ALEPH Tau Spectral Functions and QCD

—— revisited(*) ——

Michel Davier (LAL), Andreas Höcker (CERN) and Zhiqing Zhang (LAL)

Tau Workshop, September 19–22, 2006, SNS Pisa, Italy

(*) Rev. Mod. Phys. vol.78 July-Sept 2006; hep-ph/0507078

Outline

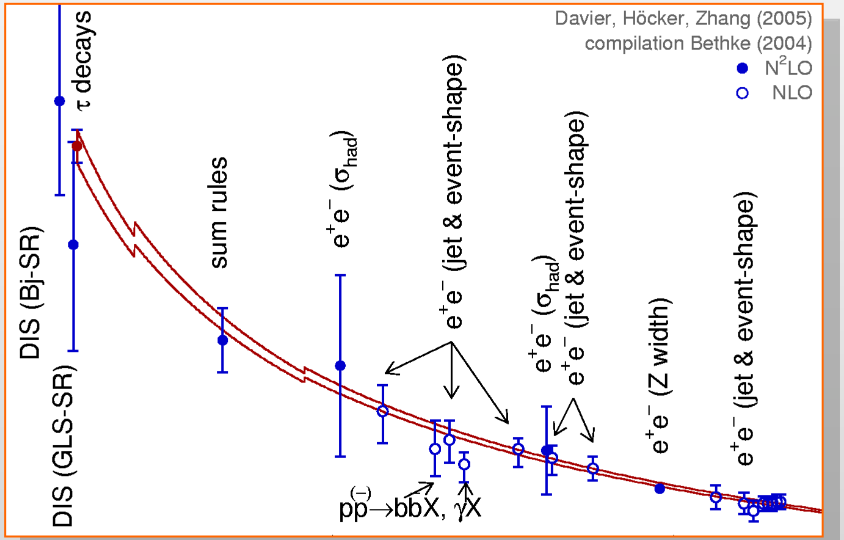
I. Revisiting Tau Hadronic Spectral Functions

II. Theoretical Framework

III. Measurement of the Strong Coupling Constant

IV. Test of Asymptotic Freedom

V. Conclusions



Tau Hadronic Spectral Functions (SFs)

- Hadronic physics factorizes in (vector and axial-vector) Spectral Functions :

$$\upsilon / a\left[\tau^- \to V^- / A^- \nu_\tau\right] \propto \underbrace{\frac{BR\left[\tau^- \to V^- / A^- \nu_\tau\right]}{BR\left[\tau^- \to e^- \bar{\nu}_e \nu_\tau\right]}}_{\text{branching fractions}} \underbrace{\frac{1}{N_{V/A}} \frac{dN_{V/A}}{ds}}_{\text{mass spectrum}} \underbrace{\frac{m_\tau^2}{\left(1 - s/m_\tau^2\right)^2 \left(1 + s/m_\tau^2\right)}}_{\text{kinematic factor (PS)}}$$

Fundamental ingredient relating long distance hadrons to short distance quarks (QCD)

- Isospin symmetry connects $I = 1$ e^+e^- cross section to vector τ spectral functions:

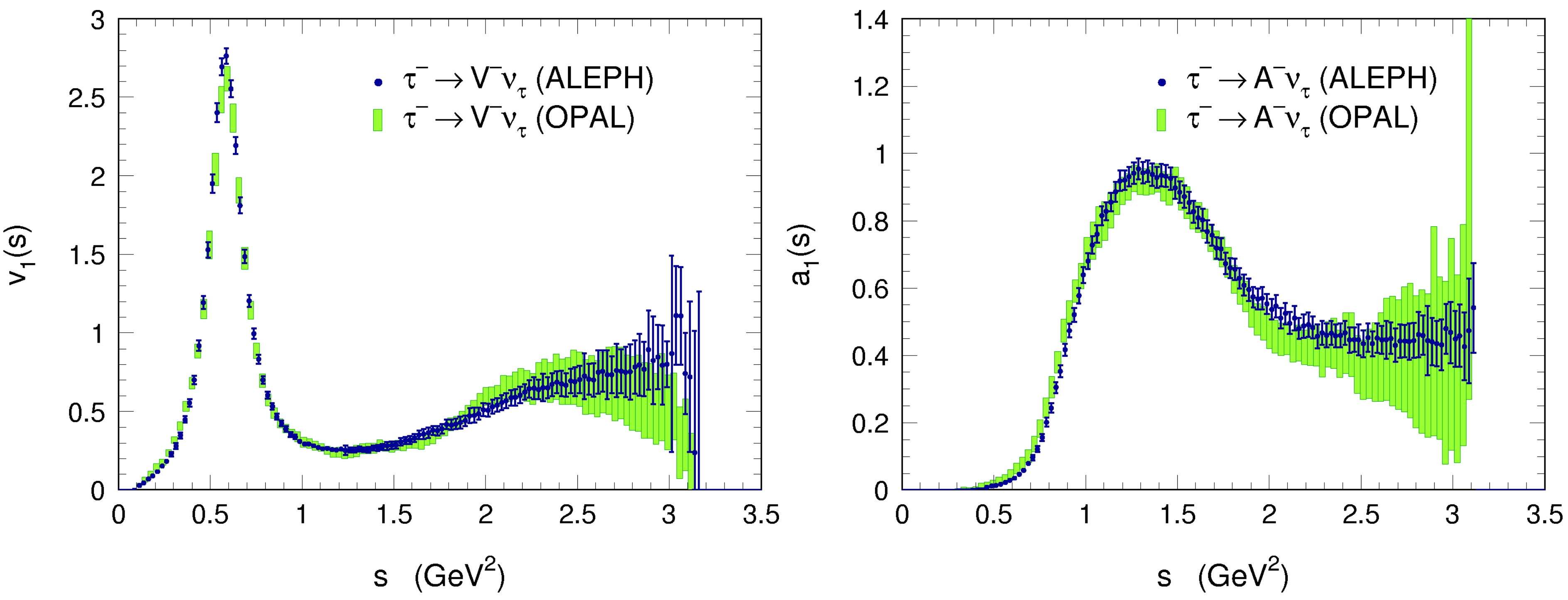
$$\boxed{\sigma^{(I=1)}\left[e^+e^- \to \pi^+\pi^-\right] = \frac{4\pi\alpha^2}{s} \upsilon\left[\tau^- \to \pi^-\pi^0\nu_\tau\right]}$$

