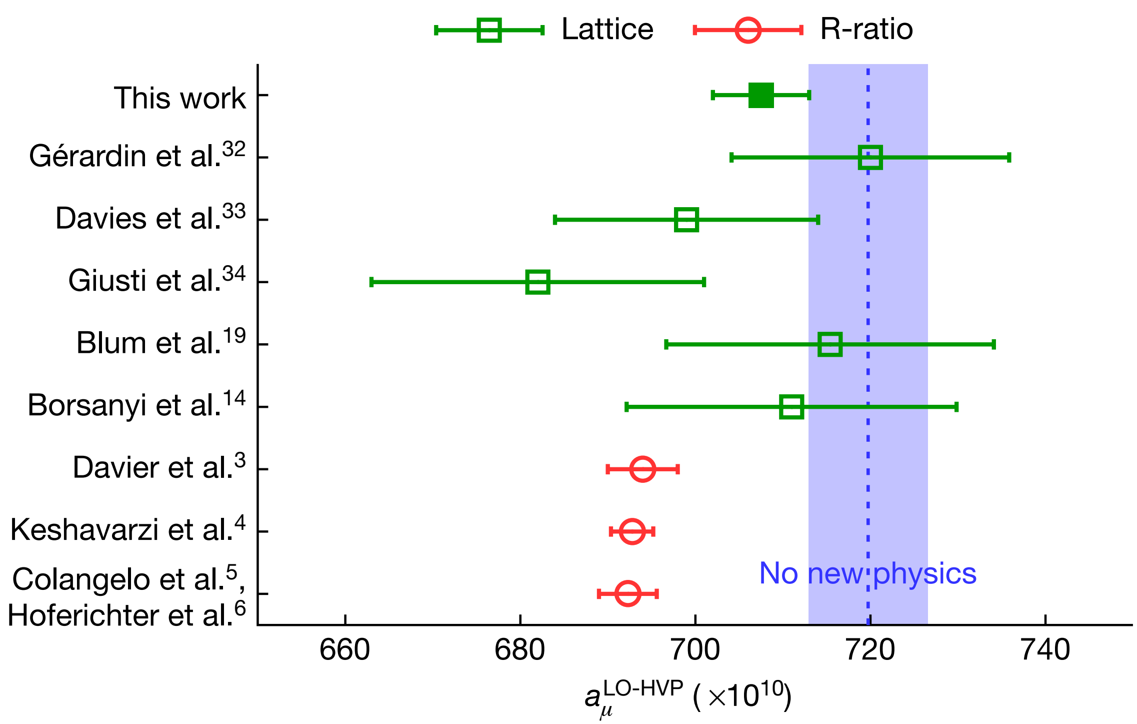
$$\Delta a_\mu = a_\mu^{\mathrm{Exp}} - a_\mu^{\mathrm{SM}} = 24.3 \pm 4.1_{\mathrm{exp}} \pm 4.3_{\mathrm{SM}} \;\; (\pm 6.0_{\mathrm{tot}})$$ → 4.1σ level, using WP 2020 it is 4.2σ

Lattice QCD allows to directly compute the real part of the two-point correlation function without invoking the resonances occurring on the imaginary axis

The required sub-percent precision is, however, a huge challenge, requiring to master systematic and extrapolation uncertainties

The BMW collaboration recently published* $a_\mu^{\mathrm{Had,LO}} = (707.5 \pm 5.5) \cdot 10^{-10}$, which reduces the discrepancy with experiment to $\Delta a_\mu = 10.8 \pm 4.1_{\mathrm{exp}} \pm 5.8_{\mathrm{SM}}\ \ (= 1.9\sigma)$

