Precision Probes of New Physics:

The Muon ($g-2$) Challenge

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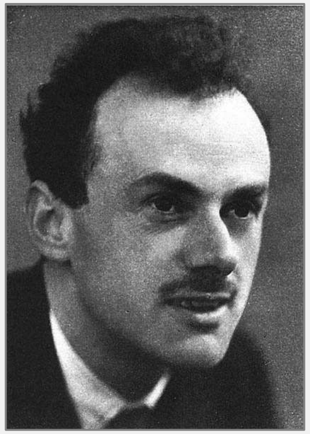
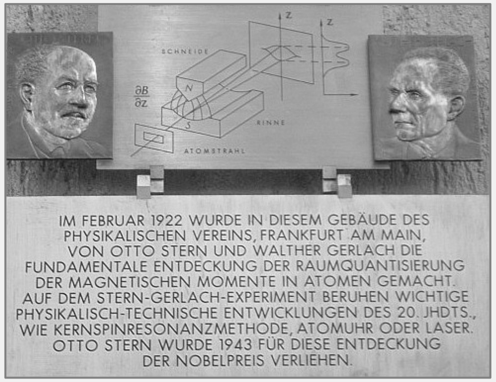
Outline

- The magnetic anomaly: motivation for a precision measurement
- The experimental program
- Confronting experiment with theory
- Hadronic vacuum polarization
- A Standard Model prediction for $(g–2)_\mu$
- Looking at New Physics …
- Appendix: BABAR's ISR program

The Magnetic Moment …of the electron



Dirac, imagining holes and seas in 1928



The Stern-Gerlach experiment:
A commemorative plaque at the Frankfurt physics institute

- Combining quantum mechanics with special relativity, and linearization of $\partial/\partial t$, leads Dirac to the equation

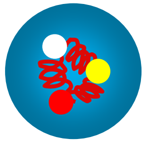
$$i\gamma^{\mu}\partial_{\mu}\psi(x,t) - m\psi(x,t) = 0$$

Apart from antimatter …

- It also accounts in a natural way for *quantized particle spin*, and describes *elementary* spin-1/2 particles
- In the classical limit, one finds the Pauli equation with magnetic moment for *elementary* spin-1/2 particles:

$$\vec{\mu}_{\ell} = g_{\ell}\frac{e}{2m_{\ell}}\vec{s} \qquad (\text{with } g_{\ell} = 2)$$ “gyromagnetic” ratio

- But $g_p/g_e \sim 2.8$ → hint that proton is not elementary

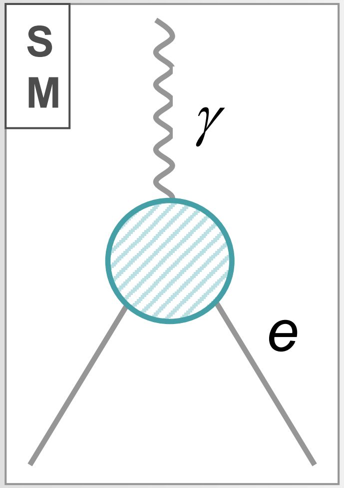


(E.g., the obviously composite He has $g_p/g_e \sim 7400$)

The *Anomalous* Magnetic Moment ...of the electron

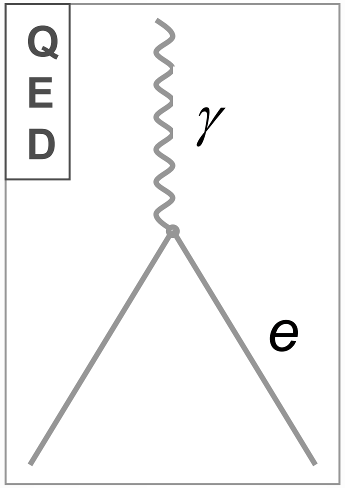
- Dirac's gyromagnetic factor corresponds to the lowest order QED graph
- However, there are corrections to it:

Schwinger 1948
(Nobel price 1965)



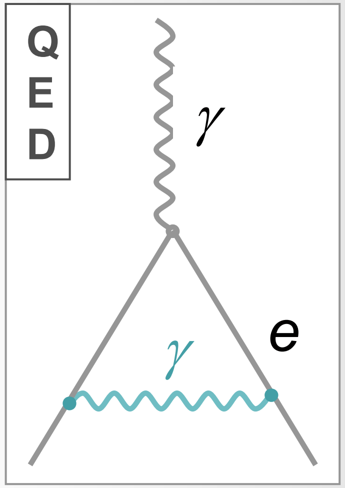
$(g–2)_e \neq 0$ (full Standard Model)

=



$(g–2)_e = 0$ (Dirac)

+

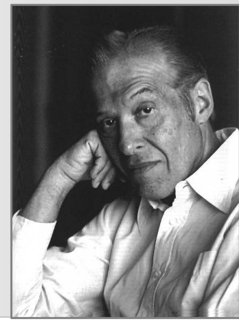


coupling to virtual fields:
$(g–2)_e \neq 0$ (1^{st} order QED)

+ ...

- Quantum fluctuations shift the gyromagnetic ratio

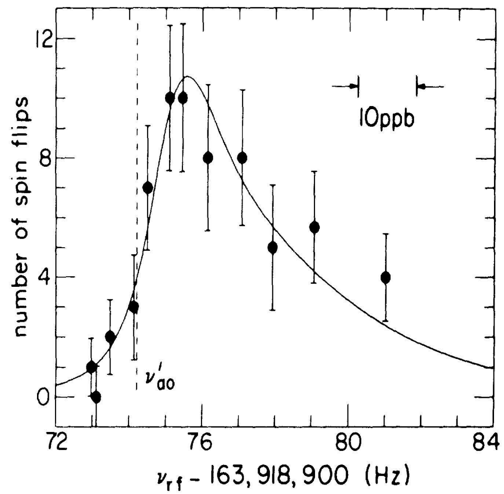
$$a_\ell \equiv \frac{g_\ell - 2}{2} = \frac{\alpha}{2\pi} + \ldots = \frac{1}{2\pi}\underbrace{\frac{e^2}{4\pi}}_{1/137.036\ldots} + \ldots = 0.001161\ldots$$



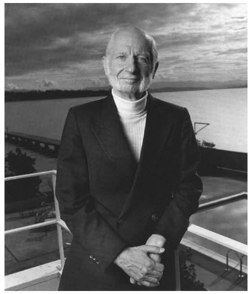
Schwinger,
finding QED easy

The Electron *Anomalous* Magnetic Moment

- Schwinger's (*et al.*) prediction agreed (and agrees) beautifully with experiment:



Electron spin resonance near 141 GHz versus the frequency of an exciting radiofrequency field.



Dehmelt, capturing single positrons for months

For the electron a_e, a series of Nobel price winning experiments, using e^-/e^+ capture in a Penning trap

$$a_e^{\text{exp}} = 0.0011596521882(30)$$

Dyck-Schwinberg-Dehmelt PRL59, 26–29 (1987)

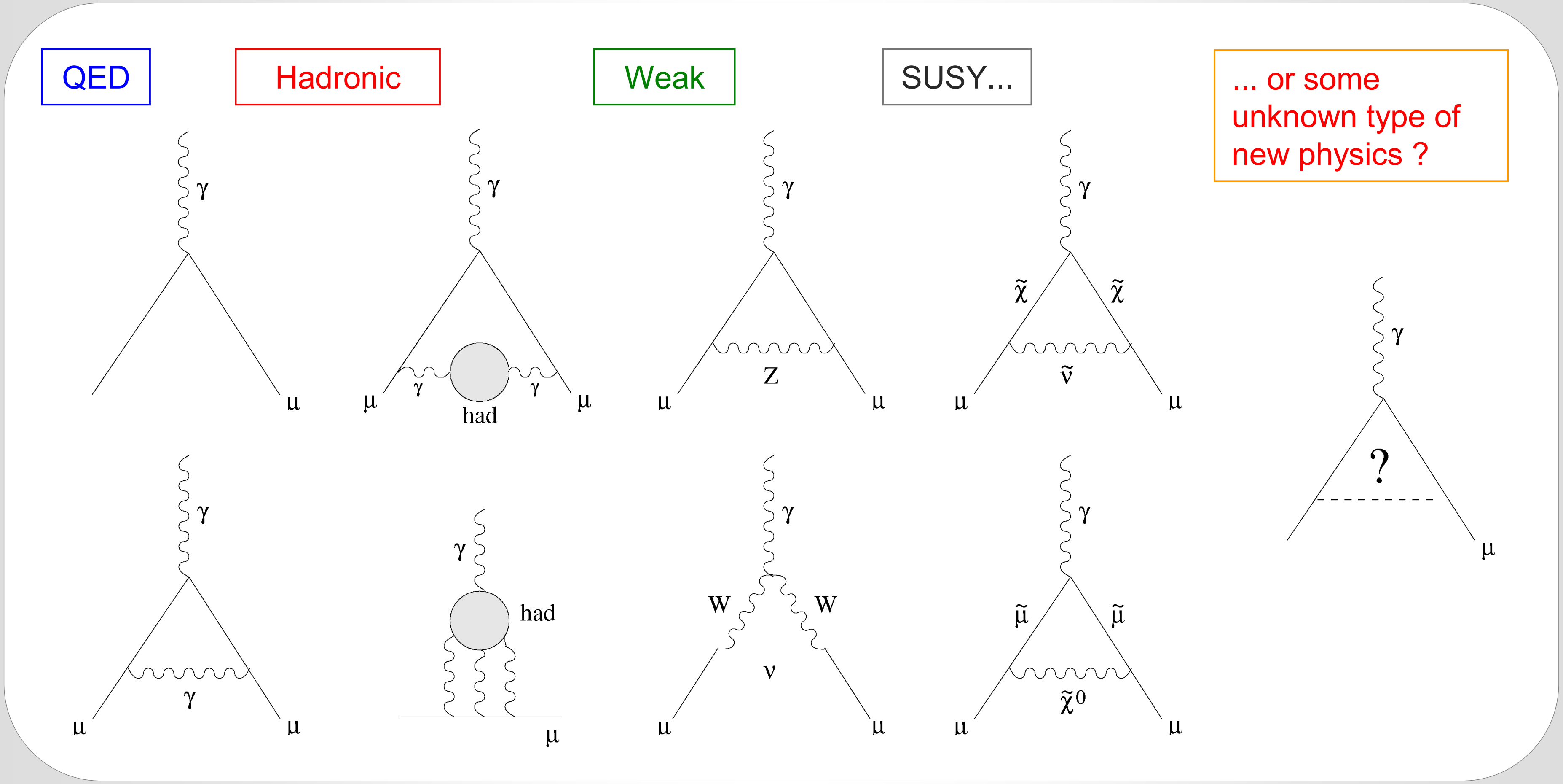
$$a_e^{\text{SM}} = \frac{\alpha}{2\pi} - 0.328478444\left(\frac{\alpha}{\pi}\right)^2 + 1.181234\left(\frac{\alpha}{\pi}\right)^3 - 1.7502\left(\frac{\alpha}{\pi}\right)^4 + \underbrace{1.7\times10^{-12}}_{\text{hadronic \& EW effects}}$$

Kinoshita-Nio, PRD 70, 113001 (2004)

most precise determination of α to date: $\alpha^{-1} = 137.03599877(40)$ (quantum Hall: 137.03600300(270))

The *Muon* Anomalous Magnetic Moment

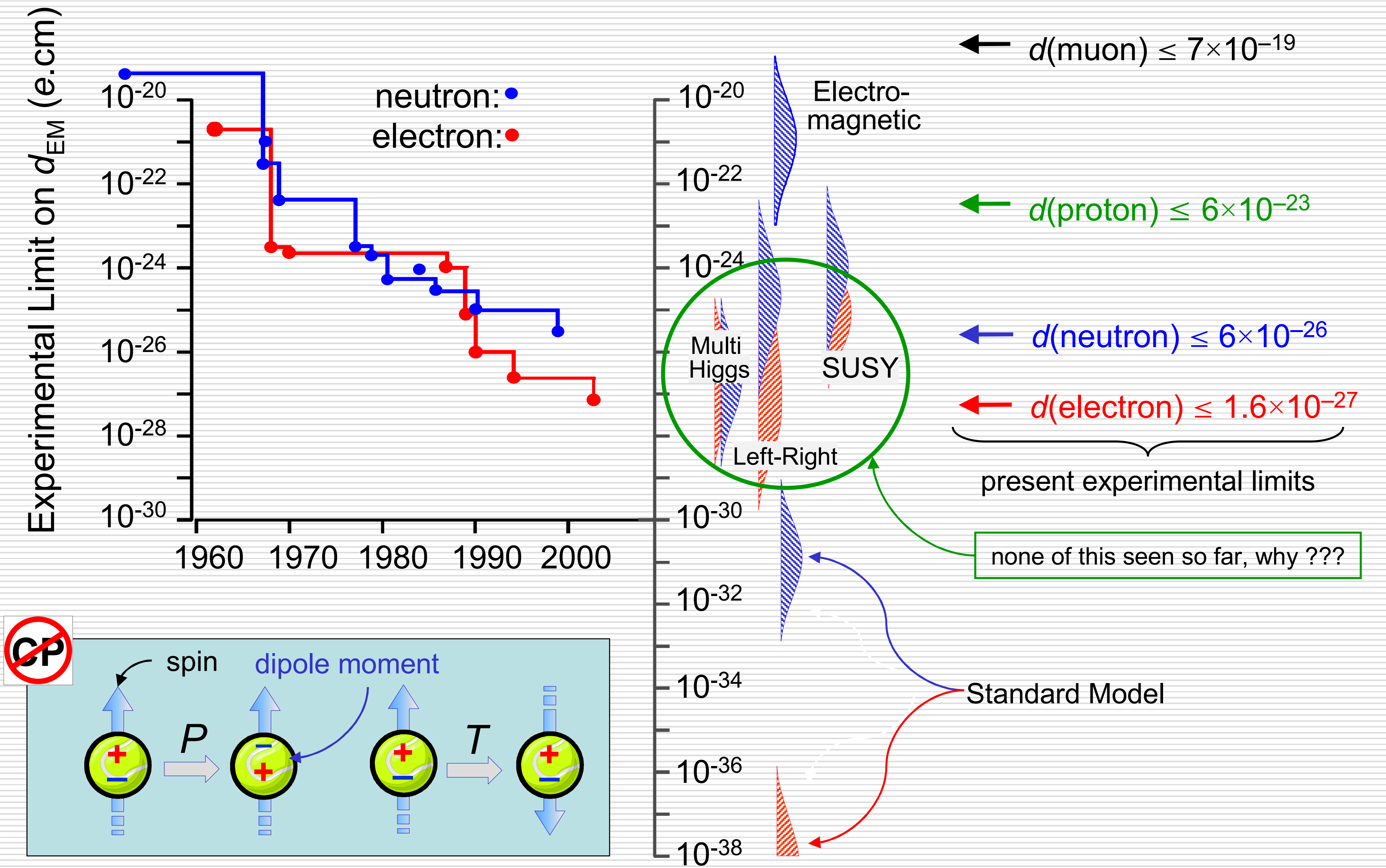
- Diagrams contributing to the magnetic moment



The Worldwide Quest for “New Physics”

- Most large-scale HEP experimental facilities(*) today search for specific signs of new physics beyond the Standard Model. WHY ?
  - Dark matter
  - The gauge hierarchy Problem (Higgs sector, scale ~ 1 TeV)
  - Baryogenesis (CKM CPV too small)
  - The strong *CP* Problem (why is $\theta \sim 0$?)
  - Grand Unification of the gauge couplings
  - Neutrino masses, cosmological constant ... many more
- Conflict between New Physics limits from flavor physics and EDM searches:
  → “generic” NP scale much larger than 1 TeV

Brief insertion: *CP*-Violating Electric Dipole Moments

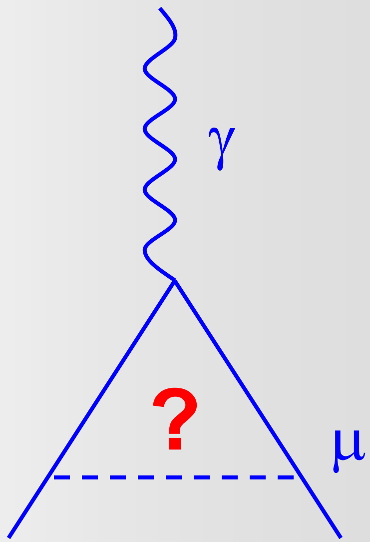


The Quest for “New Physics”

- The experimental precision for a_μ will be much worse than for a_e, so why do it ?

- From chiral symmetry expect the New Physics (NP) effects to scale ~ $m^2(e/\mu)$:

$$a_\ell^{\rm NP}(\Lambda_{\rm NP}) \propto \mathcal{O}\left(\frac{m_\ell^2}{\Lambda_{\rm NP}^2}\right) \quad \Longrightarrow \quad \frac{a_\mu^{\rm NP}}{a_e^{\rm NP}} \propto \mathcal{O}\left(\frac{m_\mu^2}{m_e^2}\right) \approx 42{,}000$$



- Expect to loose about a factor of 200 in experimental precision

$\Rightarrow$ a_μ should be roughly 200 times more sensitive to NP than a_e !

The Experimental Program

Polarized muons moving in a uniform *B* field (perp. to muon spin and orbit plane), and vertically focused in *E* quadrupole field, the observed difference between spin precession and cyclotron frequency is:

$$\vec{\omega}_a = \frac{e}{mc}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\vec{\beta} \times \vec{E}\right]$$

The *E* dependence is eliminated at "magic γ": $\gamma = 29.3 \rightarrow p_\mu = 3.09$ GeV/c

The experiment measures ($g-2$) directly, and not g !