Tau Branching Fractions measured by ALEPH

mode	B [%]		
e	17.837 ± 0.080		
μ	17.319 ± 0.077		
π^-	10.828 ± 0.105	A	
$\pi^-\pi^0$	25.471 ± 0.129	V	
$\pi^-2\pi^0$	9.239 ± 0.124	A	
$\pi^-3\pi^0$	0.977 ± 0.090	V	
$\pi^-4\pi^0$	0.112 ± 0.051	A	
$\pi^-\pi^-\pi^+$	9.041 ± 0.097	A	
$\pi^-\pi^-\pi^+\pi^0$	4.590 ± 0.086	V	
$\pi^-\pi^-\pi^+2\pi^0$	0.392 ± 0.046	A	
$\pi^-\pi^-\pi^+3\pi^0$	0.013 ± 0.010	V	estim
$3\pi^-2\pi^+$	0.072 ± 0.015	A	
$3\pi^-2\pi^+\pi^0$	0.014 ± 0.009	V	
$\pi^-\pi^0\eta$	0.180 ± 0.045	V	
$(3\pi)^-\eta$	0.039 ± 0.007	A	CLEO
$a_1^-(\to \pi^-\gamma)$	0.040 ± 0.020	A	estim
$\pi^-\omega(\to \pi^0\gamma, \pi^+\pi^-)$	0.253 ± 0.018	V	
$\pi^-\pi^0\omega(\to \pi^0\gamma, \pi^+\pi^-)$	0.048 ± 0.009	A	+ CLEO
$(3\pi)^-\omega(\to \pi^0\gamma, \pi^+\pi^-)$	0.003 ± 0.003	V	CLEO
K^-K^0	0.163 ± 0.027	V	
$K^-\pi^0K^0$	0.145 ± 0.027	(94^{+6}_{-8})%A	
$\pi^-K^0\overline{K^0}$	0.153 ± 0.035	(94^{+6}_{-8})%A	← (75±25)% used (CLEO)
$K^-K^+\pi^-$	0.163 ± 0.027	(94^{+6}_{-8})%A	
$(K\overline{K}\pi\pi)^-$	0.050 ± 0.020	(50 ± 50)% A	
K^-	0.696 ± 0.029	S	
$K^-\pi^0$	0.444 ± 0.035	S	
$\overline{K^0}\pi^-$	0.917 ± 0.052	S	
$K^-2\pi^0$	0.056 ± 0.025	S	
$K^-\pi^+\pi^-$	0.214 ± 0.047	S	
$\overline{K^0}\pi^-\pi^0$	0.327 ± 0.051	S	
$(K3\pi)^-$	0.076 ± 0.044	S	
$K^-\eta$	0.029 ± 0.014	S	
$K^-\omega$	0.067 ± 0.021	S	
$K^{*-}\eta$	0.029 ± 0.009	S	CLEO

Inclusive Vector and Axial-Vector Spectral Functions

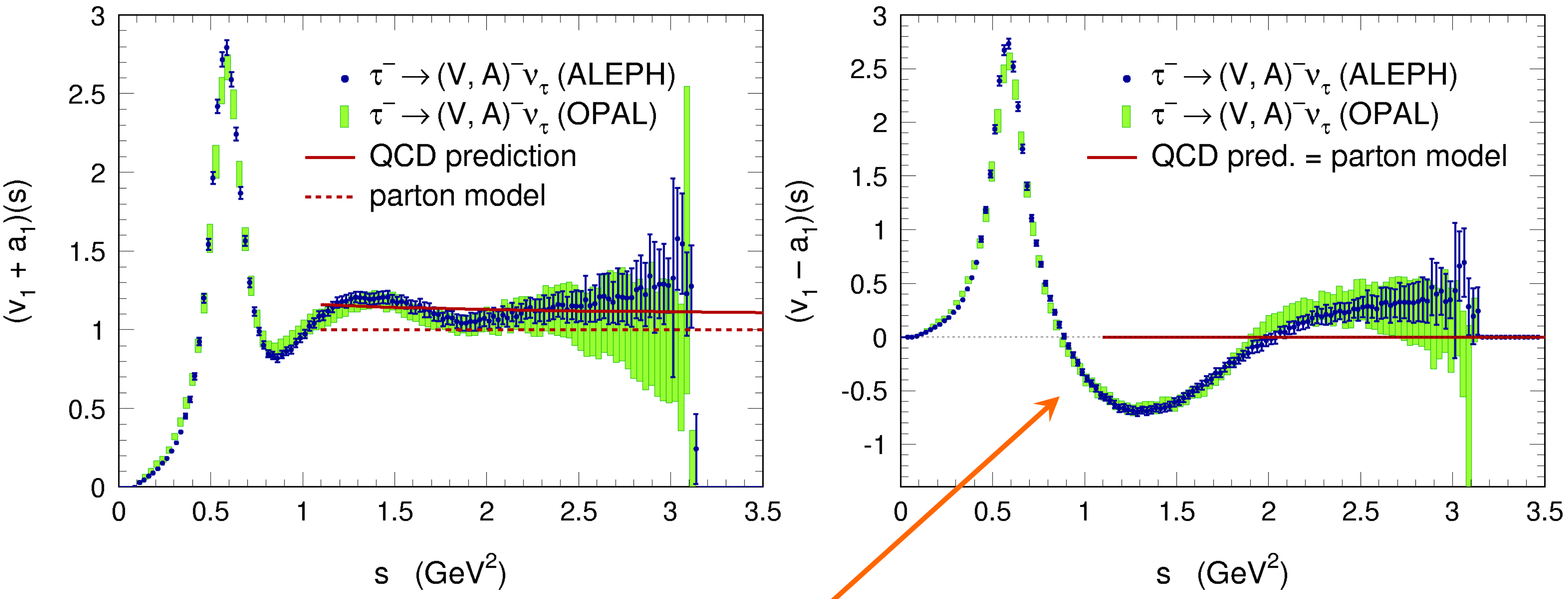
- Results from ALEPH and OPAL — and their comparison



ALEPH, Phys. Rep. 421 (2005)
OPAL, EPJ, C7, 571 (1999)

Inclusive *V+A* and *V–A* Spectral Functions

- Results from ALEPH and OPAL — and their comparison



Of purely nonperturbative origin

ALEPH, Phys. Rep. 421 (2005)
OPAL, EPJ, C7, 571 (1999)

Hadronic Tau Decays and QCD

- ALEPH, CLEO and OPAL carried out tests of QCD using the tau hadronic width
- The opportunity to precisely measure $\alpha_s(m_\tau)$ has triggered theoretical research in particular on the perturbative expansion of the tau hadronic width, affected by a truncation at NNLO = $O(\alpha_s^3)$:
  - contour improvement of fixed-order perturbation theory (FOPT)
  - effective-charge and minimal-sensitivity approaches
  - large β_0 expansion (key word: "*renormalons*")
  - … and combinations of these approaches
- The measurement of inclusive spectral functions lead to direct studies of nonperturbative hadronic properties as a function of the mass scale $s_0 < m_\tau^2$
- Nonperturbative contributions were found to be very small at m_τ^2
- Direct measurements of the running of $\alpha_s(s_0 < m_\tau^2)$ could be performed

Tau and QCD: Naïve Considerations

- The tau hadronic width:

neglecting QCD and EW corrections

$$R_\tau = \frac{\Gamma\left(\tau^- \to \text{hadrons } \nu_\tau\right)}{\Gamma\left(\tau^- \to e^- \bar{\nu}_e \nu_\tau\right)} = \frac{1 - B_e - B_\mu}{B_e} = \frac{1}{B_e^{\text{universal}}} - 1.9726 = N_C\left(|V_{ud}|^2 + |V_{us}|^2\right) \approx 3$$

- Separate into appropriate currents: $R_\tau = R_{\tau,V} + R_{\tau,A} + R_{\tau,S}$

Contribution from QCD radiative corrections ≈ 21%

- Using ALEPH branching fractions (and the WA tau lifetime):

$B_e^{\text{universal}} = (17.810 \pm 0.039)$ % [→ 2.2 per mille on absolute measurement !!!]