2.0σ towards DHMZ dispersive approach

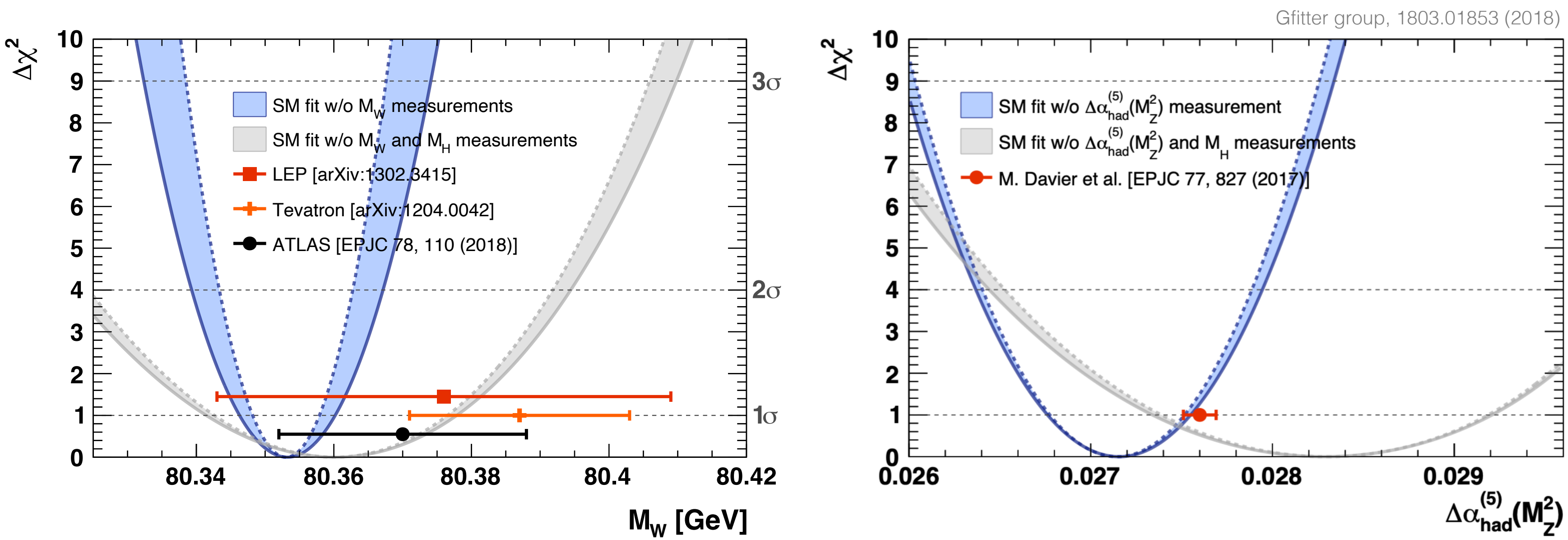


*BMW, Nature 593, 51 (2021)

Digression : Conflict with Electroweak fit ?

Same dispersive approach used to determine hadronic contribution to $\alpha_{QED}(m_Z)$ which is a critical ingredient in the global electroweak (EW) fit

If $a_\mu^{\text{Had,LO}}$ were 3–4% larger to accommodate experiment, would the EW fit break down?



– While the $\pi^+\pi^-$ channel contributes 73% to $a_\mu^{\text{Had,LO}}$ it is only 13% in the case of $\Delta\alpha_{\text{had}}(m_Z)$ → a 5% change in the former would lead to a shift in the latter value by 1.7σ

– A shift of $\Delta\alpha_{\text{had}}(m_Z)$ by 1σ leads to Δm_W = 2.4 MeV → no problem

Studies in the literature: Crivellin et al, 2003.04886; Keshavarzi et al., 2006.12666 ;de Rafael, 2006.13880; Malaescu & Schott, 2008.08107; Colangelo et al, 2010.07943

Digression : Can it be real ?