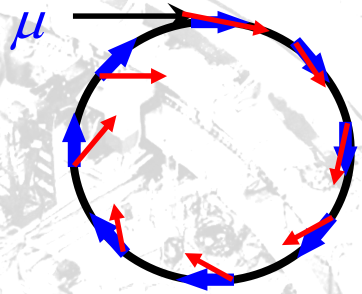
Exploit Muon Properties in Experiment

- Parity violation polarizes muons in pion decay
  pions from proton-nucleon collision (AGS)

spin orientation

$$\bar{\nu}_\mu \leftarrow \pi^- \rightarrow \mu^-_{\text{polarized}}$$

- Spin precession frequency proportional to a_μ

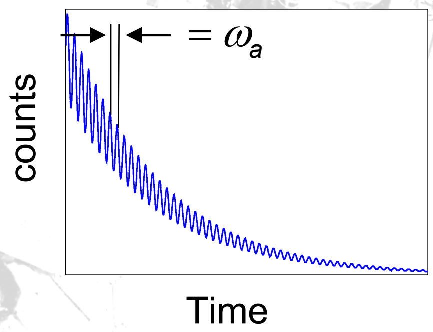


- The magic γ

$$\vec{\omega}_a = \frac{e}{mc}\left[a_\mu \vec{B} - \underbrace{\left(a_\mu - \frac{1}{\gamma^2 - 1} \right)}_{=0} \vec{\beta} \times \vec{E} \right] \cong \frac{e}{mc} a_\mu \vec{B}$$

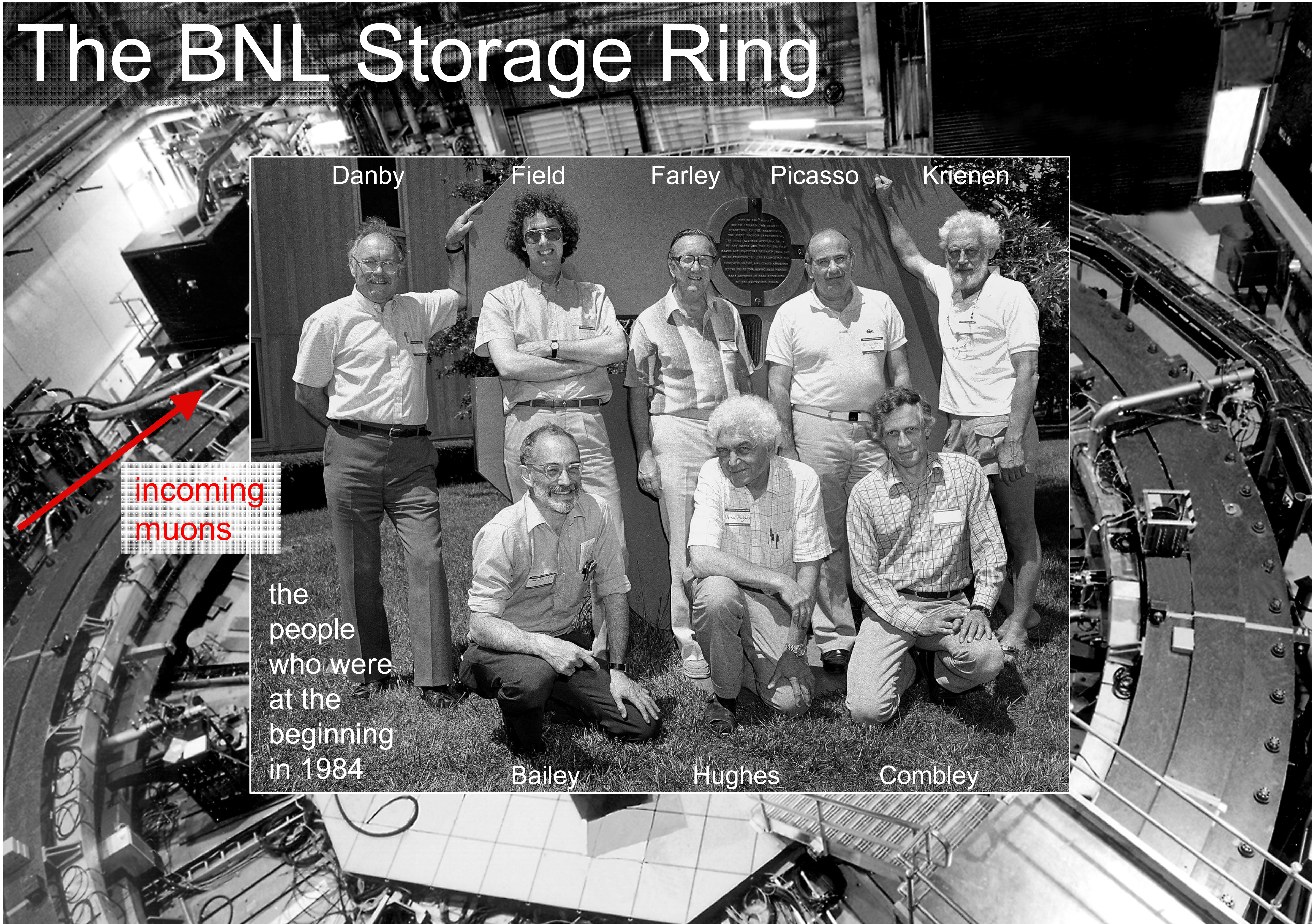
- Again parity violation in muon decay

$\mu^-_{\text{polarized}} \rightarrow e^- + \nu_\mu + \bar{\nu}_e$ fast electron emitted in direction opposite to muon spin

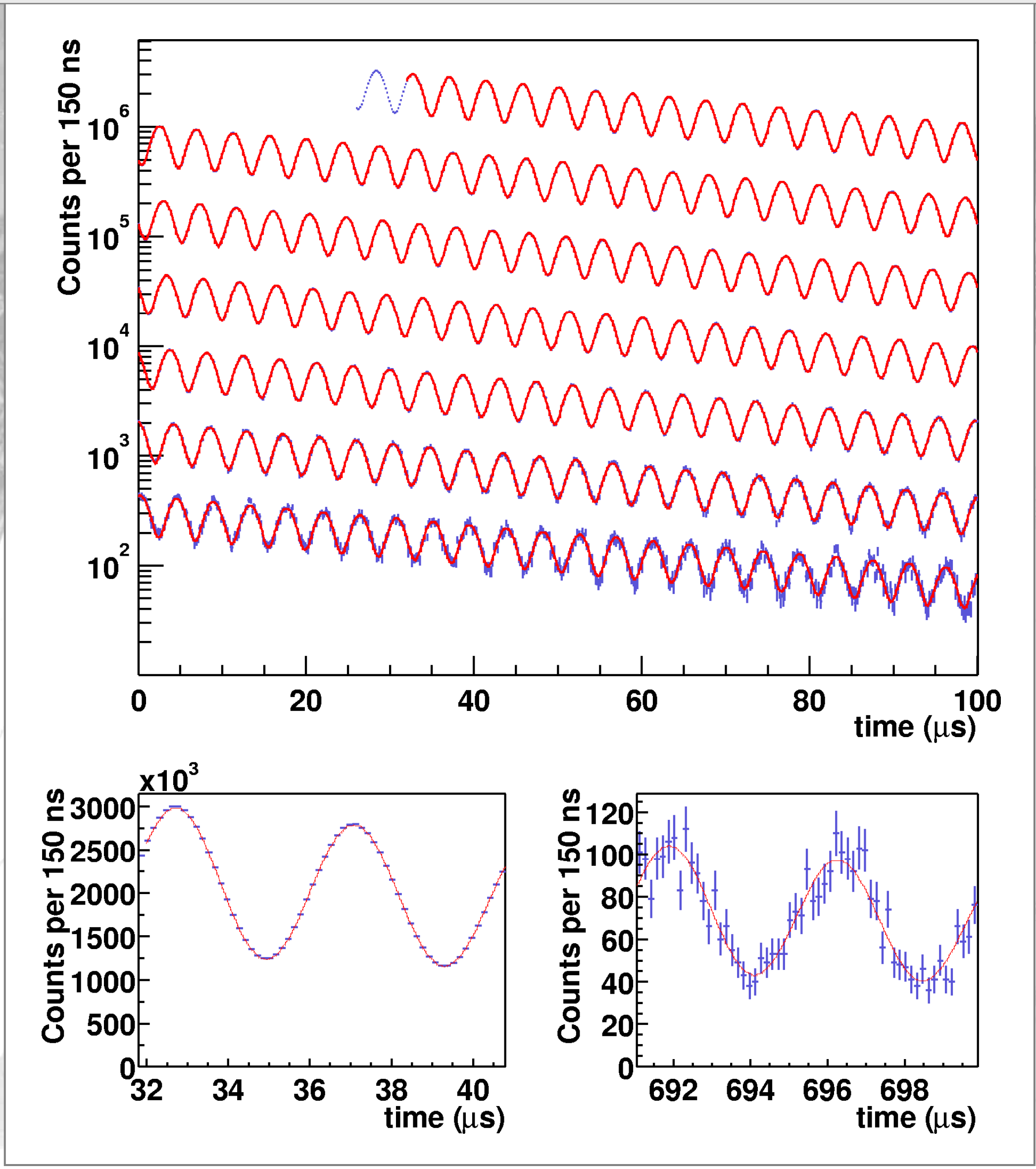


This transparency has been copied from D. Hertzog, UIUC

The BNL Storage Ring



The BNL $(g-2)_\mu$ Measurement



Observed positron rate in successive 100μs periods

These quantities are measured independently *and* blind
→ **doubly blind analysis**

Difference between spin precession and cyclotron frequency:

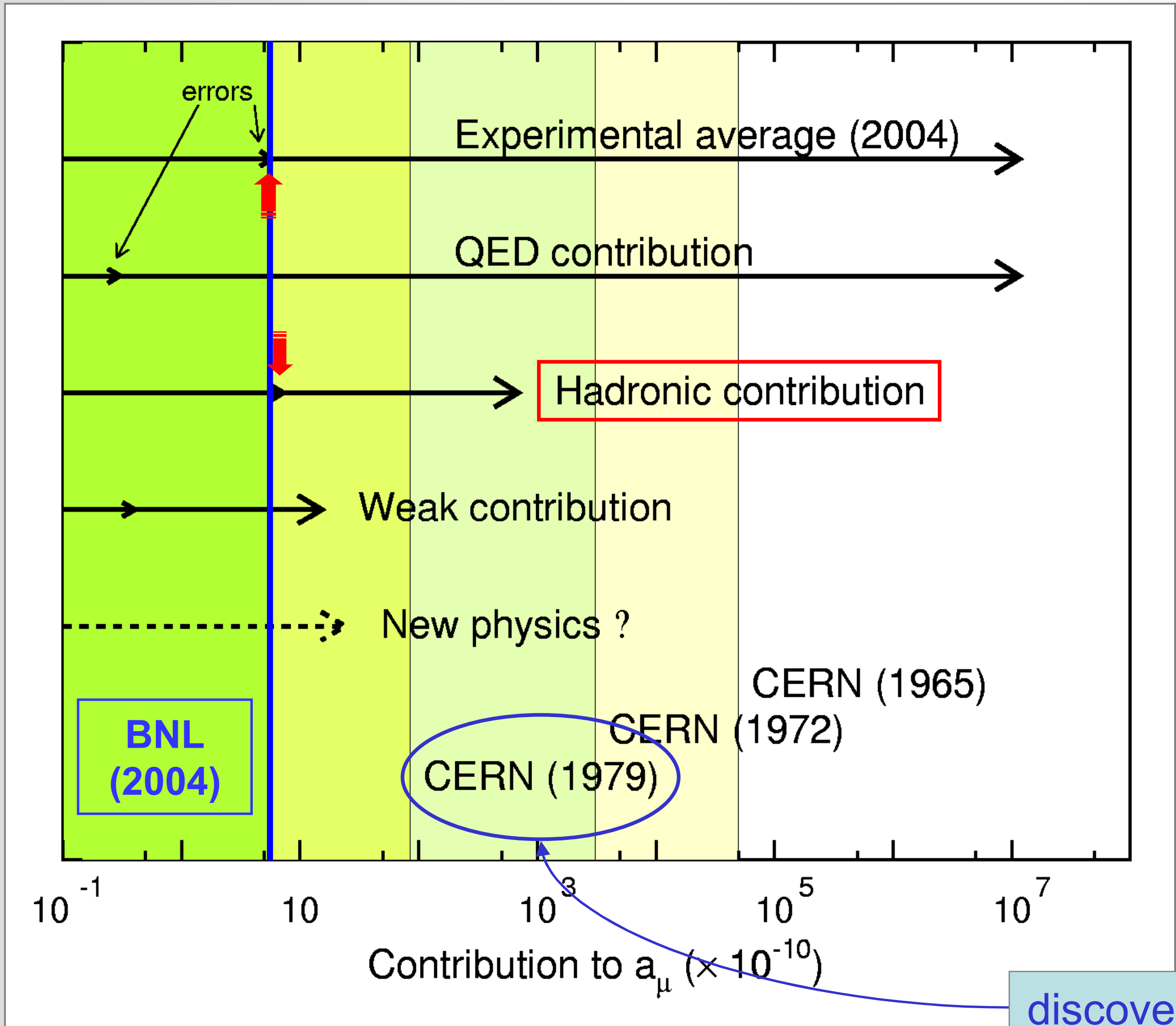
$$\vec{\omega}_a = \frac{e}{m_\mu c} a_\mu \vec{B}$$

obtained from fit to:

$$N(t) = N_0 e^{-t/\gamma\tau}\left[1 + A\sin(\omega_a t + \phi)\right]$$

plot taken from:
E821 (*g* –2), hep-ex/0202024

Experimental Progress from CERN to BNL



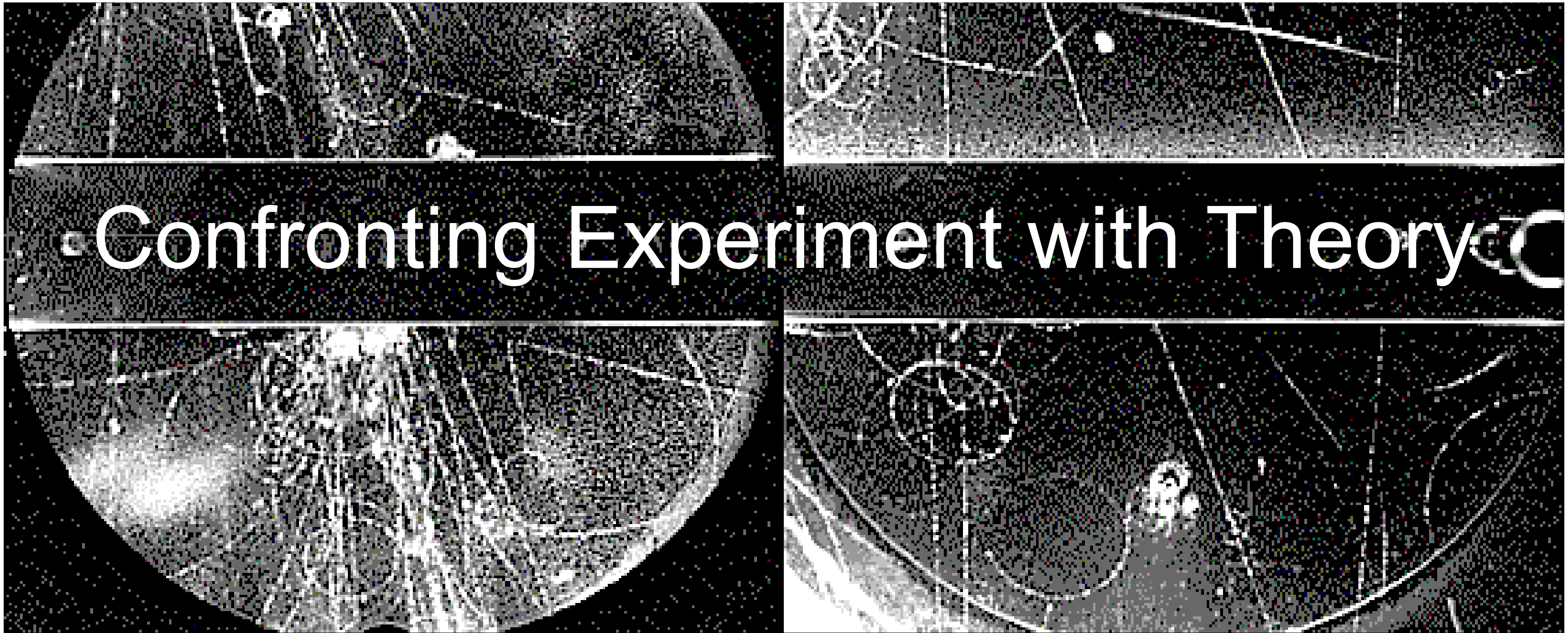
Experimental progress on precision of $(g-2)_\mu$

Outperforms theory precision on hadronic contribution

discovery of magic γ

J. Bailey et al., NP B150 1 (1979)

Confronting Experiment with Theory



The Standard Model prediction of a_μ is decomposed in its main contributions:

$$a_\mu^{\mathrm{SM}} \equiv \left(\frac{g-2}{2}\right)_\mu = a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{had}} + a_\mu^{\mathrm{weak}}$$

of which the hadronic contribution has the largest uncertainty

The Muon Magnetic Anomaly in the Standard Model

QED contribution

Computed up to 4th order
(5th order estimated)

$$a_\mu^{\rm QED} \approx \begin{pmatrix} 11{,}614{,}098.1 + 41{,}321.8 \\ + \; 3{,}014.2 + 38.1 + \underbrace{0.6}_{\text{estimated}} \end{pmatrix} \times 10^{-10} = 11{,}658{,}472.1(0.1) \times 10^{-10}$$

Kinoshita-Nio, PRD 70, 113001 (2005)

Electroweak contribution

Computed up to 2nd order

$a_\mu^{\rm weak}$ suppressed by $\dfrac{\alpha}{\pi}\dfrac{m_\mu^2}{m_W^2} \sim 10^{-9}$ (!)