$B_S = (2.85 \pm 0.11)$ %

$$R_\tau = \mathbf{3.642 \pm 0.021} \qquad R_{\tau,V+A} = 3.482 \pm 0.014$$
$$R_{\tau,V} = 1.787 \pm 0.013 \qquad R_{\tau,V-A} = 0.092 \pm 0.023$$
$$R_{\tau,A} = 1.695 \pm 0.013 \qquad R_{\tau,S} = 0.1603 \pm 0.0064$$

nonperturbative contributions < 3%

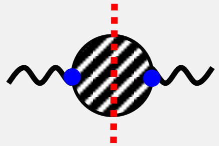
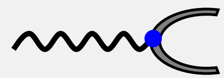
Tau and QCD: Towards a Theoretical Prediction

- The optical theorem relates $\upsilon_J/a_J(s) \propto \mathrm{Im}\,\Pi_{V/A}^{(J)}(s)$, where $\Pi_{V/A}^{(J)}(s)$ are 2-point polarization functions describing the creation of hadrons out of the QCD vacuum

- The vector current has spin J=1 (CVC), while axial-vector has J=0,1

similar in e^+e^- annihilation into hadrons:

$$12\pi\,\mathrm{Im}\,\Pi_\gamma(s) = \frac{\sigma^{(0)}[e^+e^- \to \mathrm{hadrons}]}{\sigma^{(0)}[e^+e^- \to \mu^+\mu^-]} \equiv R(s)$$

Im[] $\propto$ |  hadrons $|^2$

- The tau hadronic width for

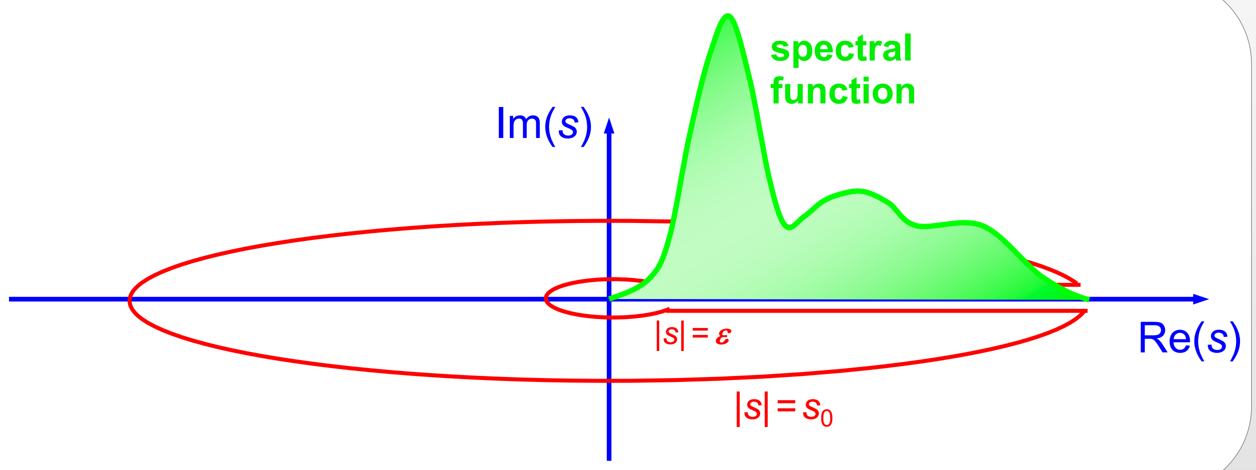
Known radiative correction of order 2%

$$R_{\tau,U}(s_0) = 12\pi S_{EW} \int_0^{s_0} \frac{ds}{s_0}\left(1-\frac{s}{s_0}\right)^2 \left[\left(1+2\frac{s}{s_0}\right)\mathrm{Im}\,\Pi_U^{(1)}(s+i\varepsilon) + \mathrm{Im}\,\Pi_U^{(1)}(s+i\varepsilon)\right]$$

Tau and QCD: The Cauchy Trick

- Problem: $\mathrm{Im}\,\Pi_{V/A}^{(J)}(s)$ contains hadronic physics that cannot be predicted in QCD
- However, owing to the analyticity of $\Pi(s)$, one can use Cauchy's theorem:

$$R(s_0) \propto \int_0^{s_0} ds\, \underbrace{w(s)}_{\text{kinematic factor}}\, \mathrm{Im}\,\Pi(s+i\varepsilon) \quad \Leftrightarrow \quad -\frac{1}{2i}\oint_{|s|=s_0} ds\, w(s)\Pi(s)$$



- The contour integral can be solved using perturbative QCD for sufficiently large s_0

Tau and QCD: The Operator Product Expansion

- Full theoretical ansatz, including nonperturbative operators via the OPE:
  (in the following: $a_s = \alpha_s / \pi$)

$$R_{\tau,U}(s_0) \propto |V_{\mathrm{CKM}}|^2 S_{\mathrm{EW}} \left(1 + \delta^{(0)} + \delta'_{\mathrm{EW}} + \delta_U^{(2,m_q)} + \sum_{D=4,6,\ldots} \delta_U^{(D)} \right)$$

EW correction:

$\simeq 0.0010$ (neglected)

Perturbative quark-mass terms:

$$\sum_{i,j=u,d,s} C_{U,ij}(s_0) \frac{m_i(s_0) \cdot m_j(s_0)}{s_0}$$

Perturbative contribution

$$-2\pi i \oint_{|s|=s_0} \frac{ds}{s_0} \left(1 - 2\frac{s}{s_0} + 2\left(\frac{s}{s_0}\right)^3 - \left(\frac{s}{s_0}\right)^4 \right) \cdot D(s)$$

where: $D(s) = \dfrac{1}{4\pi^2} \sum_{n=0}^{\infty} \tilde{K}_n(\xi) a_s^n(-\xi s)$

Adler function to avoid unphysical subtractions:

$$D(s) \equiv -s \frac{d\Pi(s)}{ds}$$

Nonperturbative contribution

$$\delta_U^{(D)} = (-1)^D \sum_{\dim O = D} C_U(s,\mu) \frac{\langle O(\mu) \rangle_U}{s_s^{D/2}}$$

The Perturbative Prediction

- Perturbative prediction of Adler function given to NNLO, but how should one best solve the contour integral $A^{(n)}(a_s)$ occurring in the prediction of R_τ?

$$\delta^{(0)} = \sum_n \tilde{K}_n(\xi) A^{(n)}(a_s) \qquad \text{(1)}$$

Perturbative coefficients of Adler function series

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} d\varphi \left(1 + 2e^{i\varphi} - 2e^{i3\varphi} - e^{i4\varphi}\right) \cdot a_s^n(\xi s_0 e^{i\varphi})$$

- Complex s dependence of a_s driven by running:

$$\frac{da_s}{d\ln s} = \beta(a_s) = -a_s^2 \sum_{n=0}^{\infty} \beta_n a_s^n$$

RGE β-function, known to n=3:

$$\beta_0 = 2.25,\ \beta_1 = 4$$
$$\beta_2(\overline{\mathrm{MS}}) \approx 10.1,\ \beta_3(\overline{\mathrm{MS}}) \approx 47.2$$

In practice, use Taylor development around fix point and step-wise (Runge-Kutta) integration to solve contour integral

$$a_s(s) = a_s(s_0) - \beta_0 \ln\frac{s}{s_0} a_s^2(s_0) + \ldots$$

Four Ways to Solve Eq. (1) : FOPT and CIPT …

- **Fixed-order perturbation theory (FOPT):** insert Taylor expansion in Eq. (1), evaluate integral, collect all terms of equal powers in a_s, and reject all powers $n > 4$, leads to:

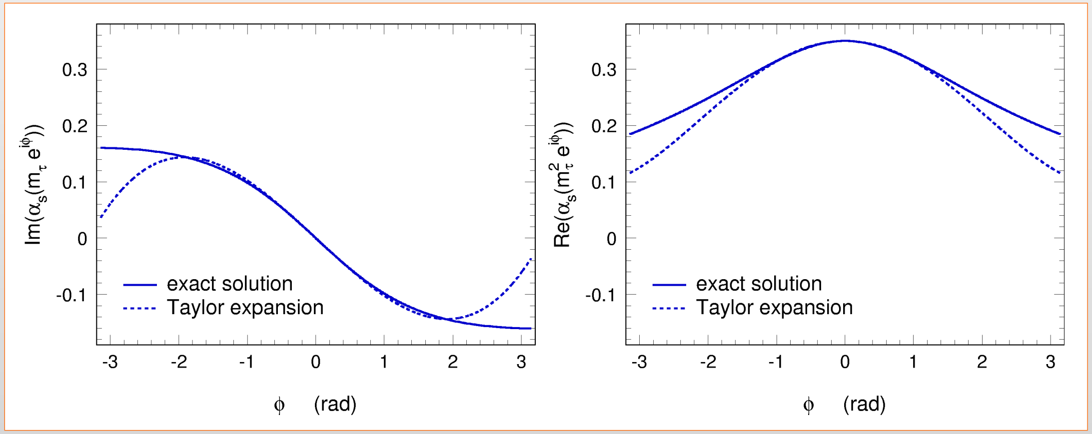
$$\delta^{(0)} = \sum_{n=0}^{4} \left(\tilde{K}_n(\xi) + g_n(\xi) \right) a_s^n(\xi s_0) \qquad (2)$$

All terms known to n=3, unknown β_4 coefficient only enters at n=6

$$\delta^{(0)}_{\xi=1} = a_s(s_0) + 5.2 \cdot a_s^2(s_0) + 26.4 \cdot a_s^3(s_0) + \left(K_4 + 78.0\right) \cdot a_s^4(s_0)$$
$$+ \left(K_5 + 14.3 K_4 - 392\right) \cdot a_s^5(s_0) + f\left(K_{4-6}, \beta_4\right) \cdot a_s^6(s_0) + \ldots$$

- **Contour-improved fixed-order perturbation theory (CIPT):** perform numerical solution of integral in Eq. (1), which corresponds to keeping all known terms in Eq. (2)

Taylor expansion inaccurate for large positive and negative angles



Four Ways to Solve Eq. (1) : … ECPT …

- Effective-charge perturbation theory (ECPT): express $\delta^{(0)}$ as physical observable ("effective charge") so that it is renormalization scheme (RS) independent:

$$\delta^{(0)} = a_\tau \qquad (3)$$

In ECPT the energy evolution of the observable is identical to β-function evolution (running) of the coupling

The evolution of the effective charge obeys the RGE:

$$\frac{da_\tau}{d\ln s} = \beta_\tau(a_\tau) = -a_\tau^2 \sum_{n=0}^{\infty} \beta_{\tau,n}\, a_\tau^n$$

where the β_τ-function is obtained via variable exchange:

$$\beta_\tau(a_\tau) = \frac{da_\tau}{da_s}\,\beta(a_s)\Big|_{a_s = a_s(a_\tau)} \qquad (4)$$

Using Eq. (2) for a_τ at l.h.s. and r.h.s. of Eq. (4) and equating all coefficients of same orders in a_s gives:

$\beta_{0/1,\tau} = \beta_{0/1}$ (RS-independent)

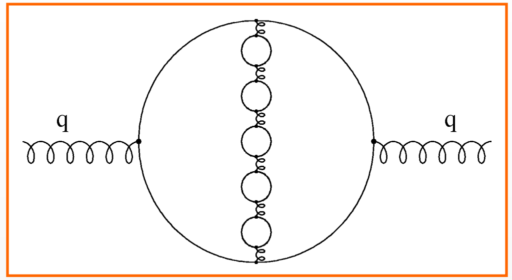
$$\beta_{2,\tau} = \beta_2 - \beta_1(K_2 + g_2)_{\xi=1} + \beta_0[-(K_2+g_2)^2 + (K_3+g_3)]_{\xi=1} \approx 12.3$$

$$\beta_{3,\tau} = \ldots \approx -183 + 4.5K_4$$

$$\beta_{4,\tau} = \ldots \approx -236 + \beta_4 - 40.3K_4 + 6.8K_5$$

Four Ways to Solve Eq. (1) : … Large-β_0 Expansion

- Consider fermion-loop insertion into qq-bar vacuum polarization diagram:



The "large-β_0 expansion" consists of resumming this chain, and only keeping lowest order $(\beta_0 a_s)^n$ terms

- Following this line, $\delta^{(0)}$ can be expanded as:

$$\delta^{(0)}(s) = 1 + a_s \sum_{n=0}^{N} a_s^n \left(d_n^\tau \beta_0^n + \delta_n^\tau \right)$$

neglected in large-β_0 expansion

Results agree not too badly; however, could be by chance: worse agreement for Adler function

$\Rightarrow$ powerful idea, but not suited to a precision analysis

comparison w/ FOPT: $\tilde{K}_1(1) + g_1 = 1, \quad \tilde{K}_2(1) + g_2 = 5.2, \quad \tilde{K}_3(1) + g_3 = 26.4, \quad \tilde{K}_4(1) + g_4 = \tilde{K}_4(1) + 78.0$

Estimating Unknown Higher-Order Coefficient: K_4

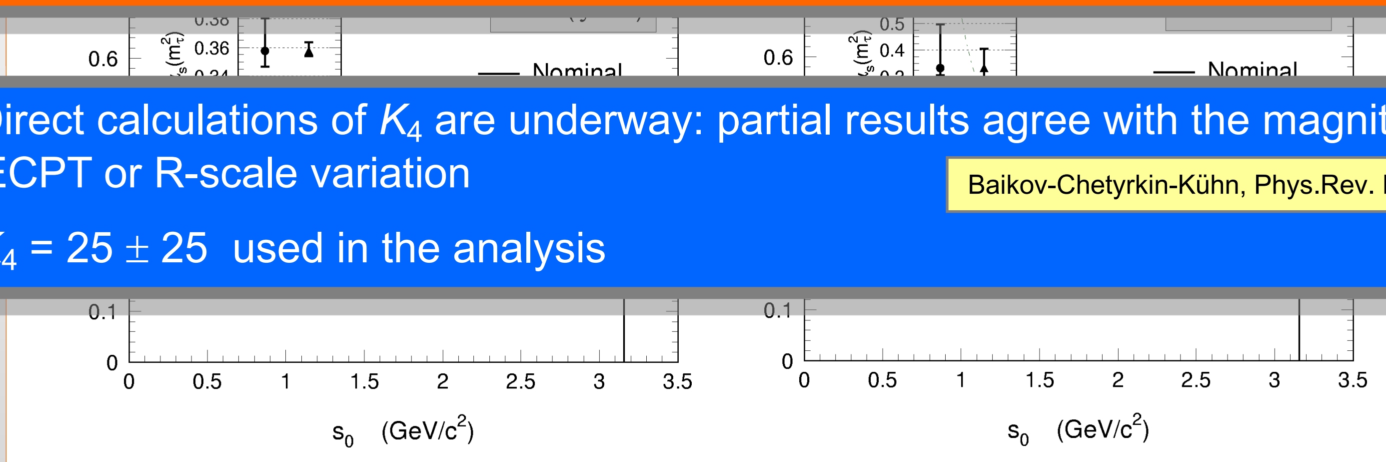
- Trick: modify R-scheme (ECPT instead of FOPT) or R-scale (ξ in FOPT or CIPT) to obtain less convergent perturbative series so that sensitivity to K_4 is enhanced, and K_4 can be obtained from comparison with experimental observable

- However: obtaining K_4 this way assumes negligible K_5 dependence → not true; even K_6 contribution is not negligible !
- The error on the ECPT prediction: K_4 ~ 27 is of similar order
- Similar conclusions for R-scale modification method (methods are <u>correlated</u>)

Kataev-Starshenko, Mod. Phys. Lett. A10, 235 (1995)

Le Diberder, Nucl. Phys. (Proc. Suppl.) B39, 318 (1995)