The absolute size of the effect $\Delta a_\mu = 24.3 \pm 6.0$ is large compared to EW contribution of 15.4
(but some cancellation among bosons in latter contribution)

- **Generic decoupling new physics** predicts: $a_\mu^{\mathrm{NP}} \sim \mathrm{C} \cdot \left(\frac{m_\mu}{m_{\mathrm{NP}}}\right)^2$ [Jegerlehner, Nyffeler, 0902.3360]

  Here: $m_{\mathrm{NP}} \sim 2\ \mathrm{TeV}$ for $C = 1$, $m_{\mathrm{NP}} \sim 100\ \mathrm{GeV}$ for $\mathrm{C} = \frac{\alpha}{\pi}$ (natural strength), $m_{\mathrm{NP}} \sim 5\ \mathrm{GeV}$ for $C = \left(\frac{\alpha}{\pi}\right)^2$

- Generic **SUSY** predicts: $a_\mu^{\mathrm{SUSY}} \sim \mathrm{sign}(\mu) \cdot (13 \cdot 10^{-10}) \cdot \left(\frac{100\ \mathrm{GeV}}{m_{\mathrm{SUSY}}}\right)^2 \cdot \tan\beta$

  – In constrained SUSY models, Δa_μ cannot be reconciled with the non-observation of strongly produced sparticles at the LHC [de Vries et al, MasterCode, 1504.03260]

  – However, general models such as the pMSSM can still accommodate Δa_μ with light neutralinos, charginos and sleptons, not fully excluded by the LHC

- A "**dark photon**" (γ') coupling to SM via mixing with photon may give: $a_\mu^{\gamma'} \sim \frac{\alpha}{2\pi}\varepsilon^2 F(m_{\gamma'})$

  – Δa_μ is accommodated for coupling strength $\varepsilon \sim 0.1\text{–}0.2\%$ and mass $m_{\gamma'} \sim 10 - 100\ \mathrm{MeV}$

  – Recent experimental constraints disfavour such a scenario if the dark photon decays primarily into charged lepton pairs

Many, many new papers since the Fermilab g–2 result, apologies for not reporting them here

Conclusions

Independently of this discrepancy, there is so much wonder in these high-precision tests, as beautifully expressed by CERN physicist Francis Farley some time ago (before the crisis):

"What the theorists do and what we experimenters do is completely different. They talk about Feynman diagrams, amplitudes, integrals, expansions and a whole lot of complex mathematics. We hook up an accelerator to beam lines and steering magnets to the device itself, which is stuffed with wires, thousands of cables, timing devices, sensors and such things. It's two totally different worlds! But, they come out with a number, we come out with a number, and these numbers agree to parts per million! It's unbelievable! That is the most astonishing thing for me!"