$$\underbrace{a_\mu^{\rm weak}}_{\text{1-loop}} = \frac{G_\mu m_\mu^2}{8\sqrt{2}\pi^2}\left(\frac{5}{3} + \frac{1}{3}\left(1 - 4\sin^2\theta_W\right) + \mathcal{O}\left(\frac{m_\mu^2}{m_W^2}\right) + \mathcal{O}\left(\frac{m_\mu^2}{m_H^2}\right)\right) = +19.5 \times 10^{-10}$$

Czarnecki *et al.*,
PRD 52, 2619 (1995);
PRL 76, 3267 (1996)

2nd order contribution surprisingly large: $\underbrace{a_\mu^{\rm weak}}_{\text{2-loop}} = -4.1(0.2) \times 10^{-10}$

(due to large logs: $\ln[m_Z/m_\mu]$)

Note that between a_μ and a_e, the same sensitivity factor as for “new physics” applies here

2002: *Second* Round of BNL Results on $(g-2)_\mu$

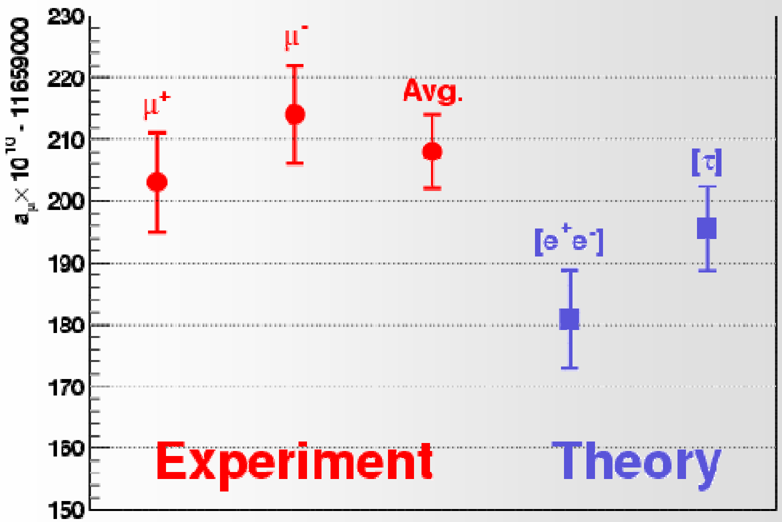
The new analysis, first presented at ICHEP'02, achieves 2 times better precision (using 4× more statistics) than the 2001 result:

$$a_\mu(\text{exp}) = 11\,659\,203(7)(5) \times 10^{-10}$$

main systematics:

- 3.6×10^{-10} from precession frequency
- 2.8×10^{-10} from magnetic field

E821 $(g-2)$
PRL 92, 161802 (2004)



BNL compares WA with SM prediction:

$$a_\mu(\text{exp}) - a_\mu(\text{SM}) = 25(10) \times 10^{-10}$$

Experimental and theoretical uncertainties now of similar order !

More work and, in particular, better data needed to achieve a more precise prediction of the hadronic contribution

The Hadronic Contribution to $(g-2)_\mu$

$$a_\mu^{\mathrm{SM}} = a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{had}} + a_\mu^{\mathrm{weak}}$$

The Situation 1995

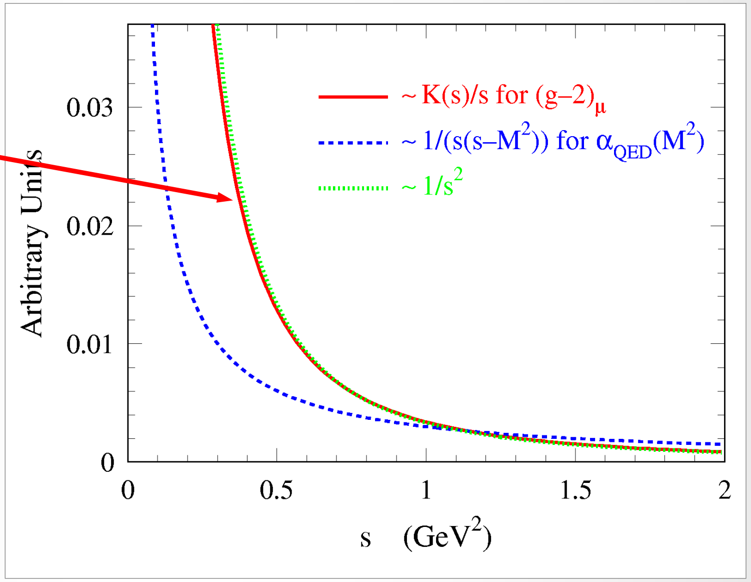
Source	$\sigma(a_\mu)$	Reference
QED	**~ 0.1 × 10^{-10}**	[Schwinger '48 & others]
Hadrons	**~ (15 ⊕ 4) × 10^{-10}**	[Eidelman-Jegerlehner '95 & others]
***Z, W* exchange**	**~ 0.2 × 10^{-10}**	[Czarnecki *et al.* '95 & others]

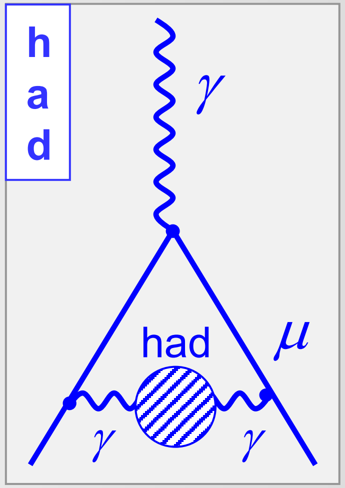
Dominant uncertainty from lowest order hadronic piece. Cannot be calculated from QCD ("first principles") – but: we can use experiment (!)

$$a_\mu^{\mathrm{had,LO}} = \frac{\alpha^2}{3\pi^2}\int_{4m_\pi^2}^{\infty} ds\, \frac{K(s)}{s} R(s)$$

Dispersion relation, uses unitarity (optical theorem) and analyticity

... (*see digression*)





digression: Vacuum polarization … and the running of α_{QED}

Define: photon vacuum polarization function $\Pi_\gamma(q^2)$

$$i\int d^4x\, e^{iqx}\langle 0|TJ^\mu_{\text{em}}(x)\left(J^\nu_{\text{em}}(0)\right)^\dagger|0\rangle = -\left(g^{\mu\nu}q^2 - q^\mu q^\nu\right)\Pi_\gamma(q^2)$$

Ward identities: only vacuum polarization modifies electron charge

$$\alpha(s) = \frac{\alpha(0)}{1-\Delta\alpha(s)}$$

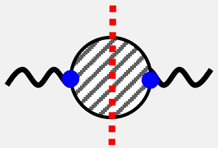
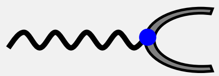
with: $\Delta\alpha(s) = -4\pi\alpha\,\text{Re}\left[\Pi_\gamma(s) - \Pi_\gamma(0)\right]$

Leptonic $\Delta\alpha_{\text{lep}}(s)$ calculable in QED. However, quark loops are modified by long-distance hadronic physics, cannot (yet) be calculated within QCD (!)

Born: $\sigma^{(0)}(s) = \sigma(s)\left(\alpha/\alpha(s)\right)^2$

Way out: Optical Theorem (*unitarity*) ...

$$12\pi\,\text{Im}\,\Pi_\gamma(s) = \frac{\sigma^{(0)}[e^+e^- \to \text{hadrons}]}{\sigma^{(0)}[e^+e^- \to \mu^+\mu^-]} \equiv R(s)$$

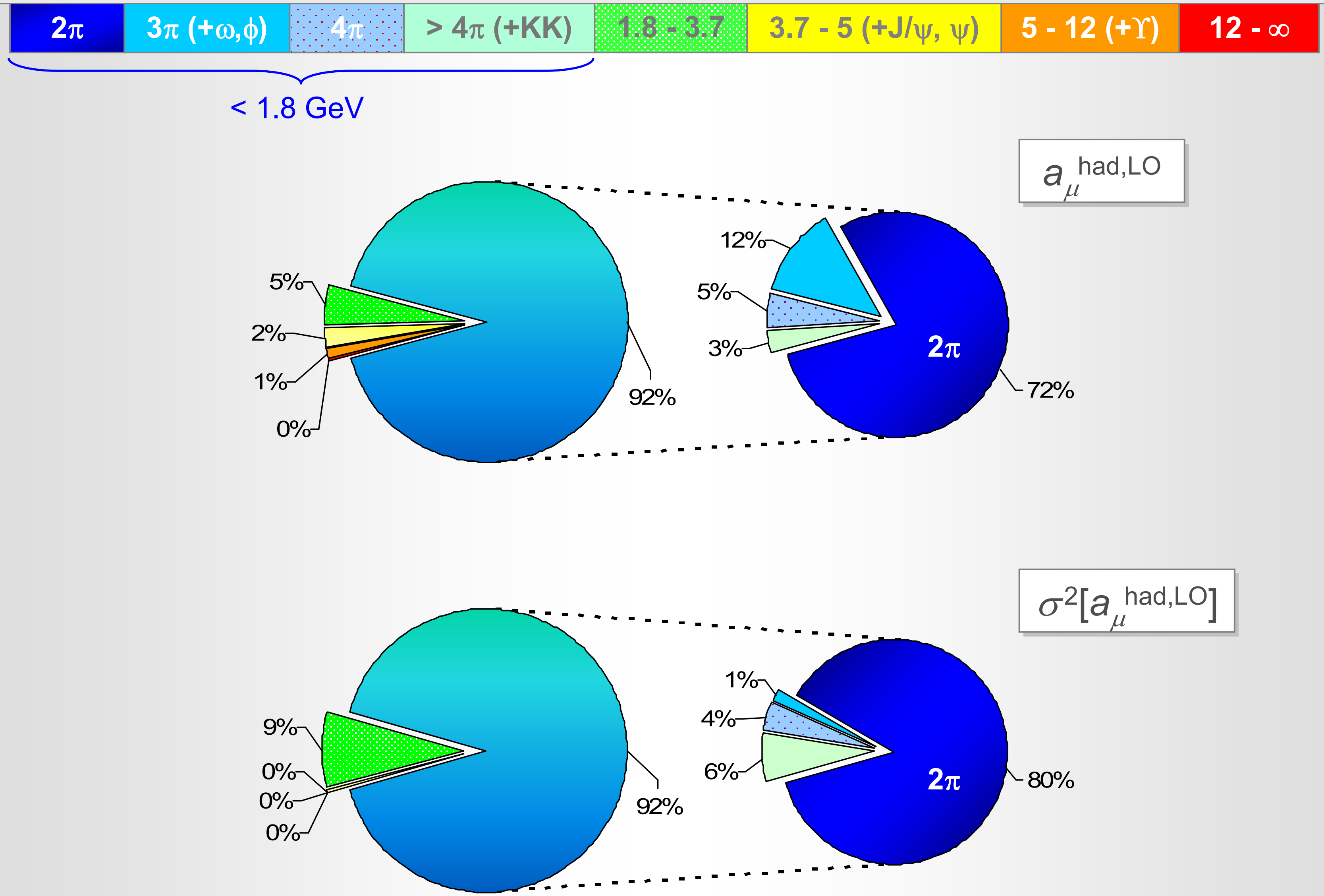
Im[] $\propto$ |  hadrons $|^2$

... and the subtracted dispersion relation of $\Pi_\gamma(q^2)$ (*analyticity*)

$$\Pi_\gamma(s) - \Pi_\gamma(0) = \frac{s}{\pi}\int_0^\infty ds' \frac{\text{Im}\,\Pi_\gamma(s')}{s'(s'-s)-i\varepsilon} \quad \Rightarrow \quad \Delta\alpha_{\text{had}}(s) = -\frac{\alpha s}{3\pi}\text{Re}\int_0^\infty ds' \frac{R(s')}{s'(s'-s)-i\varepsilon}$$

... and equivalently for a_μ[had]

Contributions to the Dispersion Integrals



Improved Determination of the Hadr. Contrib. to $(g-2)_\mu$

Eidelman-Jegerlehner, Z.Phys. C67, 585 (1995)

Since 1995, improved determination of the dispersion integral:

- better data
- extended use of QCD

Energy [GeV]	Input 1995	Input after 1998
$2m_\pi$ - 1.8	Data	Data (e^+e^- & τ) (+ QCD)
1.8 – ψ(3770)	Data	QCD
J/ψ - Υ	Data	Data (+ QCD)
Υ - 40	Data	QCD
40 - ∞	QCD	QCD

Improvement in 4 Steps:

- Inclusion of precise τ data using SU(2) (CVC)

  Alemany-Davier-H.'97, Narison'01, Trocóniz-Ynduráin'01, + later works (!)

- Extended use of (dominantly) perturbative QCD

  Martin-Zeppenfeld'95, Davier-H.'97, Kühn-Steinhauser'98, Erler'98, + others

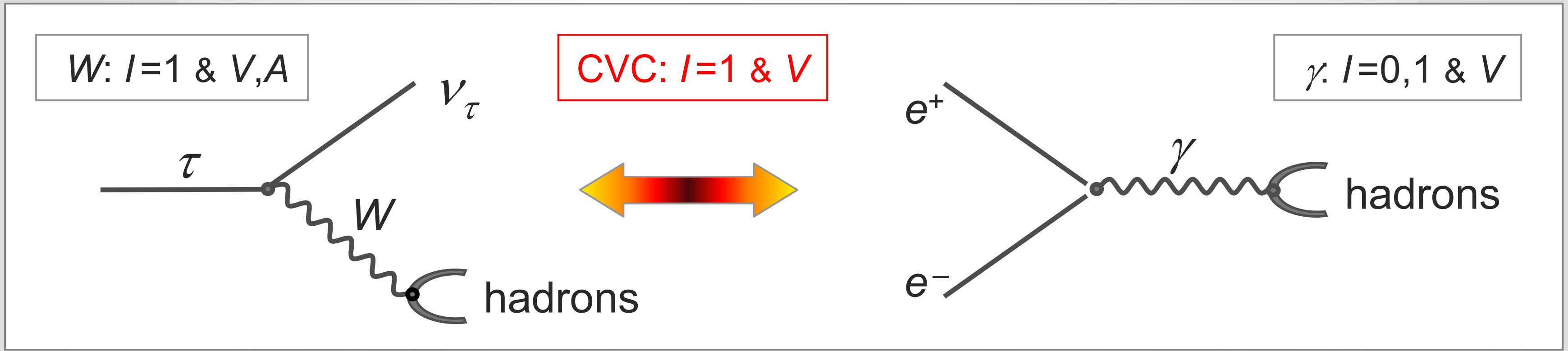
- Theoretical constraints from QCD sum rules

  Groote-Körner-Schilcher-Nasrallah'98, Davier-H.'98, Martin-Outhwaite-Ryskin'00, Cvetič-Lee-Schmidt'01, Jegerlehner et al'00, Dorokhov'04 + others

- Better data for the $e^+e^- \to \pi^+\pi^-$ cross section

  CMD-2'02, KLOE'04, SND'05 (!)

Using also Tau Data through CVC – SU(2)



Hadronic physics factorizes in Spectral Functions :

Isospin symmetry connects I =1 e^+e^- cross section to vector τ spectral functions:

$$\sigma^{(I=1)}\left[e^+e^- \to \pi^+\pi^-\right] = \frac{4\pi\alpha^2}{s}\,\upsilon\left[\tau^- \to \pi^-\pi^0\nu_\tau\right]$$

fundamental ingredient relating long distance (resonances) to short distance description (QCD)

$$\upsilon\left[\tau^- \to \pi^-\pi^0\nu_\tau\right] \propto \frac{\mathrm{BR}\left[\tau^- \to \pi^-\pi^0\nu_\tau\right]}{\mathrm{BR}\left[\tau^- \to e^-\bar{\nu}_e\nu_\tau\right]}\;\frac{1}{N_{\pi\pi^0}}\frac{dN_{\pi\pi^0}}{ds}\;\frac{m_\tau^2}{\left(1-s/m_\tau^2\right)^2\left(1+s/m_\tau^2\right)}$$

branching fractions mass spectrum kinematic factor (PS)

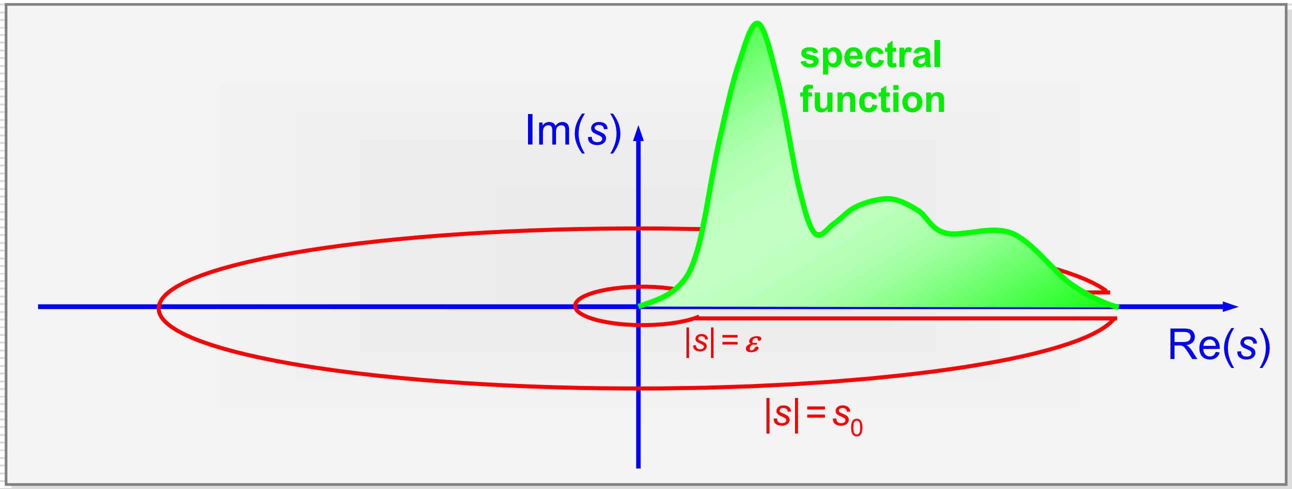
digression: Tau Spectral Functions and QCD