Detailed discussion in: Davier-Höcker-Zhang, Rev. Mod. Phys., 78 (2006)



- Direct calculations of K_4 are underway: partial results agree with the magnitude found by ECPT or R-scale variation

Baikov-Chetyrkin-Kühn, Phys.Rev. D67, 074026 (2003)

- K_4 = 25 $\pm$ 25 used in the analysis

Comparison of the Perturbative Methods

- Predictions of $\delta^{(0)}$ for the various perturbative methods and orders:

Pert. Method	$n=1$	$n=2$	$n=3$	δ^0 $(n=4)^{(*)}$	$(n=5)^{(*)}$	$(n=6)^{(*)}$	$\sum_{n=1}^{4\,(*)}$	$\sum_{n=1}^{6\,(*)}$
FOPT ($\xi=1$)	0.1114	0.0646	0.0365	0.0159	0.0010	−0.0086	0.2283	0.2208
CIPT (Taylor RGE, $\xi=1$)	0.1573	0.0317	0.0126	0.0042	0.0011	0.0001	0.2058	0.2070
CIPT (full RGE, $\xi=1$)	0.1524	0.0311	0.0129	0.0046	0.0013	0.0002	0.2009	0.2025
CIPT (full RGE, $\xi=0.4$)	0.2166	−0.0133	0.0006	−0.0007	0.0010	−0.0007	0.2032	0.2048
ECPT	0.1442	0.2187	−0.1195	−0.0344	−0.0160	−0.0120	0.2090	0.1810
Large-β_0 expansion	0.1114	0.0635	0.0398	0.0241	0.0155	0.0093	0.2388	0.2636

(*)assuming K_4=25 and geometric growth for the unknown coefficients $K_{n>4}$ and $\beta_{n>3}$

- CIPT converges faster than FOPT (minimal sensitivity may be reached by FOPT)
  [while somewhat counter intuitive: the sensitivity to K_4 is similar for FOPT and CIPT]
- the difference between FOPT and CIPT is not due to the Taylor approx. in FOPT, but because FOPT erases the known higher orders of the Taylor series
- ECPT and Large-β_0 exp. have slow convergence and hence large truncation errors
- CIPT has best convergence and should be used to predict $\delta^{(0)}$
- the difference in $\delta^{(0)}$ between CIPT and FOPT is not a useful measure of the perturbative uncertainty: it is artificially increased due to the neglect of large known terms in FOPT !

Quark-mass and Nonperturbative Contributions

- Although $\delta^{(0)}$ is the main contribution, and the one that provides the sensitivity to α_s, we must not forget the other terms in the OPE:

- D=2 (mass dimension): quark-mass terms are $\propto m_q^2/s_0$, which is negligible for $q=u,d$

- D=4: dominant contributions from gluon- and quark-field condensations (gluon condensate $\langle\alpha_s GG\rangle$ is determined from data)

- D=6: dominated by large number of four-quark dynamical operators that – assuming factorization – can be reduced to an effective scale-independent operator $\rho\alpha_s\langle q\bar{q}\rangle^2$ that is determined from data

- D=8: structure of quark-quark, quark-gluon and four-gluon condensates absorbed in single phenomenological operator $\langle O_8\rangle$ determined from data

→ **Four unknowns: $\alpha_s(s_0)$, $\langle\alpha_s GG\rangle$, $\rho\alpha_s\langle q\bar{q}\rangle^2$, $\langle O_8\rangle$**

Shifman-Vainshtein-Zakharov,
NP B147, 385, 448, 519 (1979)

Braaten-Narison-Pich,
NP B373, 581 (1992)

Spectral Moments

- Exploit shape of spectral functions to obtain additional experimental information:

$$R_{\tau,U}^{k\ell}(s_0) = \int_0^{s_0} ds \left(1 - \frac{s}{s_0}\right)^k \left(\frac{s}{s_0}\right)^\ell \frac{dR_{\tau,U}(s_0)}{ds}$$

Le Diberder-Pich,
PL B289, 165 (1992)

high-mass tail of spectral functions suppressed (enhanced) by large k (ℓ) moments; the moments exhibit strong correlations, which have to be taken into account in the fit

- Theory prediction very similar to R_τ:

$$R_{\tau,U}^{k\ell}(s_0) \propto \left|V_{\mathrm{CKM}}\right|^2 S_{\mathrm{EW}} \left(1 + \delta^{(0,k\ell)} + \delta'_{\mathrm{EW}} + \delta_U^{(2,m_q,k\ell)} + \sum_{D=4,6,\ldots} \delta_U^{(D,k\ell)}\right)$$

with corresponding perturbative and nonperturbative OPE terms

- For practical reasons it is convenient to normalize the spectral moments:

$$D_{\tau,U}^{k\ell}(s_0) = \frac{R_{\tau,U}^{k\ell}(s_0)}{R_{\tau,U}(s_0)}$$

ALEPH Fit Results

- The combined fit of R_τ and spectral moments (k=1, ℓ=0,1,2,3) gives (at $s_0=m_\tau^2$):

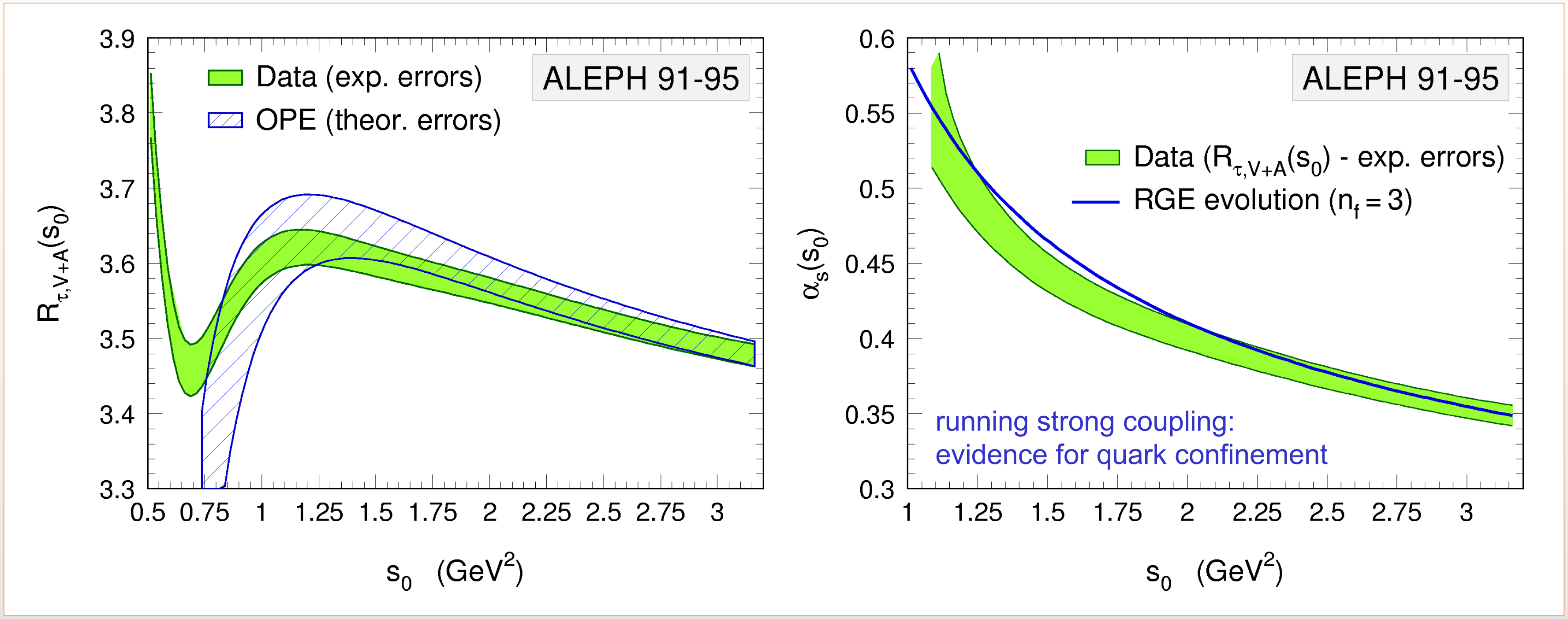
Parameter	Vector (V)	Axial-Vector (A)	$V + A$
$\alpha_S(m_\tau^2)$ (CIPT)	$0.355 \pm 0.008 \pm 0.009$ (*)	$0.333 \pm 0.009 \pm 0.009$	$0.350 \pm 0.005 \pm 0.009$
$\alpha_S(m_\tau^2)$ (FOPT)	$0.331 \pm 0.006 \pm 0.012$	$0.327 \pm 0.007 \pm 0.012$	$0.331 \pm 0.004 \pm 0.012$
$\delta^{(2)}$ (CIPT)	$(-3.3 \pm 3.0) \times 10^{-4}$	$(-5.1 \pm 3.0) \times 10^{-4}$	$(-4.4 \pm 2.0) \times 10^{-4}$
$\langle a_s GG \rangle$ (GeV^4) (CIPT)	$(0.4 \pm 0.3) \times 10^{-2}$	$(-1.3 \pm 0.4) \times 10^{-2}$	$(-0.5 \pm 0.3) \times 10^{-2}$
$\delta^{(4)}$ (CIPT)	$(4.1 \pm 1.2) \times 10^{-4}$	$(-5.7 \pm 0.1) \times 10^{-3}$	$(-2.7 \pm 0.1) \times 10^{-3}$
$\delta^{(6)}$ (CIPT)	$(2.85 \pm 0.22) \times 10^{-2}$	$(-3.23 \pm 0.26) \times 10^{-2}$	$(-2.1 \pm 2.2) \times 10^{-3}$
$\delta^{(8)}$ (CIPT)	$(-9.0 \pm 0.5) \times 10^{-3}$	$(8.9 \pm 0.6) \times 10^{-3}$	$(-0.3 \pm 4.8) \times 10^{-4}$
Total δ_{NP} (CIPT)	$(1.99 \pm 0.27) \times 10^{-2}$	$(-2.91 \pm 0.20) \times 10^{-2}$	$(-4.8 \pm 1.7) \times 10^{-3}$
χ^2/DF (CIPT)	0.52	4.97	3.66