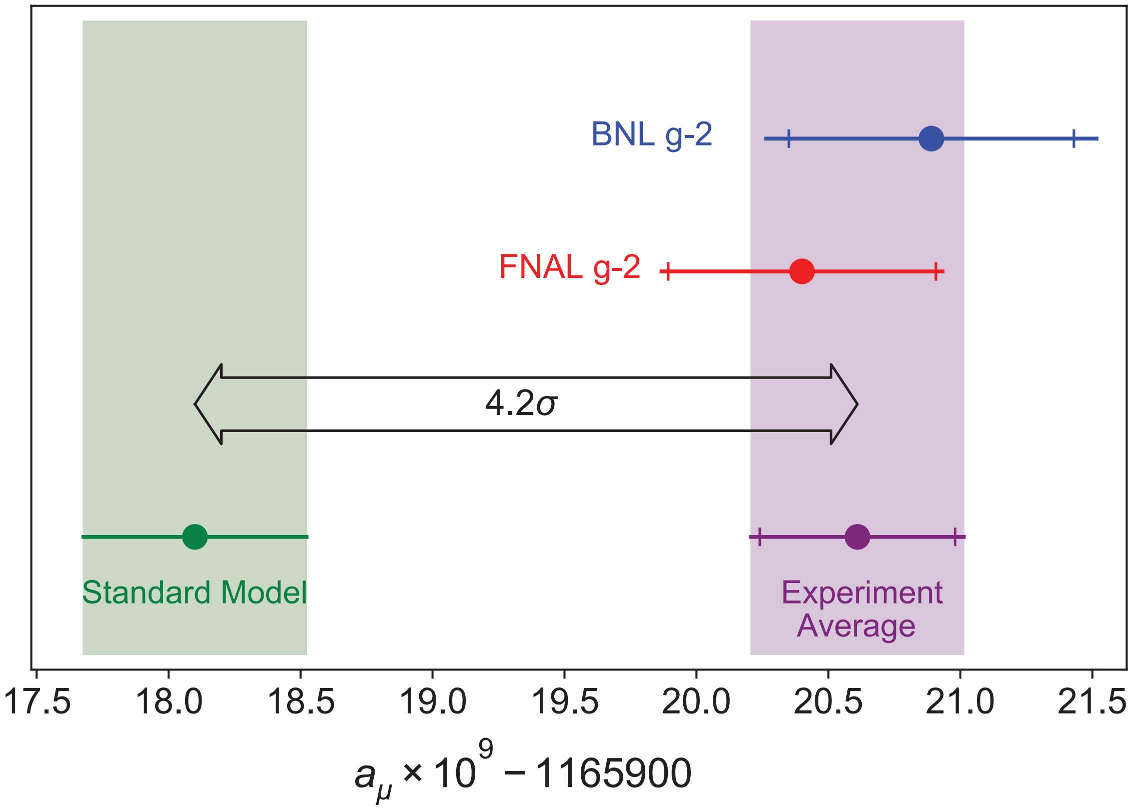
Quote reported by Robert P. Crease in Scientific American Online, 2 May 2021

https://www.scientificamerican.com/article/the-fermilab-muon-measurement-may-or-may-not-point-to-new-physics-but1/