(1) Optical theorem $\upsilon(s) \propto \mathrm{Im}\,\Pi(s)$

(2) Apply Cauchy's theorem for "save" (*i.e.*, sufficiently large) s_0:

$$R(s_0) \propto \int_0^{s_0} ds\, f(s)\, \mathrm{Im}\,\Pi(s) \quad \Leftrightarrow \quad -\frac{1}{2i} \oint_{|s|=s_0} ds\, f(s)\Pi(s)$$

kinematic factor

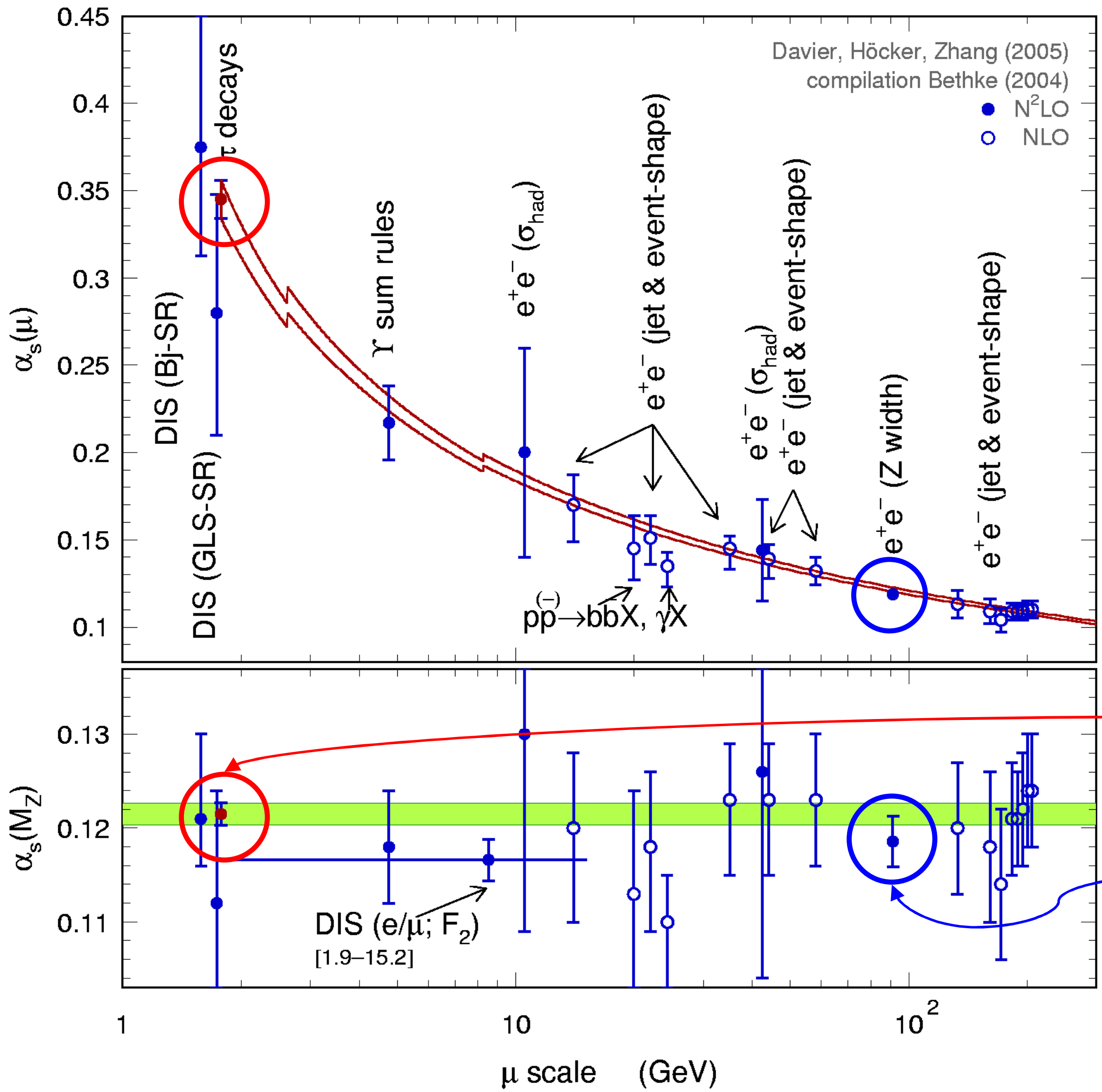


(3) Use the Adler function to remove unphysical subtractions: $D(s) = s\dfrac{d\Pi(s)}{ds}$

(4) Use global quark-hadron duality in the framework of the Operator Product Expansion (OPE) to predict: $D(s) \sim D_{\mathrm{pert}}(s) + D_{\mathrm{q\text{-}mass}}(s) + D_{\mathrm{non\text{-}pert}}(s)$

(5) Use analytical moments $f_n(s) = f(s)\cdot\mathrm{poly}_n(s)$ to fix non-perturbative parameters of the OPE ... and then fit $\alpha_s(m_\tau)$

digression: QCD Results from Hadronic Tau Decays



Evolution of $\alpha_s(m_\tau)$, measured using τ decays, to M_Z using RGE (4-loop QCD β-function & 3-loop quark flavor matching) shows the excellent compatibility of τ result with EW fit !

Davier-Höcker-Zhang
hep-ph/0507078

$\alpha_s(M_Z)$ = 0.1215 ± 0.0012
(DHZ'05, theory dominated)

$\alpha_s(M_Z)$ = 0.1186 ± 0.0027
(LEP'00, exp. dominated)

LEP EW WG,
LEPEWWG 2002-01

SU(2) Breaking

Multiplicative SU(2) corrections applied to $\tau^- \to \pi^-\pi^0 \nu_\tau$ spectral function:

- **Radiative corrections:**

S_{EW} ~ 2% (short distance),
$G_{EM}(s)$ (long distance)

- **Charged/neutral mass splitting:**

$m_{\pi^-} \neq m_{\pi^0}$, ρ-ω mixing, $m/\Gamma_{\rho^-} \neq m\Gamma_{\rho^0}$

- **Electromagnetic decays:**

$\rho \to \pi\pi\gamma$, $\rho \to \pi\gamma$, $\rho \to \eta\gamma$, $\rho \to l^+l^-$

- **Quark mass difference:**

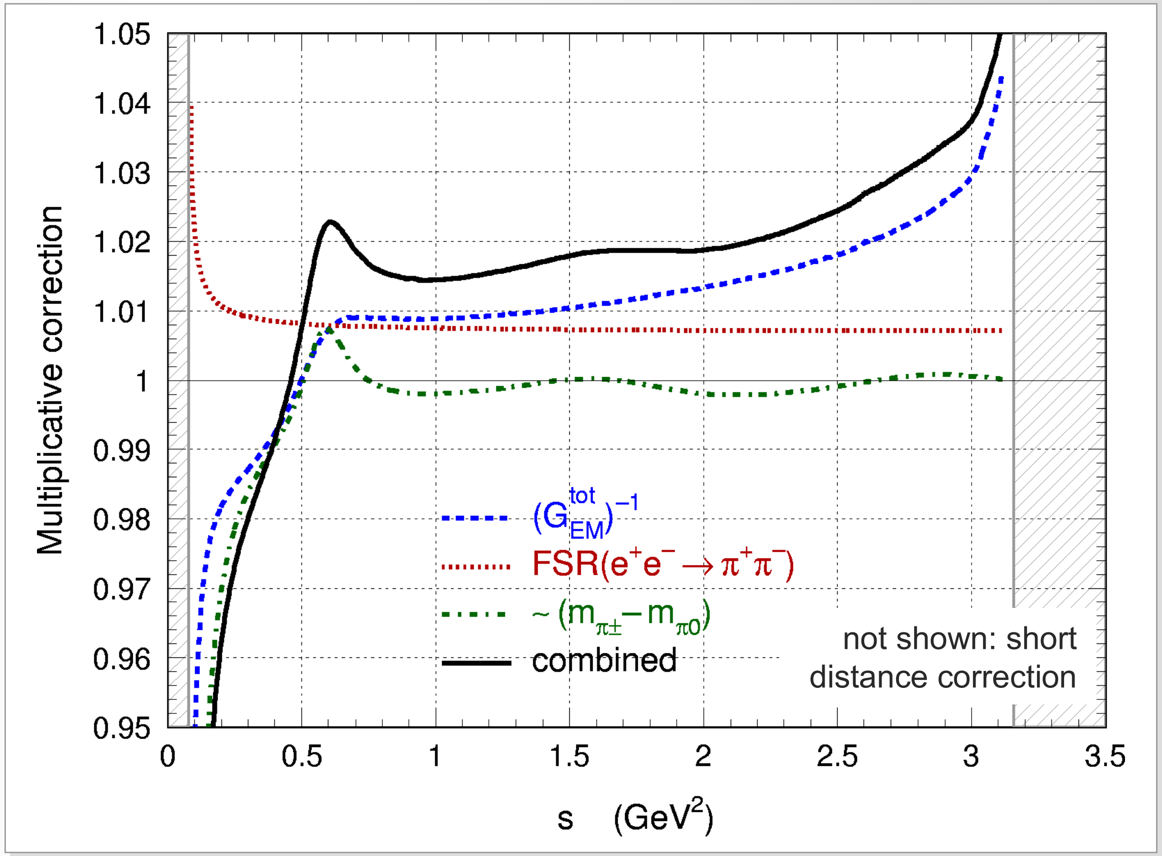
$m_u \neq m_d$ negligible

Marciano-Sirlin' 88

Braaten-Li' 90

Cirigliano-Ecker-Neufeld' 02

Alemany-Davier-H. 97, Czyż-Kühn' 01

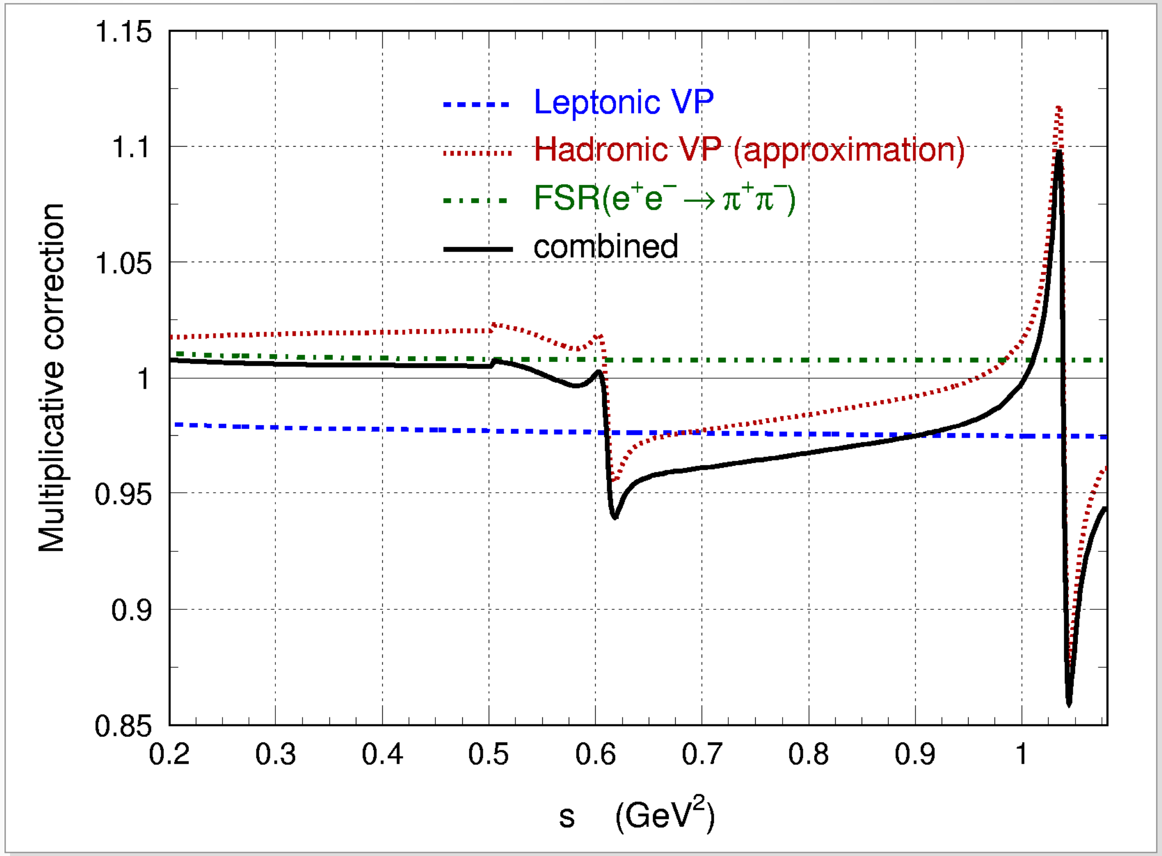


e^+e^- Radiative Corrections

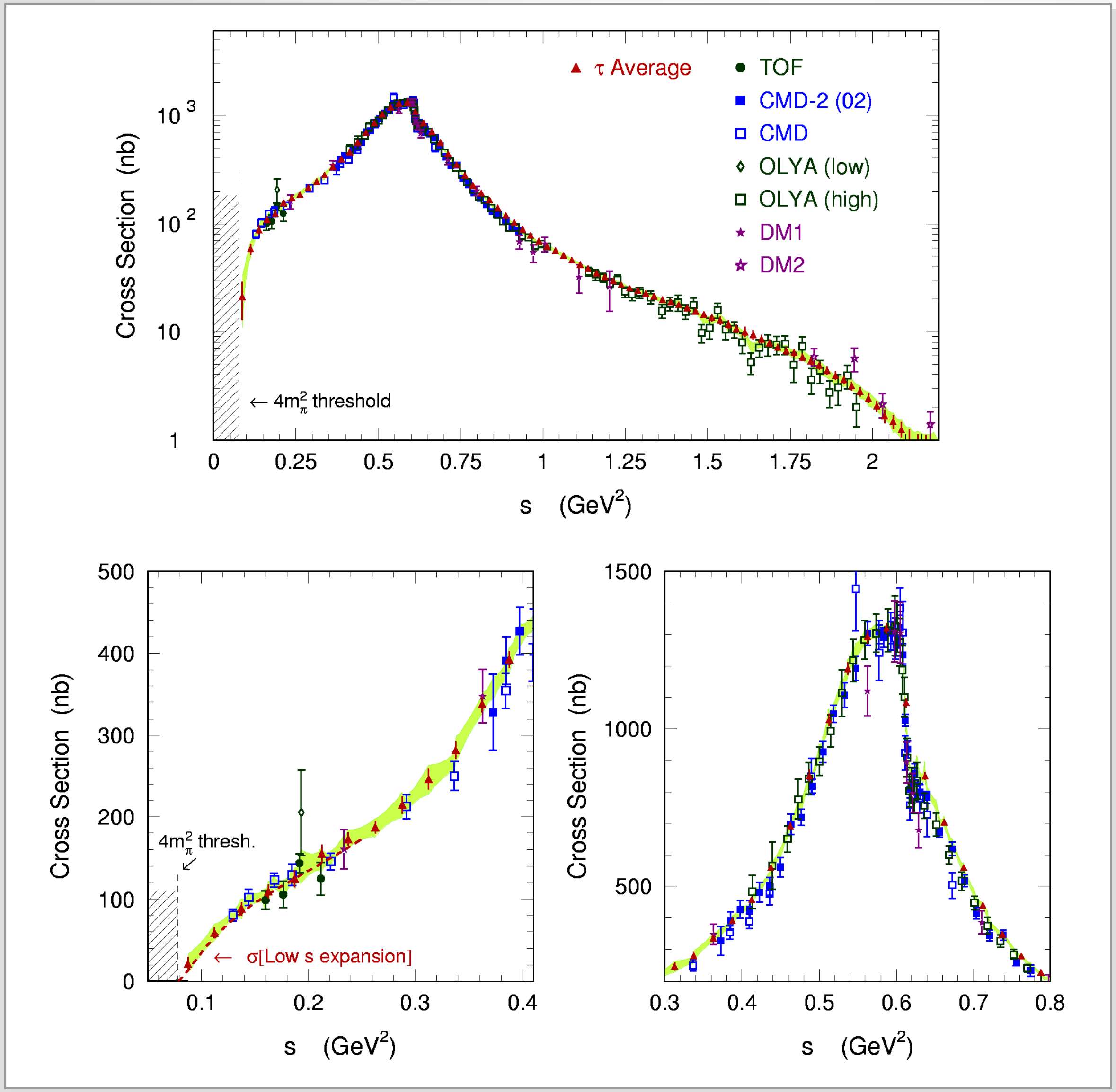
Multiple radiative corrections are applied on measured e^+e^- cross sections

- Vacuum polarization (VP) in the photon propagator
- Initial state radiation (ISR)
- Final state radiation (FSR)

[we need $e^+e^- \to$ hadrons (γ) in dispersion integral]



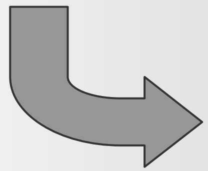
Comparing $e^+e^- \to \pi^+\pi^-$ and $\tau \to \pi^-\pi^0\nu_\tau$



Correct τ data for missing ρ-ω mixing (taken from BW fit) and all other SU(2)-breaking sources