(*)second errors given are theoretical

- correlations between α_s and nonperturbative parameters between 0.3 and 0.6
- theoretical error on α_s dominated by unknown K_4 (used: 25 ± 25) and R-scale ($m_\tau^2 \pm 2$ GeV^2)
- still keep additional syst error from difference between FOPT and CIPT
- agreement between independent vector and axial-vector fits
- nonperturbative contributions negligible for V+A, significant, but small (and opposite) for V, A

Running within the Tau Spectrum

- The spectral functions allow to measure $R_\tau(s_0 < m_\tau^2)$; the previous fit allows us to compare the measurement to the theoretical expectation assuming RGE running



- assuming quark-hadron duality to hold, this is a direct evidence for running strong coupling
- also: test of stability of OPE prediction, and hence of trustworthiness of $\alpha_s(m_\tau^2)$ fit result

Assessment of $\alpha_s(m_\tau^2)$ and Evolution to M_Z^2

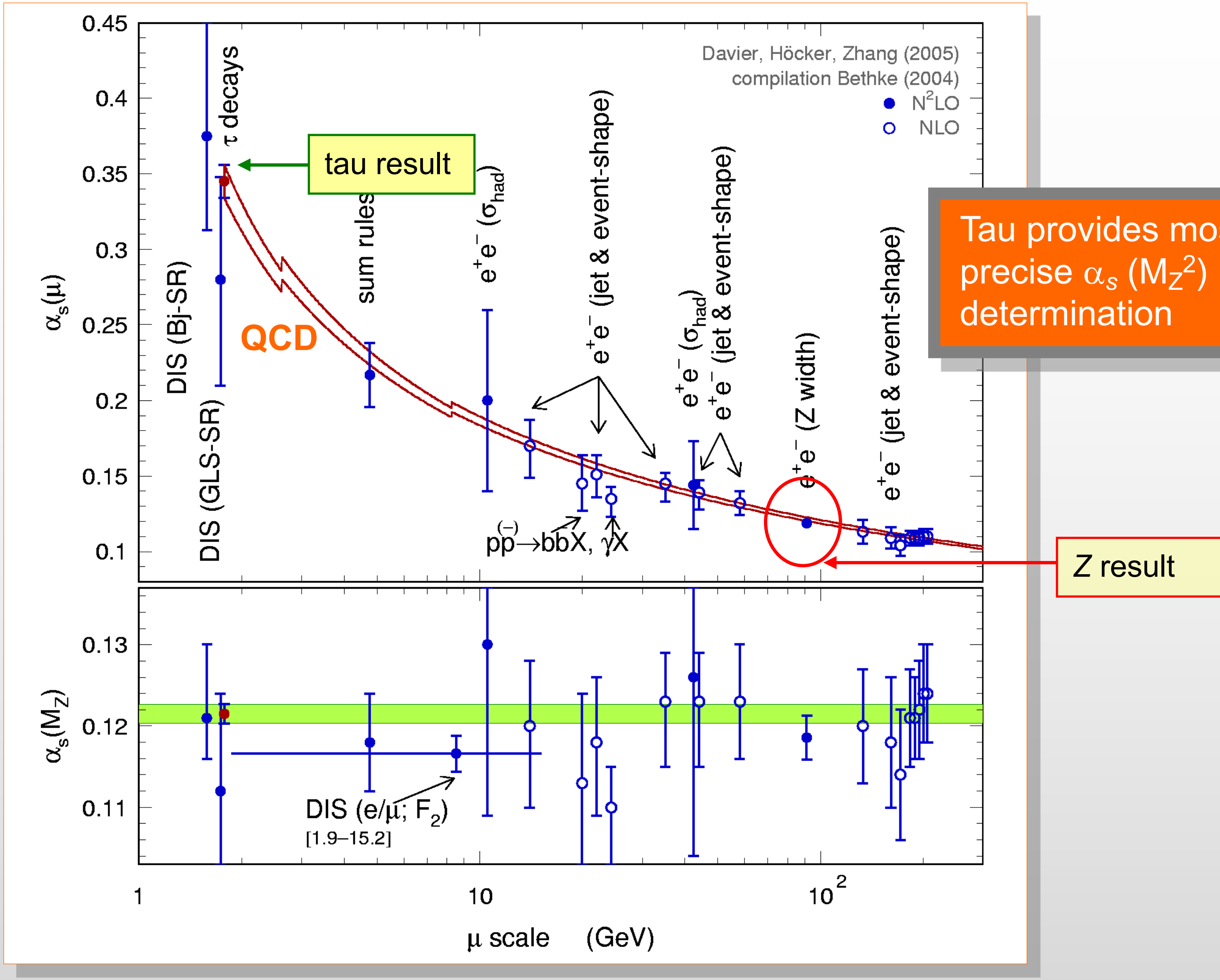
- Experimental input (1): B_e^{uni} = (17.818 ± 0.032) % WA / ALEPH
  (2): $R_{\tau,S}$ = 0.1686 ± 0.0047 WA / ALEPH
  ⇒ $R_{\tau,V+A}$ = 3.471 ± 0.011
  still use ALEPH spectral functions
- Theory framework: tests ⇒ CIPT method preferred, no CIPT-vs.-FOPT syst.
- The fit to the *V+A* data gives:

$$\alpha_s(m_\tau^2) = 0.345 \pm 0.004_{\text{exp}} \pm 0.009_{\text{theo}}$$

- Using 4-loop QCD β-function and 3-loop quark-flavour matching yields:

$$\begin{aligned}\alpha_s(M_Z^2) &= 0.1215 \pm 0.0004_{\text{exp}} \pm 0.0010_{\text{theo}} \pm 0.0005_{\text{evol}} \\ &= 0.1215 \pm 0.0012\end{aligned}$$