Francis Farley, pioneer of muon g–2 measurements

Conclusions

But, we do have a problem to solve …

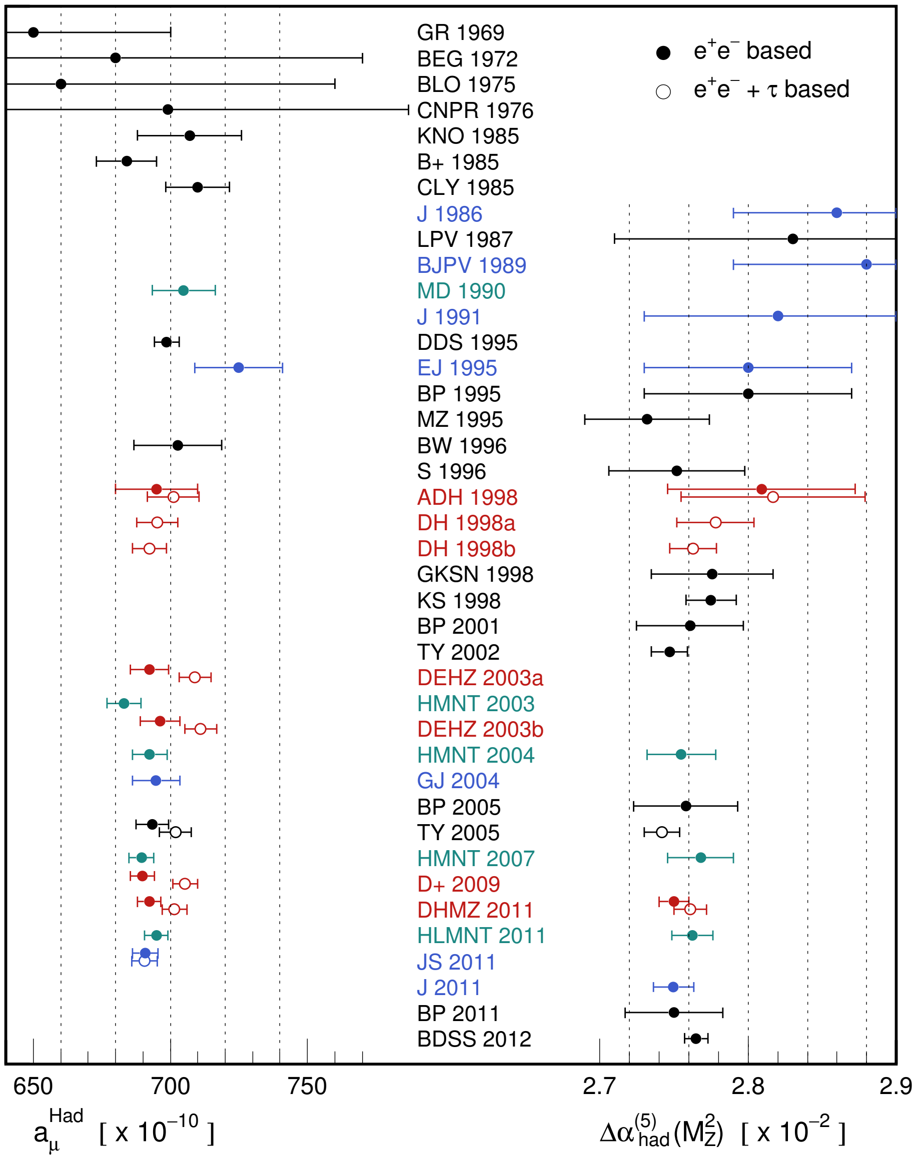


Additional slides

Long history of $a_\mu^{\mathrm{Had,LO}}$ determinations involving theorists and experimentalists

$$a_\mu^{\mathrm{Had,LO}} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_{m_\pi^2}^{\infty} ds \frac{K(s)}{s} R(s)$$

- Improvement mostly driven by better $e^+e^- \to$ hadrons data (intermittently also hadronic tau decays used to improve over insufficient-quality low-mass e^+e^- data)
- The understanding of the data and the treatment of their uncertainties improved over time
- Sum-rule tests allowed to expand the use of perturbative QCD to predict $R(s)$
- Fairly consistent picture reached