Remarkable agreement

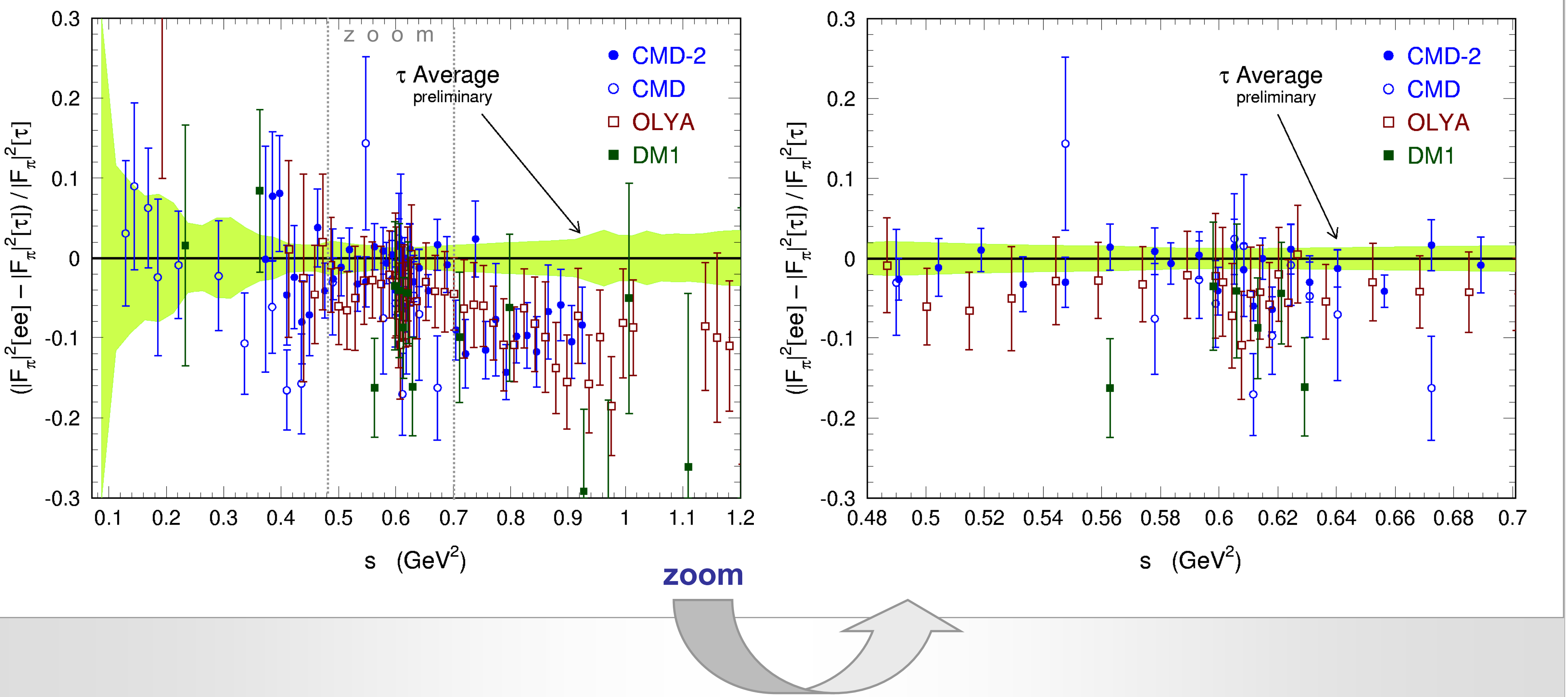
<u>But:</u> not good enough...



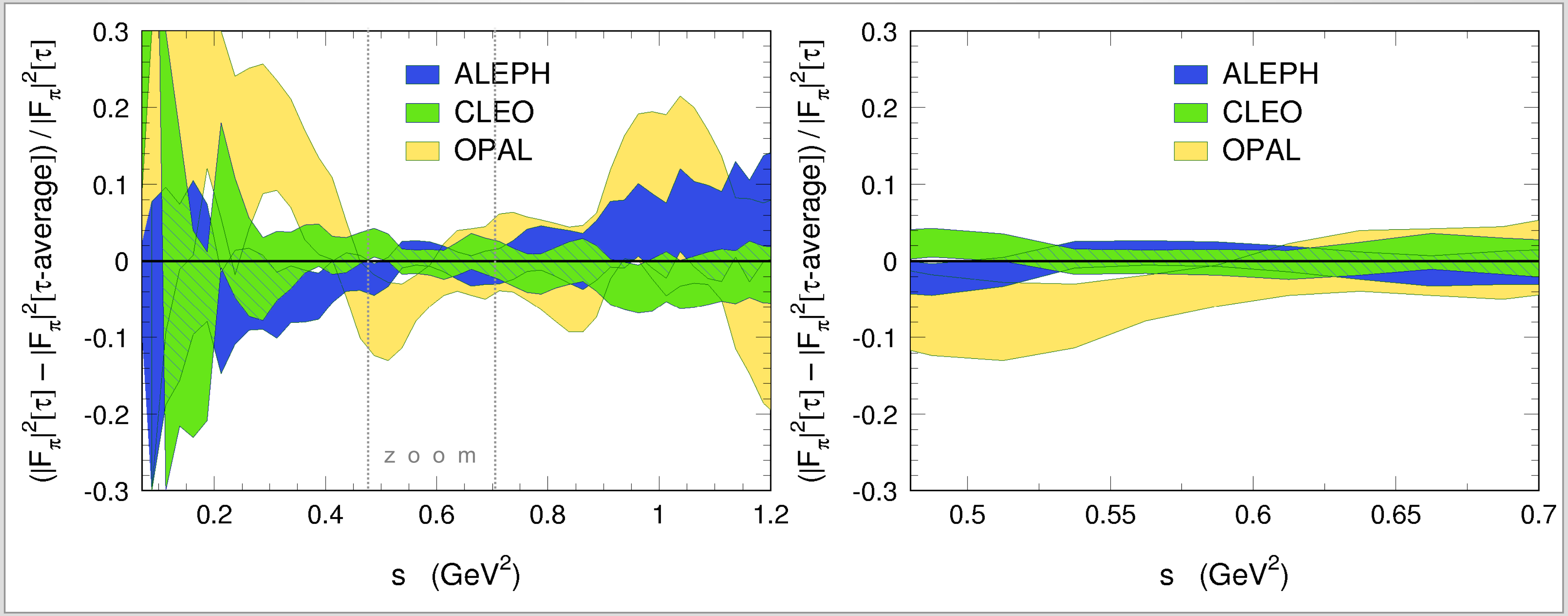
...

The Problem

Relative difference between τ and e^+e^- data:



$\tau^- \to \pi^-\pi^0\nu_\tau$: Comparing ALEPH, CLEO, OPAL



- Good agreement observed between ALEPH and CLEO
- ALEPH more precise at low *s*
- CLEO better at high *s*

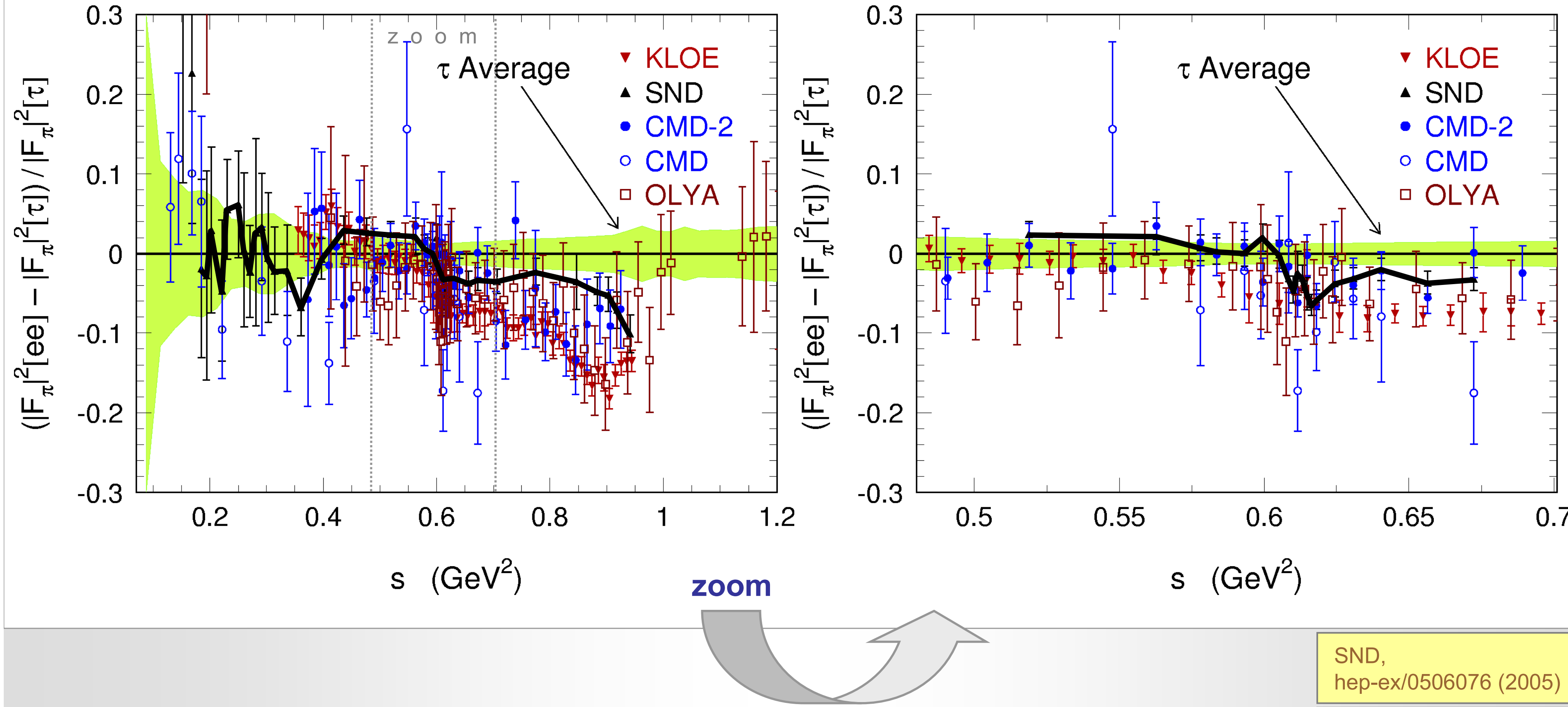
ALEPH,
Phys. Rept. 421, 191 (2005)

CLEO,
PRD 61, 112002 (2000)

OPAL,
EPJ C35, 437 (2004)

New $e^+e^- \to \pi^+\pi^-$ Data from KLOE ("radiative return") & SND

Relative difference between τ and e^+e^- data:



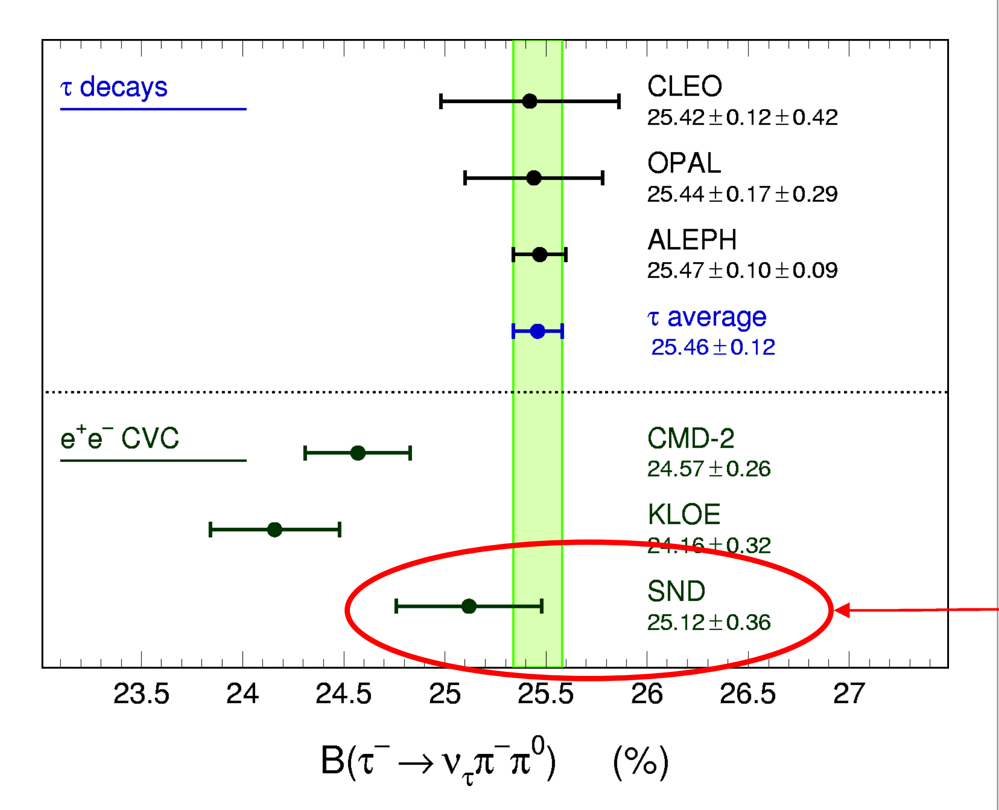
SND, hep-ex/0506076 (2005)

KLOE, PL B606, 12 (2005)

Another Way to Look at the Data

Compute τ branching fractions from e^+e^- data:

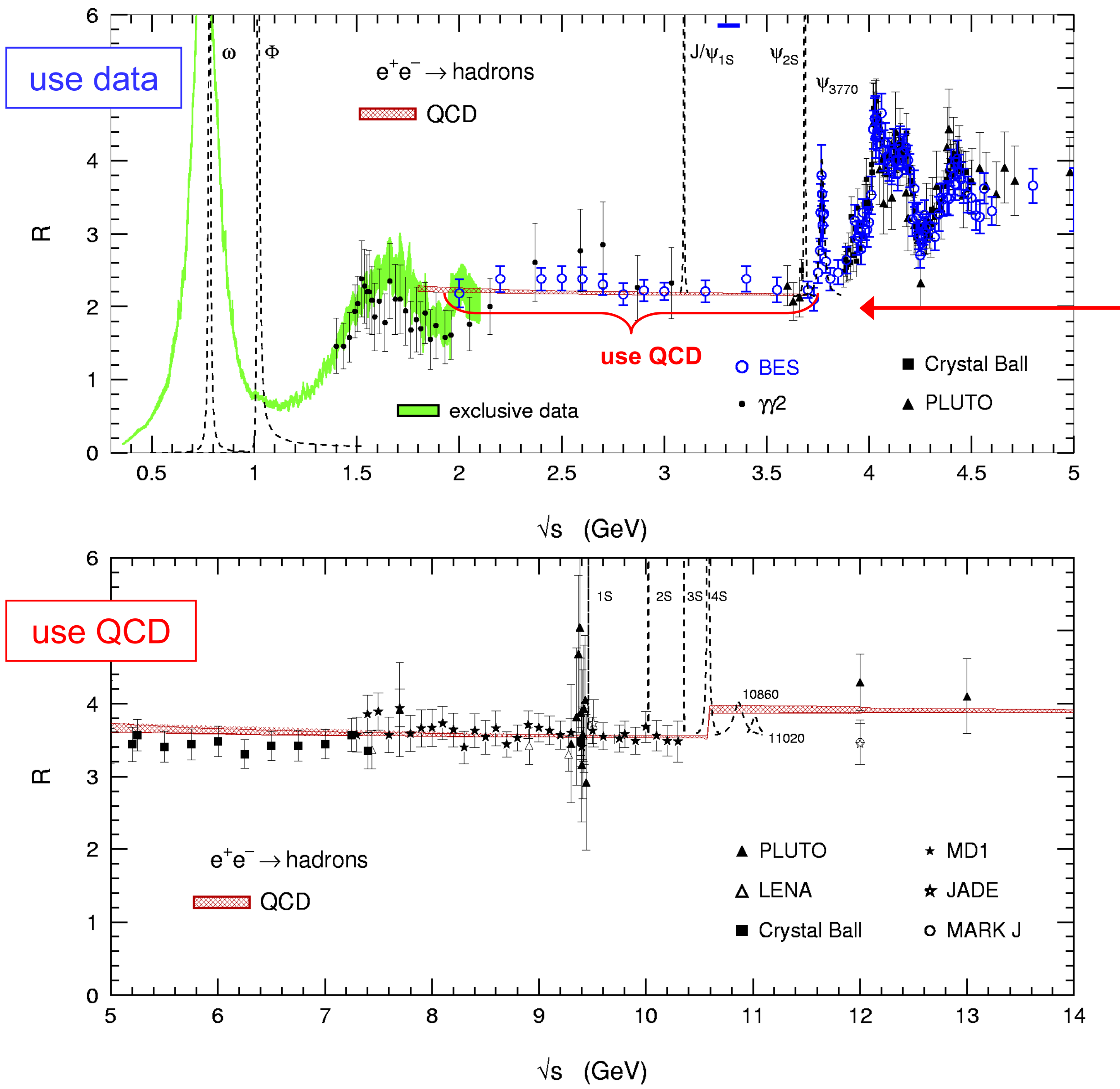
$$\mathrm{BR}_{\mathrm{CVC}}\left(\tau^- \to \pi^-\pi^0\nu_\tau\right) = \frac{6\pi\,|V_{ud}|^2\,S_{EW}}{m_\tau^2}\int_0^{m_\tau^2} ds\ \mathrm{kin}(s)\cdot \upsilon^{\mathrm{SU(2)\text{-}corrected}}_{e^+e^-\to\pi^+\pi^-}(s)$$



Does this solve the problem ?