Overall Comparison



Tau provides most precise α_s (M_Z^2) determination

Test of Asymptotic Freedom

- Need precise determinations of α_s at widely different scales: here m_τ and M_Z

- Tau hadronic width: $\alpha_s(m_\tau^2) = 0.345 \pm 0.010$ error dominated by theory

  Z hadronic width: $\alpha_s(M_Z^2) = 0.1186 \pm 0.0027$ global EW fit; experiment

  ➔ **$\alpha_s(Q^2)$ runs**

- RGE evolved tau result $\alpha_{s,\tau}(M_Z^2) = 0.1215 \pm 0.0012$

  ➔ **$\alpha_s(Q^2)$ runs exactly as predicted by QCD**

- Defining evolution estimator $r(s_1,s_2) = 2\,(\alpha_s^{-1}(s_1) - \alpha_s^{-1}(s_2)) \,/\, (\ln s_1 - \ln s_2)$

  $r_{\rm exp}(m_\tau^2, M_Z^2) = 1.405 \pm 0.053$

  $r_{\rm QCD}(m_\tau^2, M_Z^2) = 1.353 \pm 0.006$

  ➔ **Most precise test of asymptotic freedom in QCD**

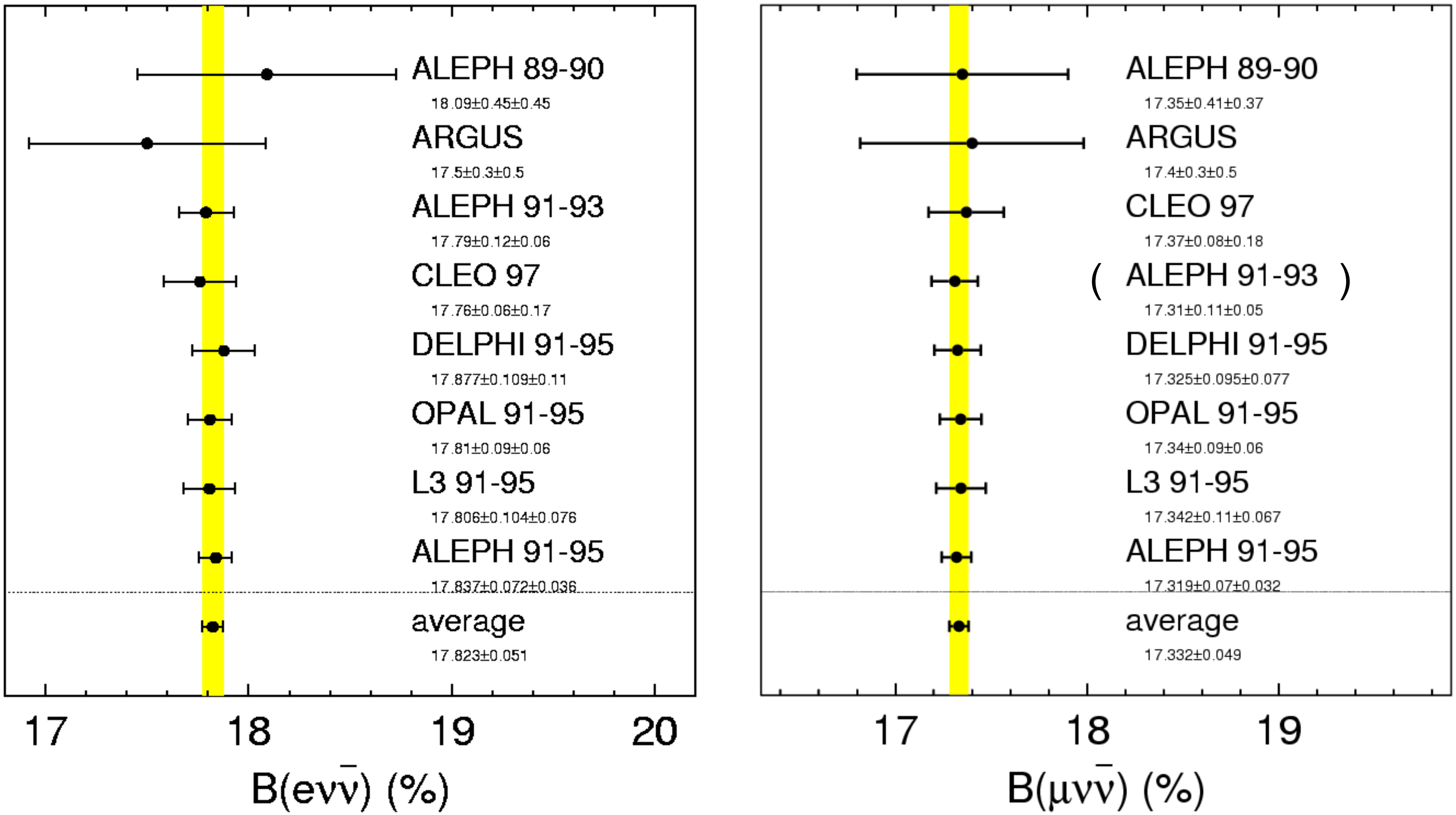
Conclusions

- Hadronic tau decays: one of the most powerful testing ground for QCD
- Despite low scale, dominated by perturbative QCD with good fortune:

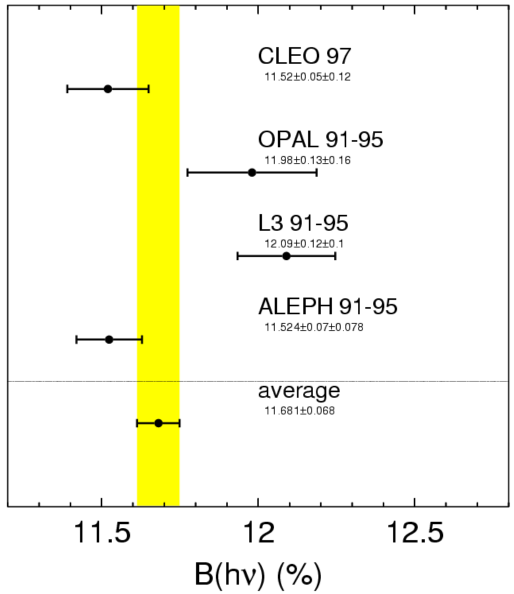
  analyticity, weight factors, inclusiveness, *V*, *A*, N2LO + good estimate of N3LO, systematic treatment of nonperturbative contributions (OPE), lowest-order NP terms suppressed
- Detailed studies on the perturbative series and its truncation: CIPT method preferred
- Good quality data for the spectral functions and their normalising branching fractions (ALEPH). Data from other experiments used to improve precision
- Joint QCD fits of total rate and spectral moments permit the extraction of the strong coupling constant and the main NP contributions
- $\alpha_s(m_\tau^2)$, extrapolated at M_Z scale, yields the most precise value of $\alpha_s(M_Z^2)$
- $\alpha_s(m_\tau^2)$ and $\alpha_s(M_Z^2)$ from Z decays provide the most precise test of asymptotic freedom in QCD