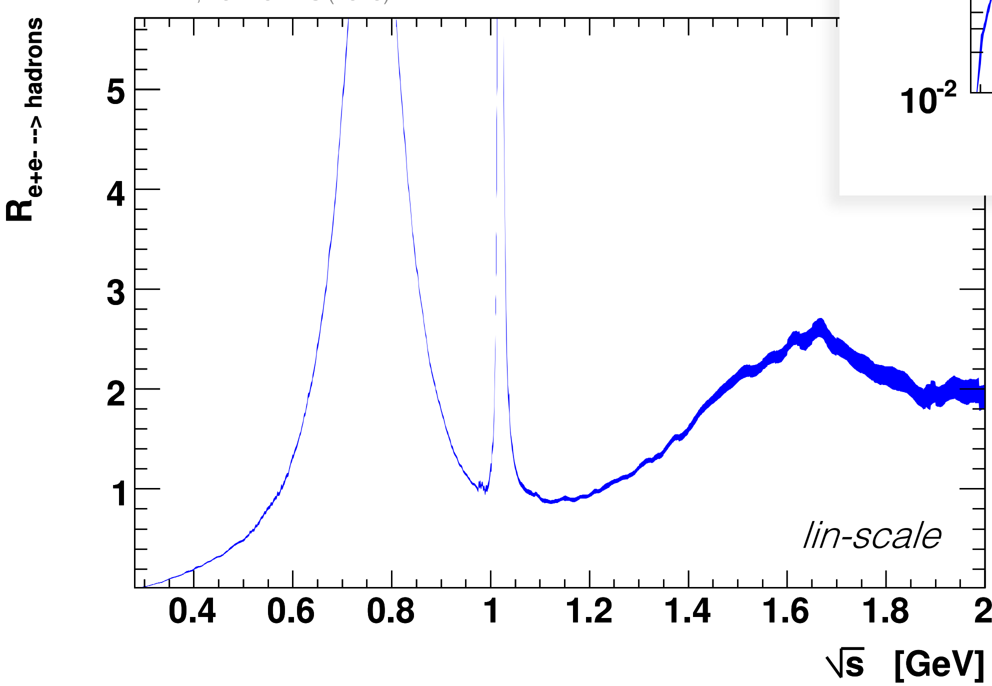


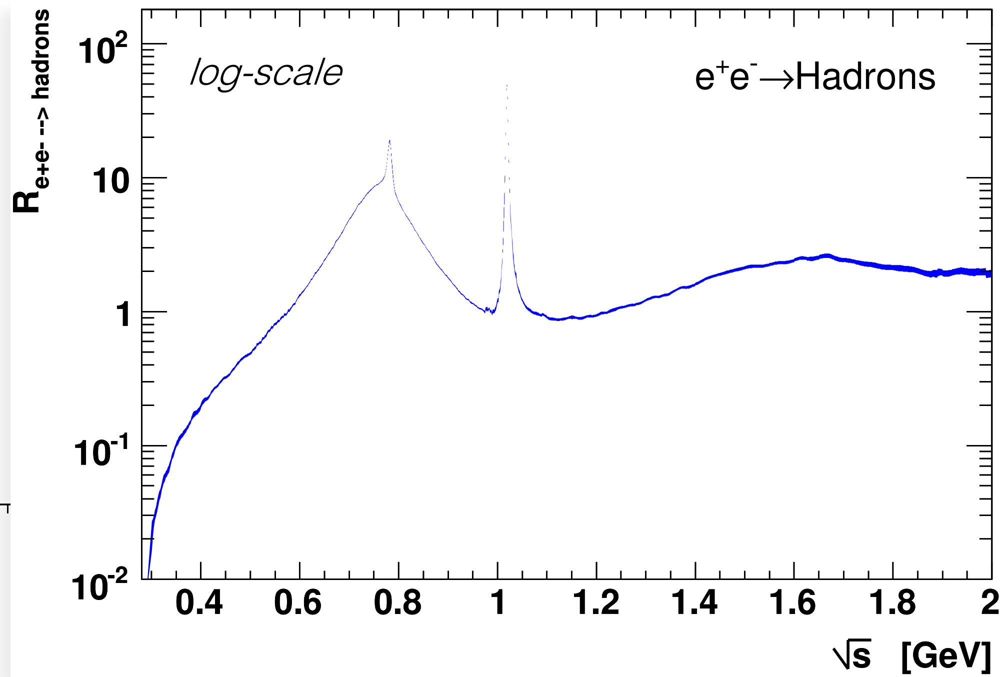
Full combination of all exclusive modes: $R(s)$ [$\sqrt{s} \leq 2$ GeV]

Resulting $R(s)$ function is finely binned with uncertainties mapped by covariance matrix

Accuracy of combination tested with pseudo-experiments

DHMZ, 1612.02743 (2016)





$R(s)$ spectrum dominated by BABAR ISR data measuring almost all exclusive channels until 2 GeV