Davier-Höcker-Zhang
hep-ph/0507078

Evaluating the Dispersion Integral



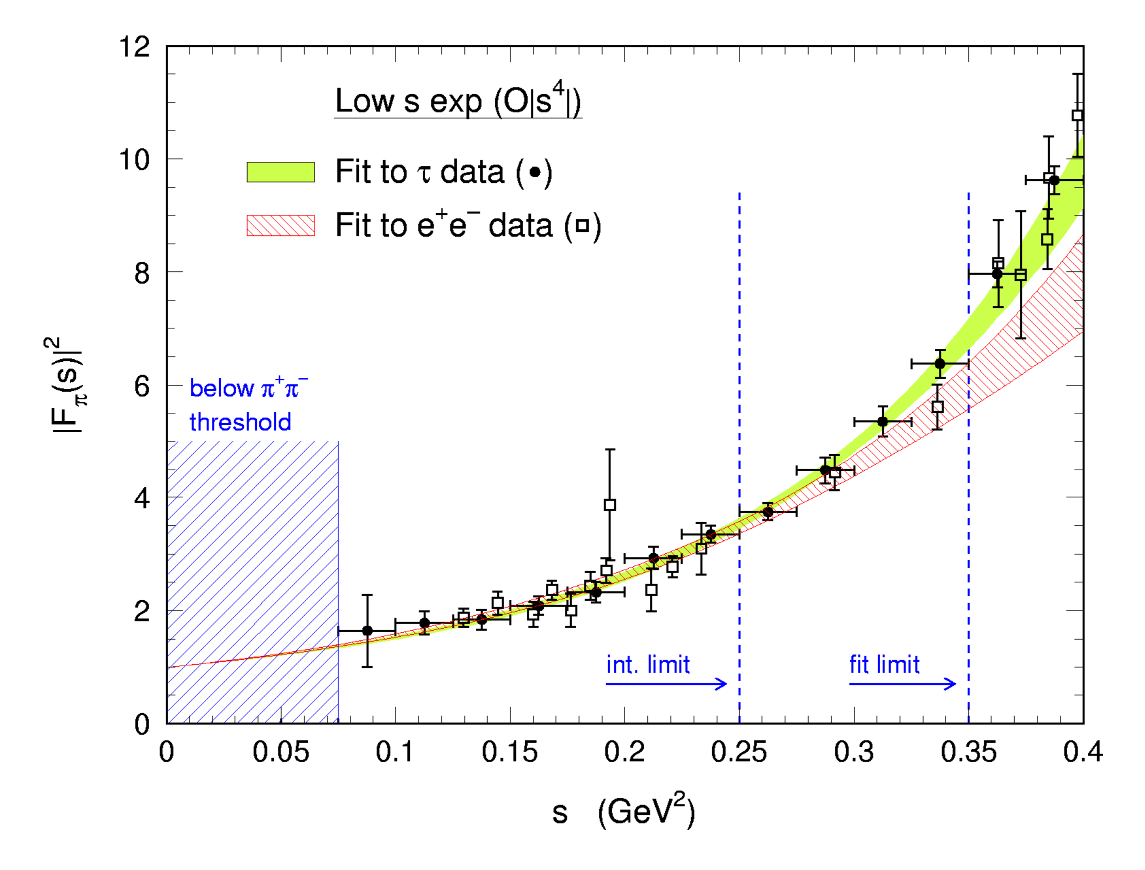
$$a_\mu^{\mathrm{had,LO}} = \frac{\alpha^2}{3\pi^2} \int_{4m_\pi^2}^{\infty} ds \frac{K(s)}{s} R(s)$$

Agreement between Data (BES) and pQCD (within *correlated* systematic errors)

digression: Specific Contributions: Threshold

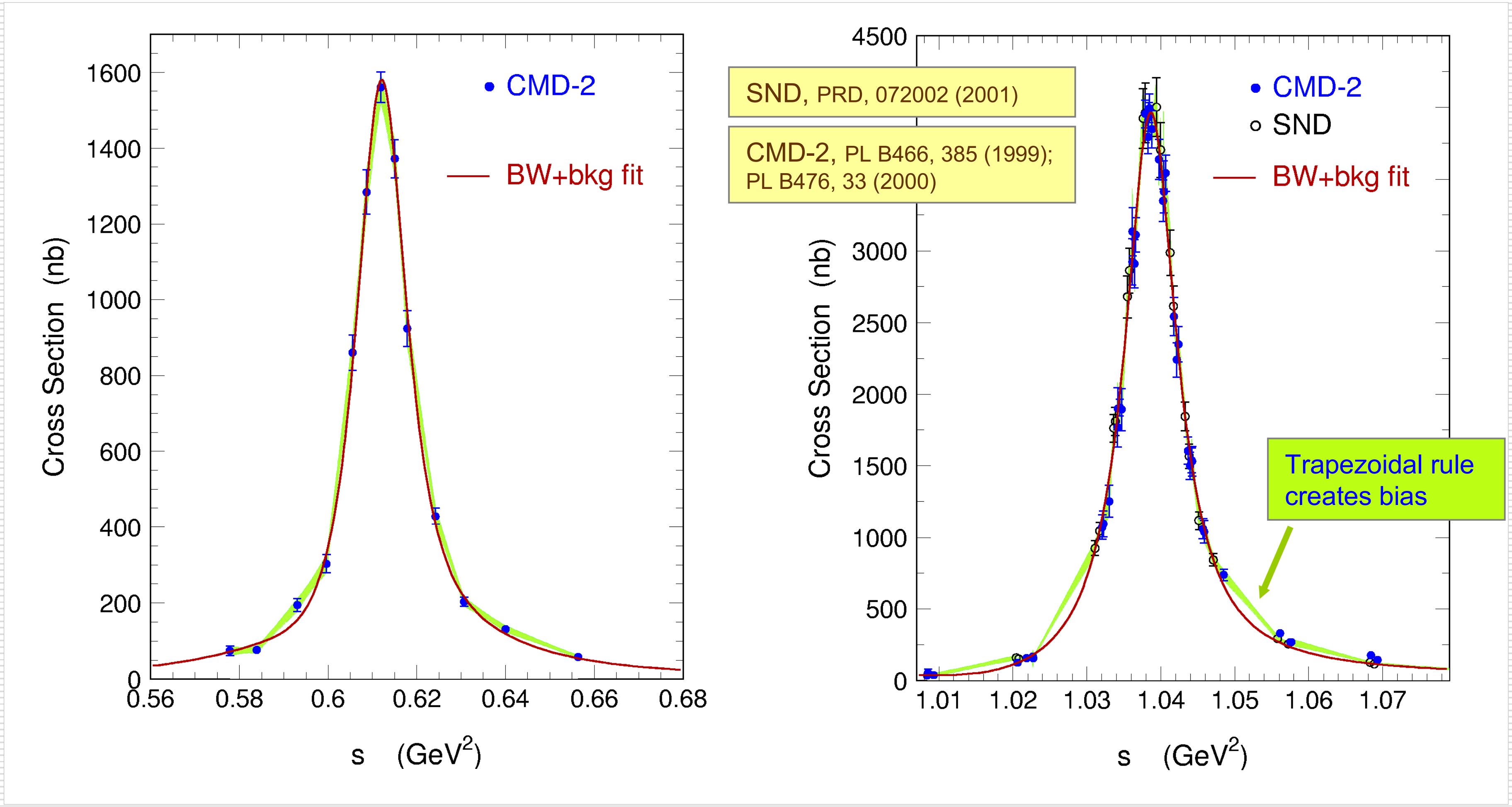
Use expansion for $\pi^+\pi^-$ threshold inspired by chiral perturbation theory:

$$\sigma_{\pi\pi} = \frac{\pi\alpha^2\beta^3}{3s}\left|F_\pi\right|^2 \qquad \text{and :} \qquad F_\pi = 1 + \frac{1}{6}\left\langle r^2\right\rangle_\pi s + c_1 s^2 + c_2 s^3 + O(s^4)$$



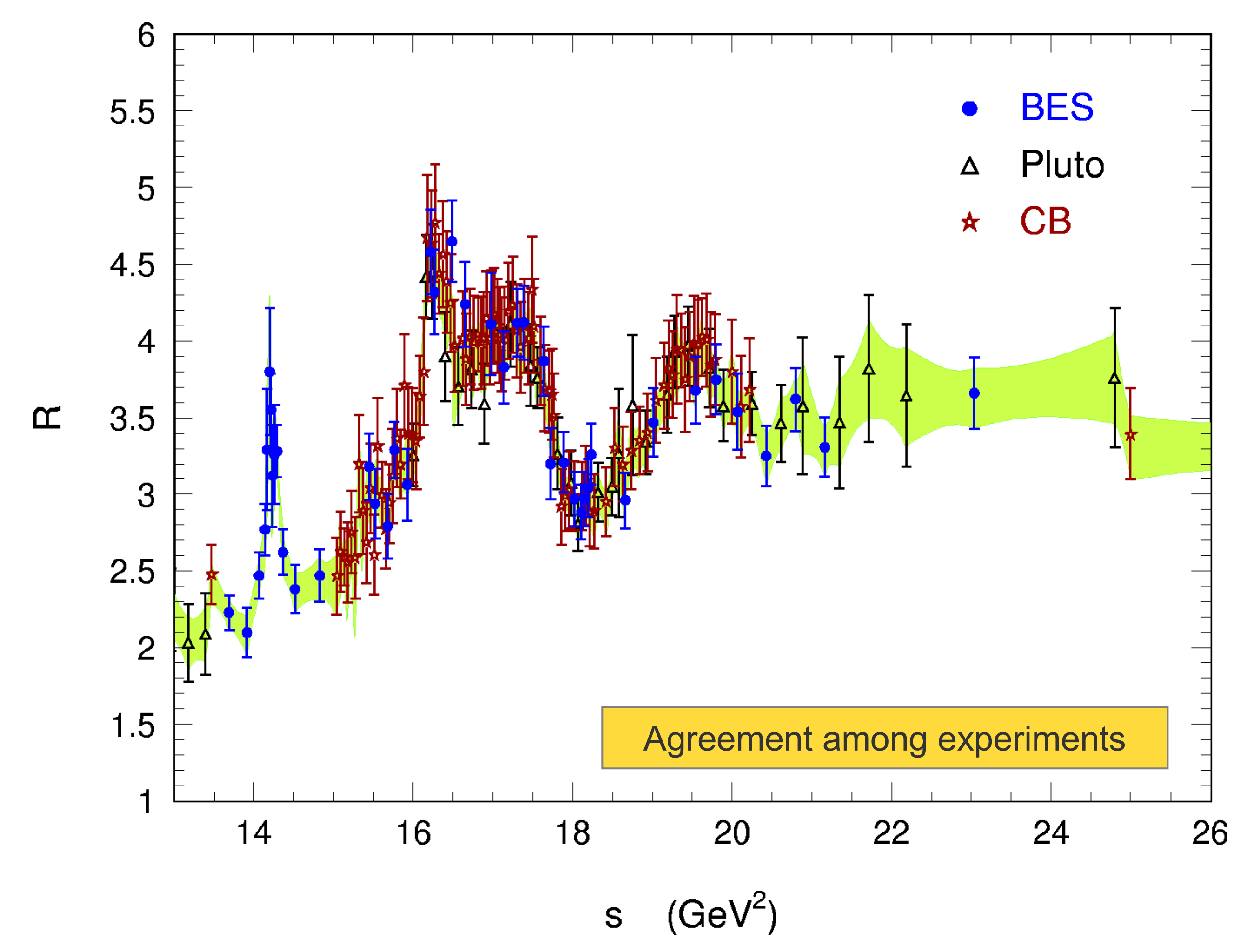
digression: Specific Contributions: Narrow Light Resonances

Use direct data integration for $\omega(782)$ and $\phi(1020)$ to account for non-resonant contributions.



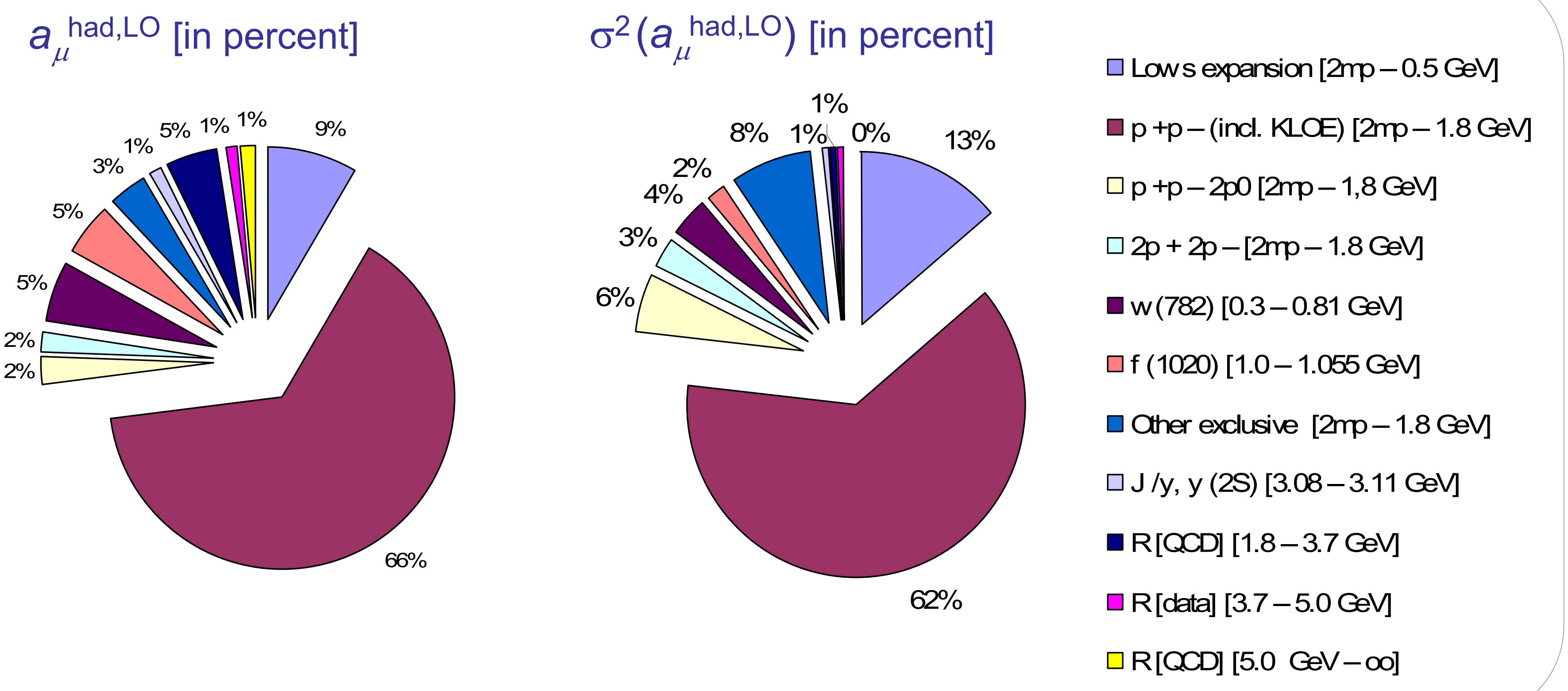
digression: Specific Contributions: Charm Threshold

New precise BES data improve $c\bar{c}$ resonance region:



Results: the Compilation (including KLOE, but not (yet) SND)

Contributions to $a_\mu^{\mathrm{had,LO}}$ [in 10^{-10}] from the different energy domains:



Results: the Compilation (including KLOE, but not (yet) SND)

Contributions to $a_\mu^{\text{had,LO}}$ [in 10^{-10}] from the different energy domains:

Modes	Energy [GeV]	e^+e^-	τ
Low s expansion	$2m_\pi - 0.5$	$58.0 \pm 1.7 \pm 1.2_{\text{rad}}$	$56.0 \pm 1.6 \pm 0.3_{\text{SU(2)}}$
[$\pi^+\pi^-$ (DEHZ'03)]	$2m_\pi - 1.8$	[$450.2 \pm 4.9 \pm 1.6_{\text{rad}}$]	$464.0 \pm 3.0 \pm 2.3_{\text{SU(2)}}$
$\pi^+\pi^-$ (incl. KLOE)	$2m_\pi - 1.8$	$448.3 \pm 4.1 \pm 1.6_{\text{rad}}$	–
$\pi^+\pi^- 2\pi^0$	$2m_\pi - 1.8$	$16.8 \pm 1.3 \pm 0.2_{\text{rad}}$	$21.4 \pm 1.3 \pm 0.6_{\text{SU(2)}}$
$2\pi^+ 2\pi^-$	$2m_\pi - 1.8$	$14.2 \pm 0.9 \pm 0.2_{\text{rad}}$	$12.3 \pm 1.0 \pm 0.4_{\text{SU(2)}}$
ω (782)	0.3 – 0.81	$38.0 \pm 1.0 \pm 0.3_{\text{rad}}$	–
ϕ (1020)	1.0 – 1.055	$35.7 \pm 0.8 \pm 0.2_{\text{rad}}$	–
Other exclusive	$2m_\pi - 1.8$	$24.0 \pm 1.5 \pm 0.3_{\text{rad}}$	–
J/ψ, ψ(2S)	3.08 – 3.11	$7.4 \pm 0.4 \pm 0.0_{\text{rad}}$	–
R [QCD]	1.8 – 3.7	$33.9 \pm 0.5 \pm 0.0_{\text{rad}}$	–
R [data]	3.7 – 5.0	$7.2 \pm 0.3 \pm 0.0_{\text{rad}}$	–
R [QCD]	$5.0 - \infty$	$9.9 \pm 0.2_{\text{theo}}$	–
Sum (incl. KLOE)	$2m_\pi - \infty$	$693.4 \pm 5.3 \pm 3.5_{\text{rad}}$	$711.0 \pm 5.0 \pm 0.8_{\text{rad}} \pm 2.8_{\text{SU(2)}}$

The Full Hadronic Contribution

- Hadronic leading order contribution

$$a_\mu^{\text{had,LO}}[e^+e^-] = (693.4 \pm 5.3 \pm 3.5_{\text{rad}}) \times 10^{-10}$$

- Hadronic next-to-leading order contributions

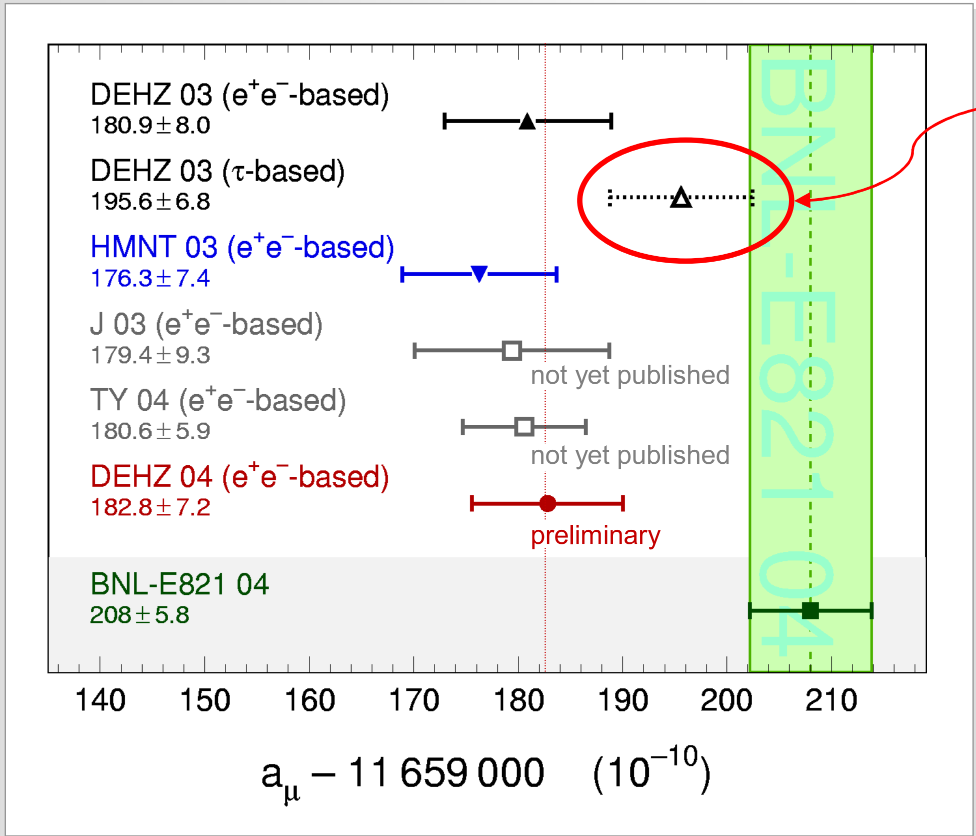
  - Vacuum polarization (1-loop) + additional photon or VP insertion:
    - computed akin to LO part via dispersion integral with modified kernel function
    - $a_\mu^{\text{had,NLO}} = -9.8(0.1) \times 10^{-10}$
  - Light-by-light scattering :
    - dispersion relation approach not possible (4-point function)
    - no first-principle calculation yet (*e.g.*, on the lattice)
    - model calculations using short dist. quark loops, π^0, $\eta^{(\prime)}$, … pole insertions and $\pi^\pm$ loops in the large-N_C limit
    - $a_\mu^{\text{had,LBL}} = +12.0(3.5) \times 10^{-10}$

based on:
Melnikov-Vainshtein, PRD 70, 113006 (2004)

And the Complete Result

$$a_\mu^{SM}[e^+e^-] = (11{,}659{,}182.8 \pm 6.3_{\text{had,LO}} \pm 3.5_{\text{LBL}} \pm 0.3_{\text{QED+weak}}) \times 10^{-10}$$

DEHZ (ICHEP 2004)



However, with SND'05, the Tau might be back in the game ?!

SM (2004):

$a_\mu^{\text{exp}} = (11{,}659{,}182.8 \pm 7.2)\ 10^{-10}$

BNL E821 (2004):

$a_\mu^{\text{exp}} = (11{,}659{,}208.0 \pm 5.8)\ 10^{-10}$

Observed Difference with Experiment:

$$\boldsymbol{a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (25.2 \pm 9.2)} \times 10^{-10}$$

➡ 2.7 "standard deviations"

And the Complete Result

$$a_\mu^{SM}[e^+e^-] = (11{,}659{,}182.8 \pm 6.3_{had,LO} \pm 3.5_{LBL} \pm 0.3_{QED+weak}) \times 10^{-10}$$

DEHZ 03 (e^+e^-
180.9 ± 8.0
DEHZ 03 (τ-ba
195.6 ± 6.8
HMNT 03 (e^+e^-
176.3 ± 7.4
J 03 (e^+e^--base
179.4 ± 9.3
TY 04 (e^+e^--ba
180.6 ± 5.9
DEHZ 04 (e^+e^-
182.8 ± 7.2
BNL-E821 04
208 ± 5.8
140 150

HZ (ICHEP 2004)
the Tau
e game ?!
7.2) 10^{-10}
5.8) 10^{-10}
Experiment:
9.2) $\times 10^{-10}$
iations"

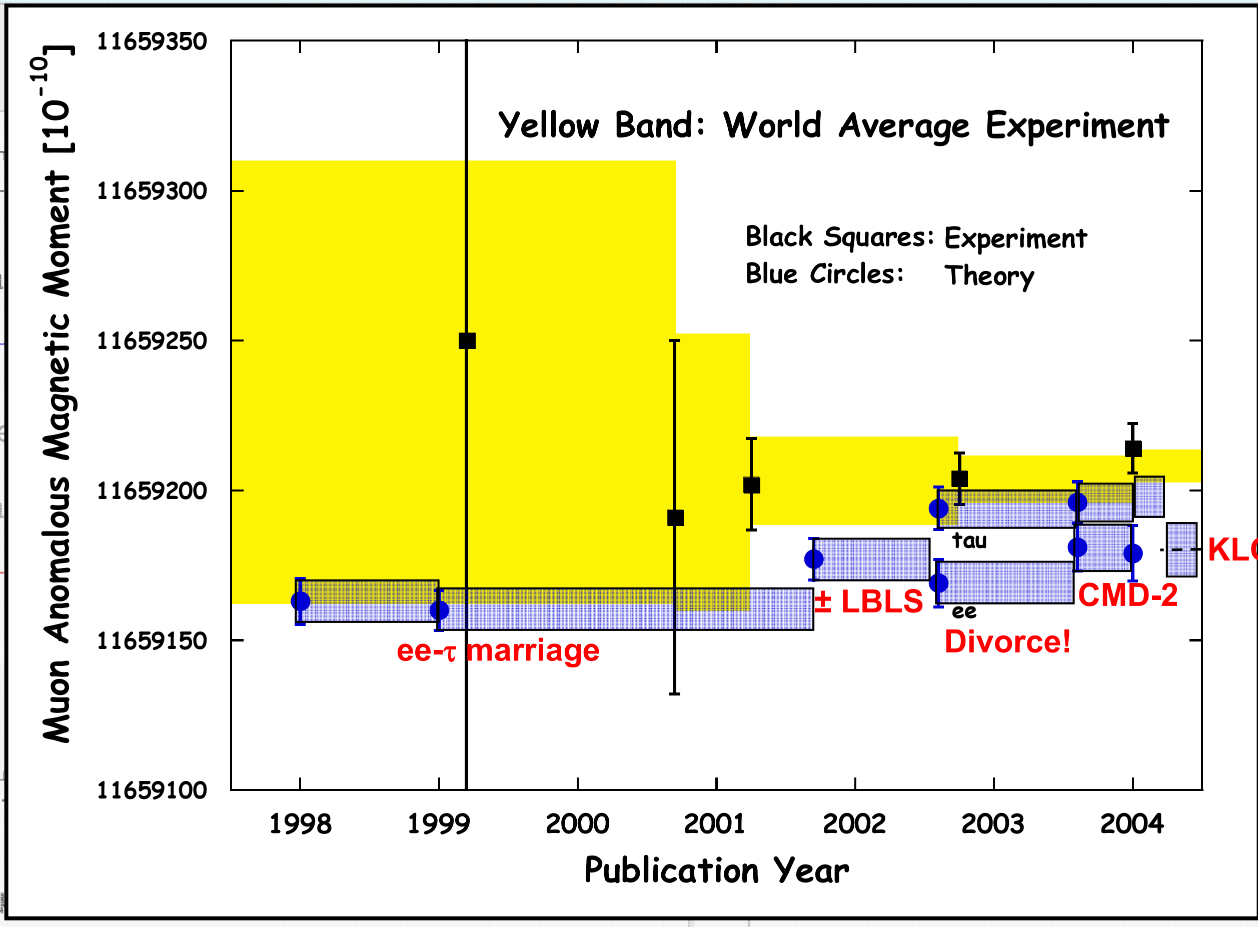


Figure copied from D. Hertzog, UIUC

What can we do with it ?

- 2.7"σ" discrepancy :
  - (a) statistical fluctuation ?
  - (b) experimental systematic ?
  - (c) theory value incorrect ?
  - (d) new physics (NP) ?

The points (a-c) were the subject of this talk.

- Concerning (d) :

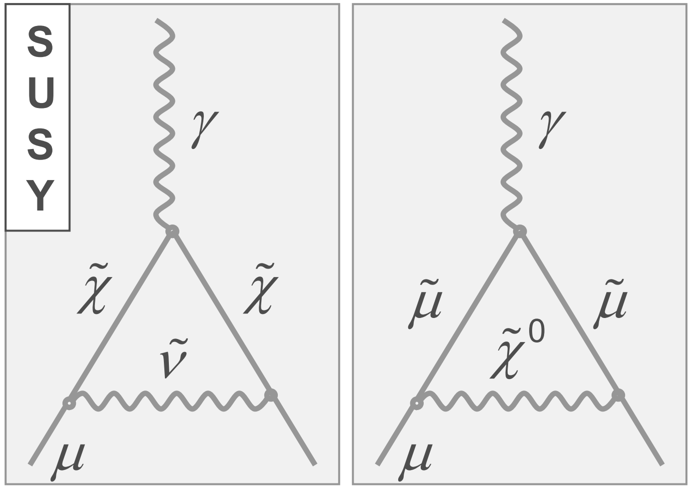
Leading contender for NP : SUSY

Fayet 80, Grifols-Mendez 82, Ellis-Hagelin-Nanopoulos 82, Barbieri-Maiani 82, ...

Involves $(\tilde{\mu}, \tilde{\chi}^0)$ and $(\tilde{\chi}, \tilde{\nu}_\mu)$ loops at mass scale $m_{\rm SUSY}$

Moroi 96, Ibrahim-Nath 98, Kosower-Krauss-Sakai (83), …

$$a_\mu^{\rm SUSY} \simeq {\rm sgn}(\mu) \times a_\mu^{\rm weak} \left(\frac{90\ {\rm GeV}}{m_{\rm SUSY}}\right)^2 \tan\beta$$



What can we do with it ?

- 2.7"σ" discrepancy :
  - (a) statistical fluctuation ?
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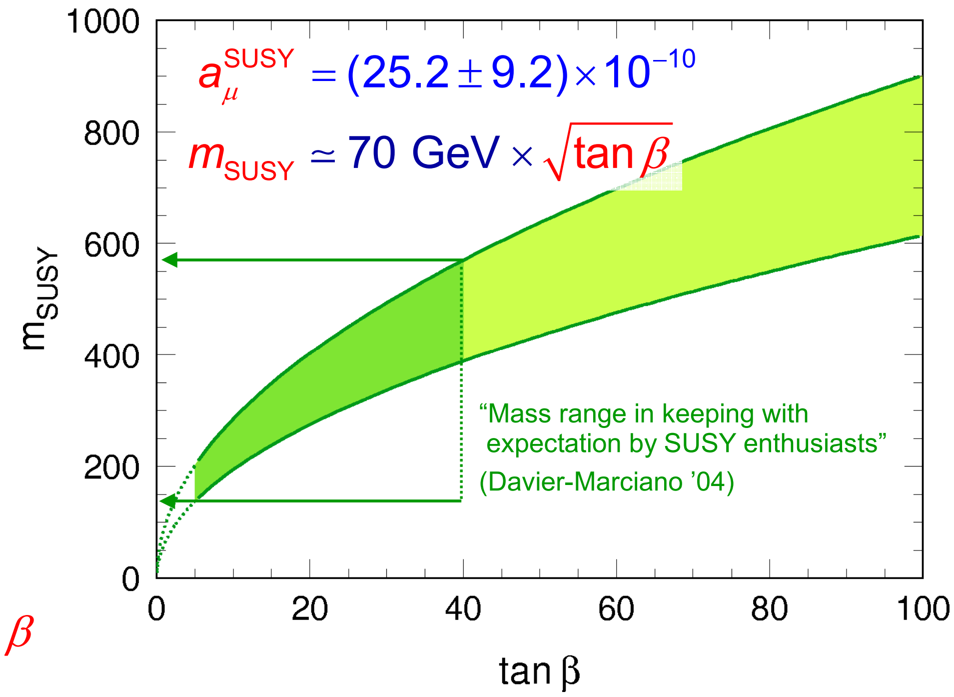
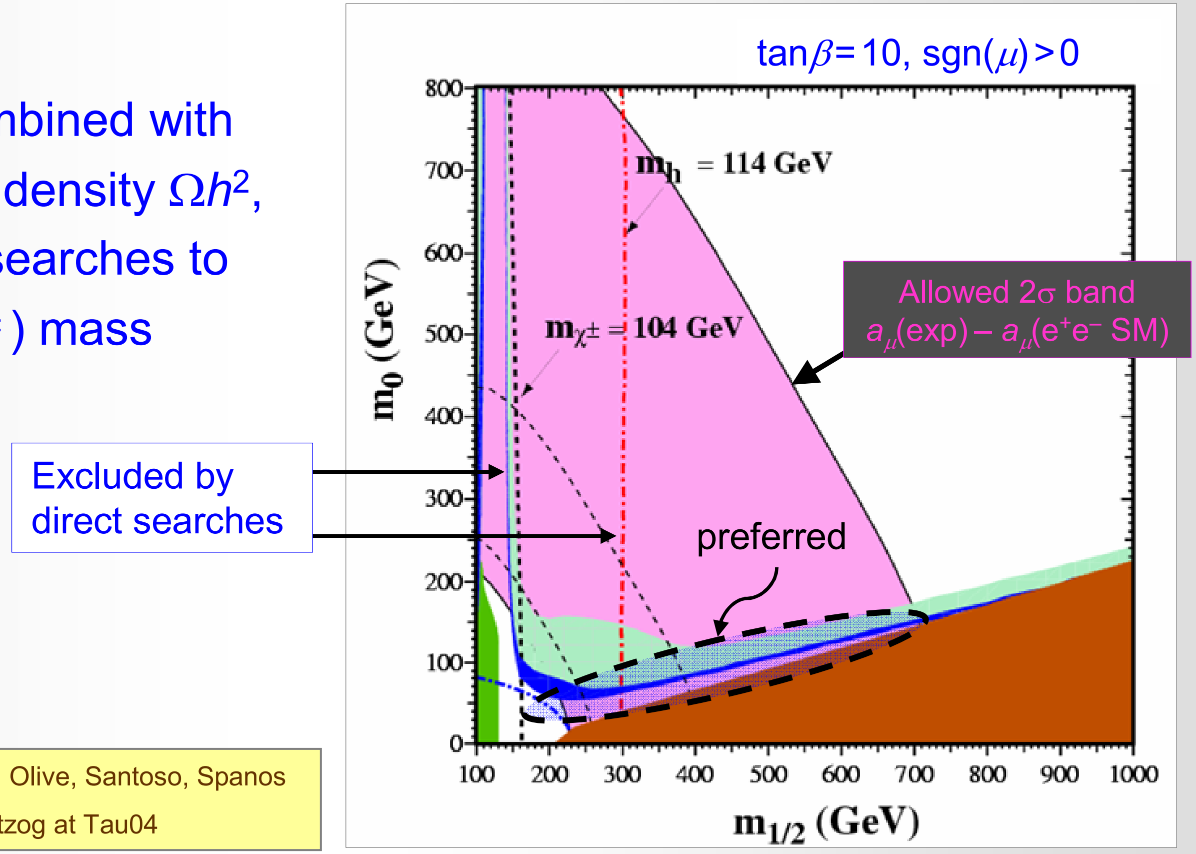


The New Physics Option

- More generally: a_μ is sensitive to a large number of Standard Model extensions:
  - SUSY, Leptoquarks, radiative light fermion masses, … $a_\mu^{\rm NP} \approx {\rm sgn}(...) \times O(1) \times \underbrace{(m_\mu / \Lambda_{\rm NP})^2}_{\Lambda_{\rm NP} \approx O(1-2\ {\rm TeV})}$
  - for some, deviation could be too large: W_R, Z', anomalous W, …

- In CMSSM, a_μ can be combined with $b \to s\gamma$, cosmological relic density Ωh^2, (WMAP) and LEP Higgs searches to constrain the chargino ($\chi^\pm$) mass