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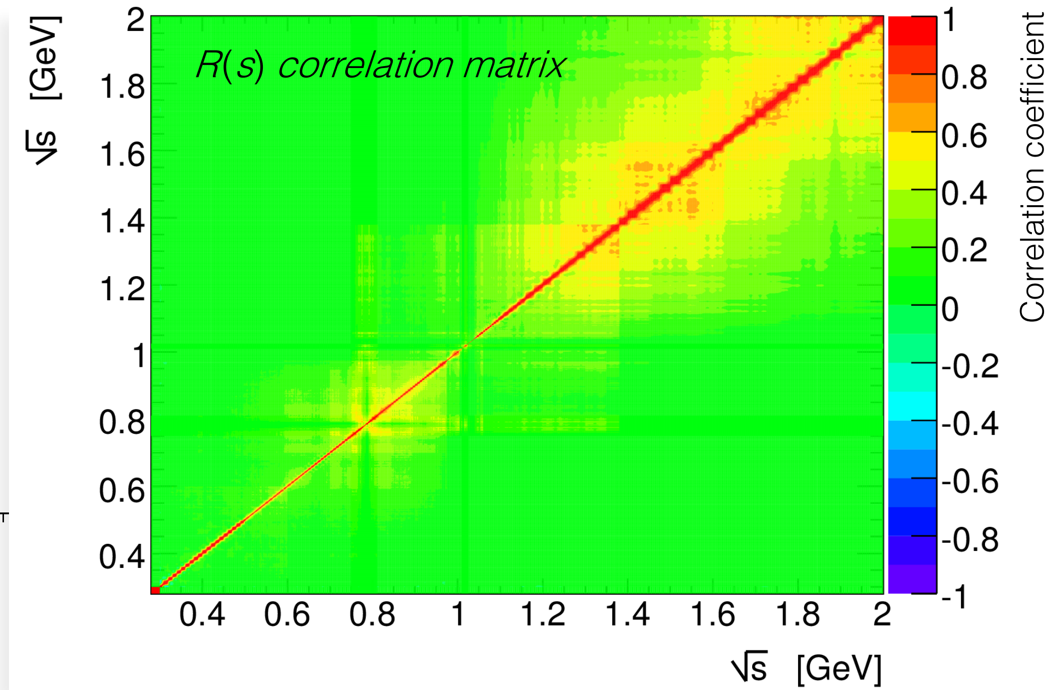

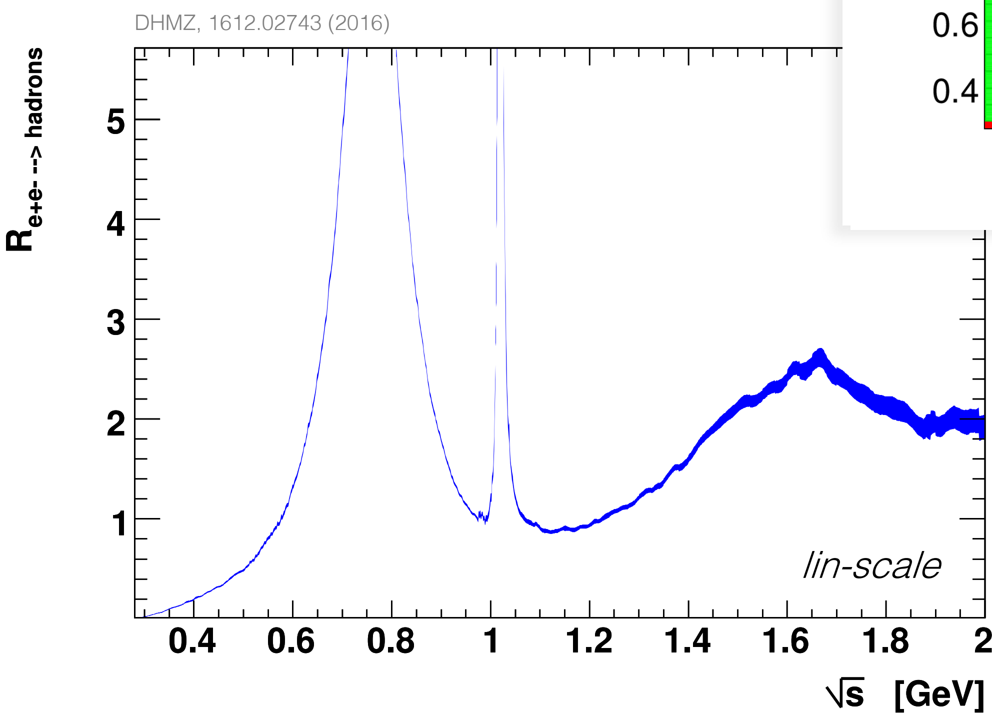

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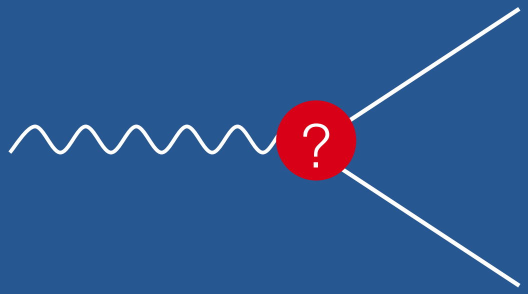