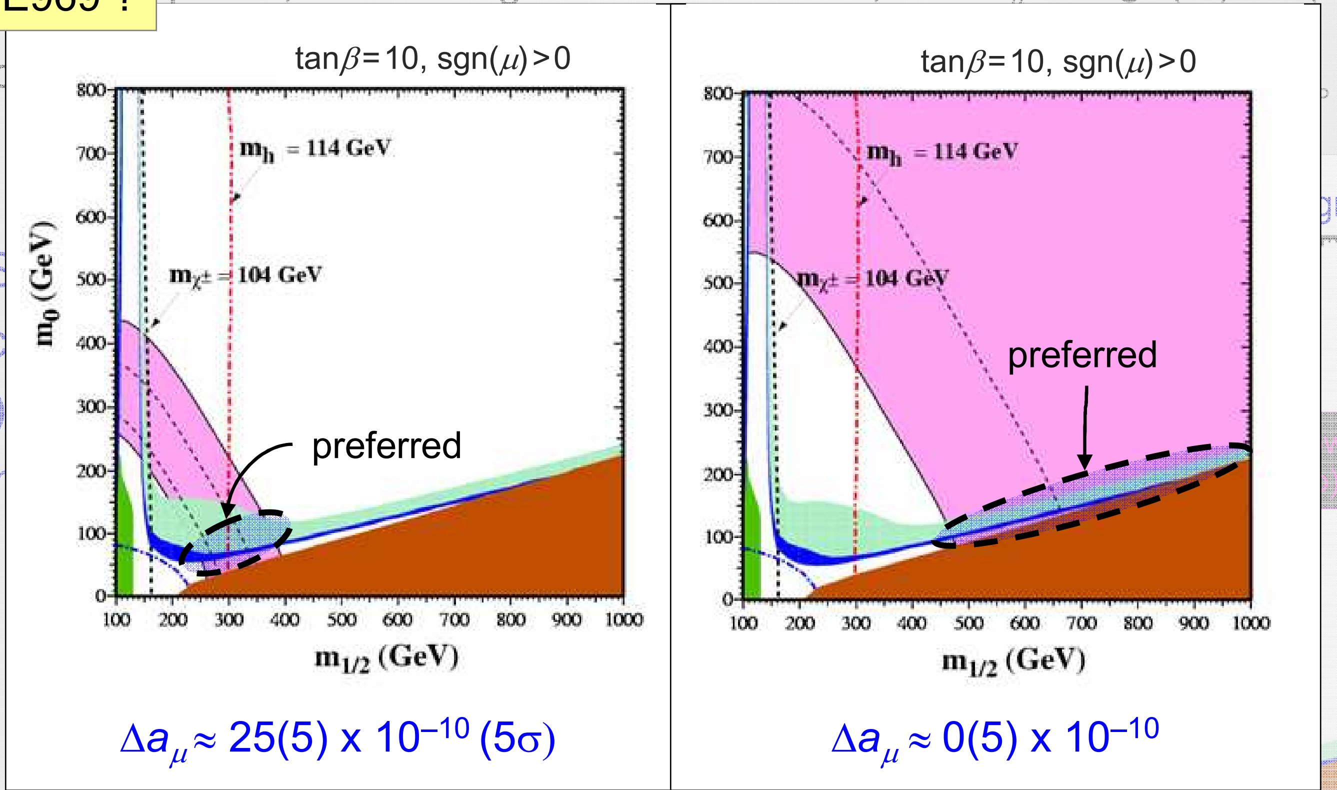


K.Olive based on Ellis, Olive, Santoso, Spanos
plot taken from D. Hertzog at Tau04

The New Physics Option

- More generally: a_μ is sensitive to a large number of Standard Model extensions:
  - … quarks, radiative light fermion masses, … $a_\mu^{NP} \approx \mathrm{sgn}(\ldots) \times O(1) \times (m_\mu / \Lambda_{NP})^2$
  - for s… $\approx O(1-2\ \mathrm{TeV})$
- In CMSS… $b \to s\gamma$, c… (WMAP) constrai…

With E969 ?



$\Delta a_\mu \approx 25(5) \times 10^{-10}$ (5σ)

$\Delta a_\mu \approx 0(5) \times 10^{-10}$

Figures taken from D. Hertzog (UIUC) @ TAU04

K.Olive based on Ellis, Olive, Santoso, Spanos plot taken from D. Hertzog at Tau04

conclusions

and perspectives

conclusions

- Phenomenal experimental progress with BNL (E821) $g-2$ measurement
- Theory progress much less impressive → mostly due to missing high quality data
- Strong disagreement between KLOE and SND data sets. Also disagreement with CMD-2
- Tau data all in mutual agreement; SND in fair agreement with Tau data
- Hadronic vacuum polarization stays dominant systematics for SM value of the muon $g-2$
- Difference between experiment and theory is within range of possible NP contributions

and perspectives

Future experimental input expected from:

- New CMD-2 results forthcoming, especially at low and large $\pi^+\pi^-$ masses
- BABAR ISR: $\pi^+\pi^-$ spectral function over full mass range, multihadron channels
- New BNL experiment E969(*) planned, aiming at twice better precision
- Ambitious muon $g-2$ project at J-PARC(**), aiming at $(0.1 – 0.2) \times \sigma$(BNL-E821)
- At long term hadronic LBL scattering contribution will limit theory precision

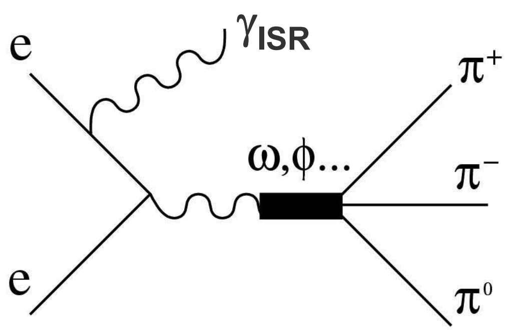
(*) for information, see for example: http://g2pc1.bu.edu/~roberts/ (**) http://g2pc1.bu.edu/~roberts/g2jparc/

appendix

ISR with BABAR

Radiative Return Cross Section Results from BABAR

- The *Radiative Return*: benefit from huge luminosities at *B* and ϕ Factories to perform continuous cross section measurements



$$\frac{d\sigma(s,x)}{dx\,d(\cos\theta)} = H(s,x,\theta)\cdot\,\sigma_0\big(s(1-x)\big)$$

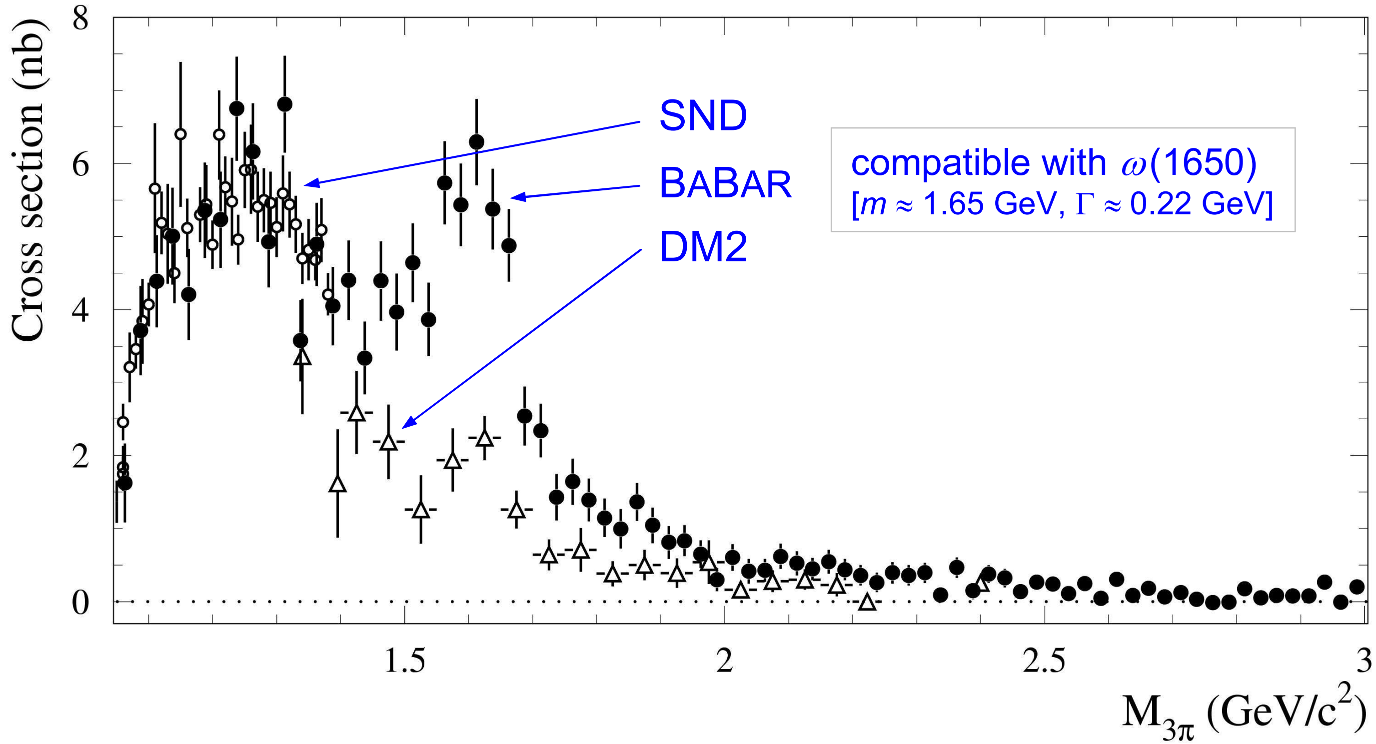
$$H(s,x,\theta) = \frac{\alpha}{\pi x}\left(\frac{2-2x+x^2}{\sin^2\theta}-\frac{x^2}{2}\right),\quad x=\frac{2E_\gamma}{\sqrt{s}}$$

H is radiation function

- High PEP-II luminosity at $\sqrt{s}$ = 10.58 GeV → precise measurement of the e^+e^- cross section σ_0 at low c.m. energies with BABAR
- Comprehensive program at BABAR
- Results for $\pi^+\pi^-\pi^0$, preliminary results for $2\pi^+2\pi^-$, $K^+K^-\pi^+\pi^-$, $2K^+2K^-$ from 89.3 fb^{-1}

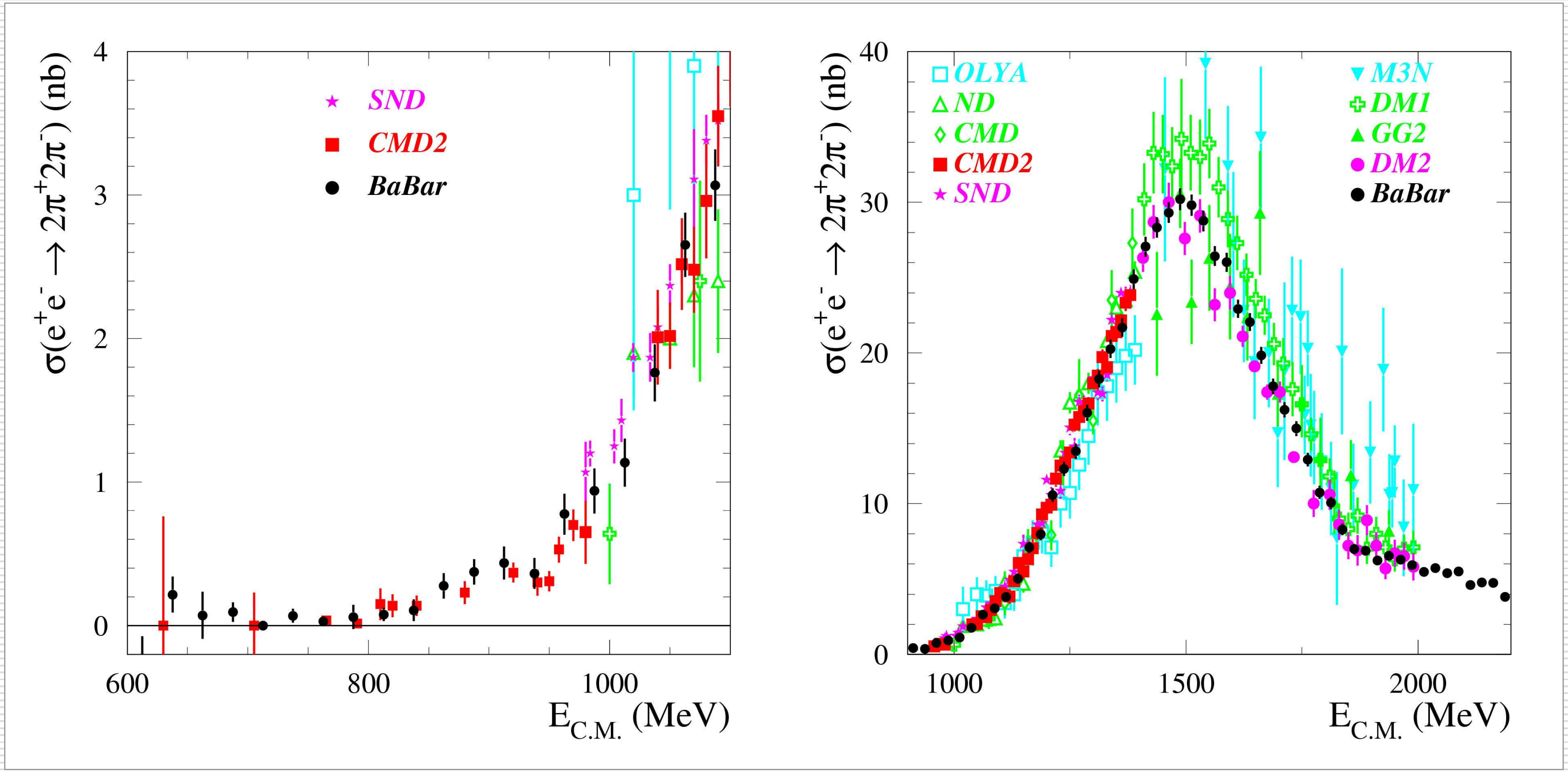
The $e^+e^- \to \pi^+\pi^-\pi^0$ Cross Section

- Cross section above ϕ resonance : DM2 missed a resonance **!**



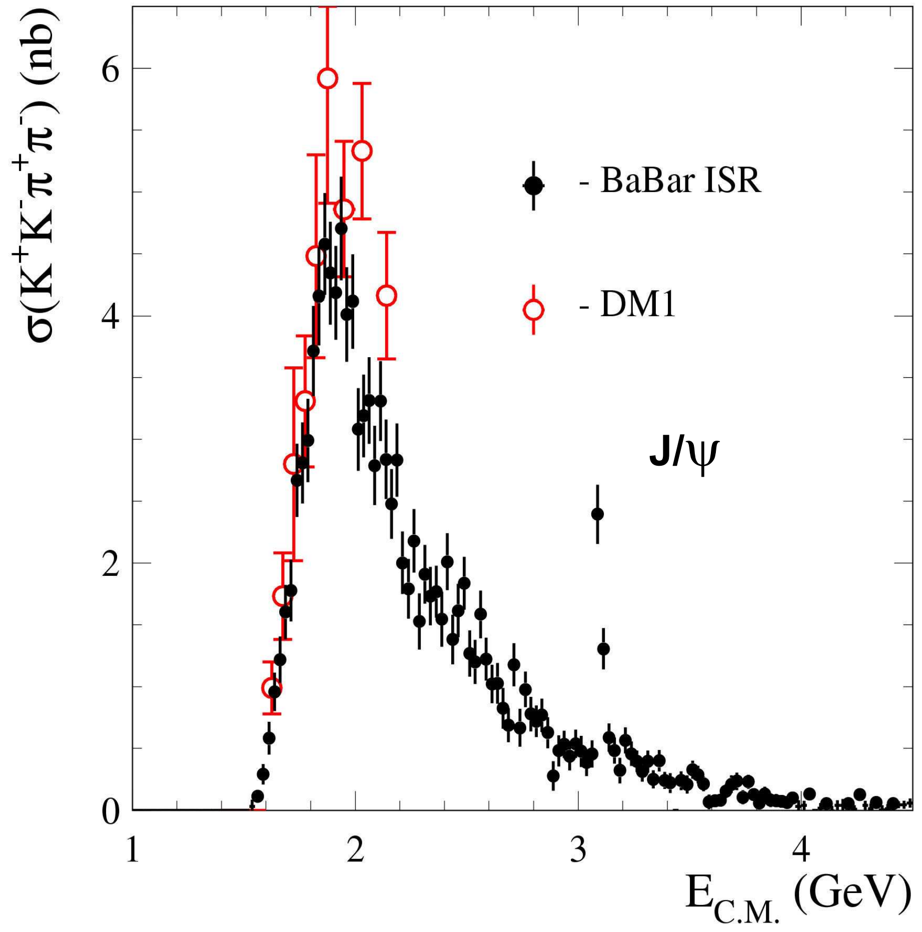
- Coverage of wide region in one experiment
- Consistent with SND data $E_{\text{C.M.}}$ < 1.4 GeV
- Inconsistent with DM2 results
- Overall normalization error ~ 5% up to 2.5 GeV

The $e^+e^- \to 2\pi^+2\pi^-$ Cross Section



- Good agreement with direct $e^+ e^-$ measurements
- Most precise result above 1.4 GeV

The $e^+e^-\to K^+K^-\pi^+\pi^-$ Cross Section



Systematic error – 15% (mainly model dependence)

Much more precise than previous measurement

Substantial resonance sub-structures observed:

- $K^*(890)K\pi$ dominant
- $\phi\pi\pi$, ρKK contribute strongly
- $K^*_2(1430)K\pi$ seen

Impact on $(g-2)_\mu$

- Contributions to a_μ^{had} ($\times 10^{-10}$)
  - from $2\pi^+ 2\pi^-$ ($\sqrt{s}$ = 0.56 – 1.8 GeV) :
    - from all e^+e^- exp. : $14.21 \pm 0.87_{\text{exp}} \pm 0.23_{\text{rad}}$
    - from all τ data : $12.35 \pm 0.96_{\text{exp}} \pm 0.40_{\text{SU(2)}}$
    - from BABAR : $12.95 \pm 0.64_{\text{exp}} \pm 0.13_{\text{rad}}$
  - from $\pi^+\pi^-\pi^0$ ($\sqrt{s}$ = 1.055 – 1.8 GeV) :
    - from all e^+e^- exp. : $2.45 \pm 0.26_{\text{exp}} \pm 0.03_{\text{rad}}$
    - from BABAR : $3.31 \pm 0.13_{\text{exp}} \pm 0.03_{\text{rad}}$
- Many more modes to come; aim for systematic errors $\leq$1% (in $\pi^+\pi^-$)
- Several $B(J/\psi \to X)$ measurements better than current world average