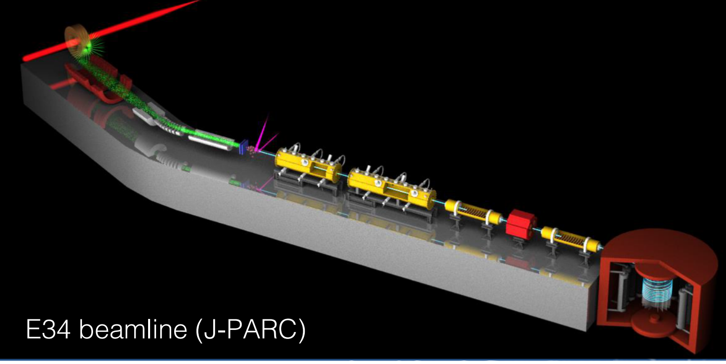
SM perspectives
($\Delta a_\mu = 27.4$)

Long standing 3-ish sigma discrepancy between data and SM on a_μ. On the SM side:

- BABAR-KLOE discrepancy in $\pi^+\pi^-$ channel unresolved. New data from CMD-3 expected, and a new BABAR analysis with the full data sample (0.3% systematic uncertainty may be reachable, current BABAR: 0.5% on peak)
- The $\pi^+\pi^-\pi^0$ channel should be further improved (need BABAR data at and below $\phi(1020)$)
- The K^+K^- data from CMD-2/3/SND must be scrutinized and understood
- Need to compare BABAR and forthcoming CMD-3/SND results in the 1–2 GeV range (so far they agree)
- More results will come from BES-III and Belle-2
- Alternative $a_\mu^{\mathrm{Had,LO}}$ determinations: (1) Lattice calculations; (2) Proposal via a dispersion integral in the spacelike region by an ultra-precise $\mu e \to \mu e$ differential cross section measurement [Abbiendi et al, 1609.08987]

Uncertainty on $a_\mu^{\mathrm{Had,LO}}$ improved by factor of 2 during the last 13 years. Now twice smaller than exp. uncertainty. LBL scattering, estimated from hadronic models, has an uncertainty of similar size that currently appears irreducible. Lattice QCD calculations may provide the way forward.

The recent & future SM improvements pave the road of a full exploitation of the next generation $g-2$ experiments at Fermilab and J-PARC



E34 beamline (J-PARC)

New experiments

J-PARC E34

Entirely new, compact “zero *E*-field” approach (off magic γ)

- Replace electric quadrupole field focusing in E821 by low-emittance, low-momentum (“cold muon”) beam
- Positive muons from pion decays are stopped in aerogel target where they form muonium ($e^-\mu^+$) atoms
- The muonium atoms are laser ionised, leaving cold muons of momentum 3 keV in average
- These are reaccelerated (to increase lifetime) in a linac to 300 MeV
- Muons are injected into a compact, ultra-precise (iron shimmed) 3 T storage magnet with 66 cm orbit diameter (14 m in E821 / E989)
- The decay positrons are measured in silicon strip tracking detector (not a calorimeter) needed because of dense muon decay environment
- Target: 4.5 at stage-one, and 1.2 final on a_μ [$\cdot\ 10^{-10}$], and ~10^{-21} e cm final μEDM sensitivity

E34 was approved among the future priority projects by KEK. Detector partially funded, moving ahead with construction. Fascinating project!