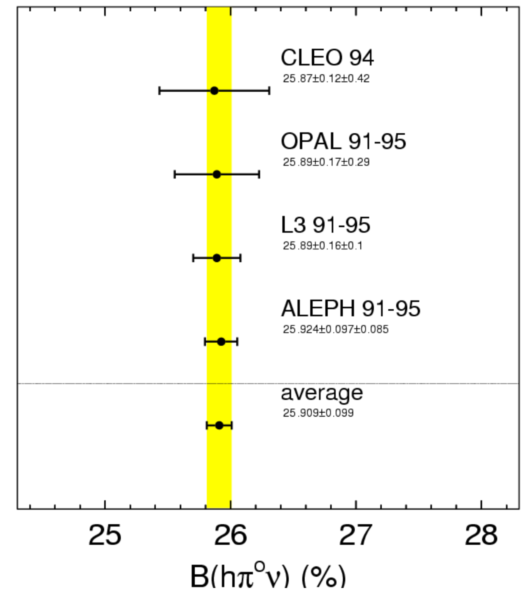
backup

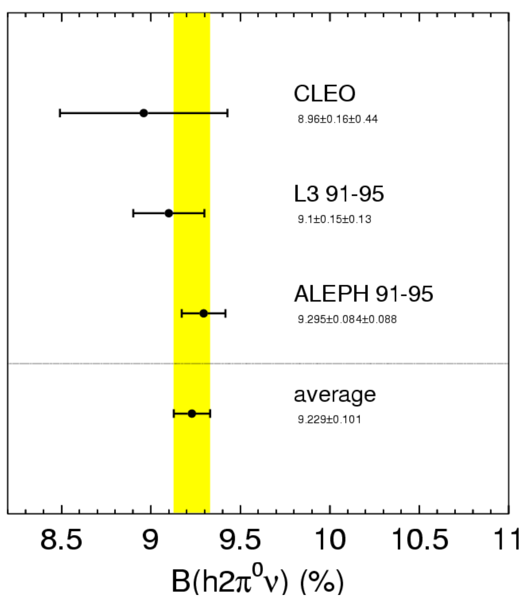
Tau Branching Fractions: Leptonic modes

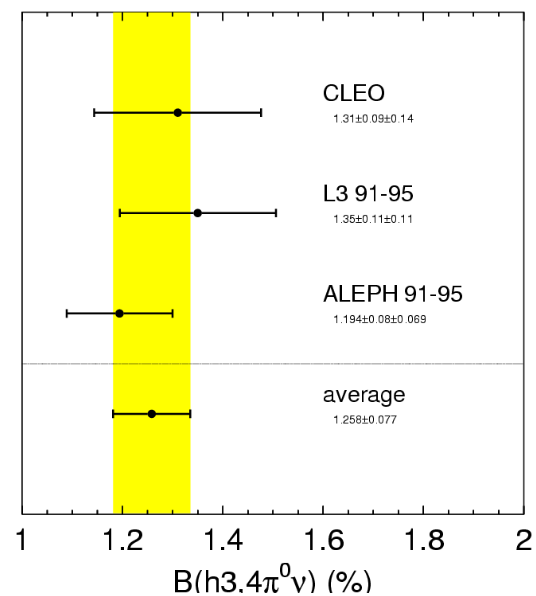


Hadronic modes (1-prong)

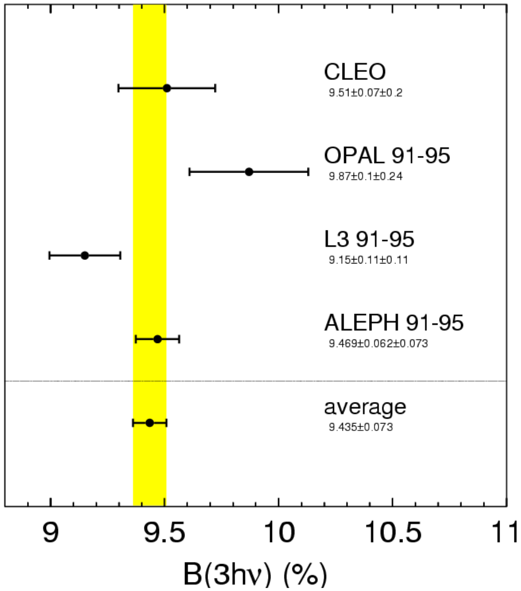


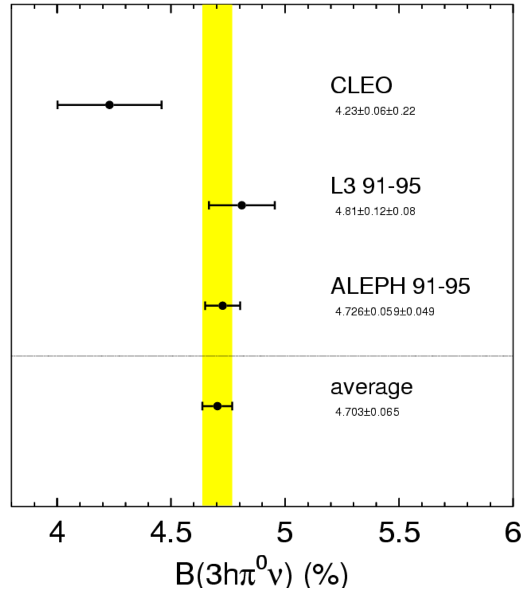


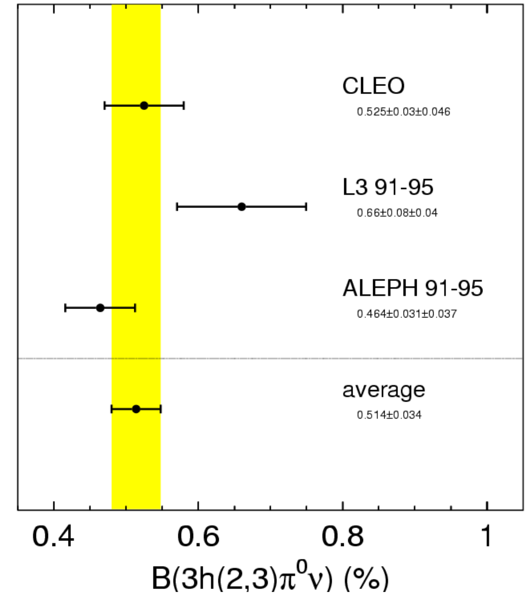


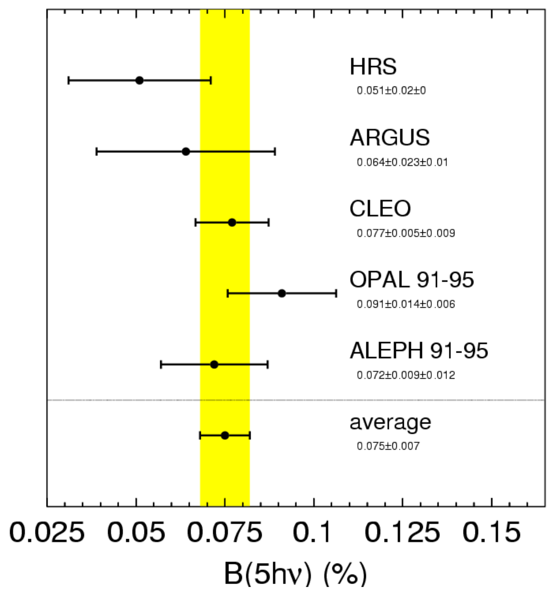


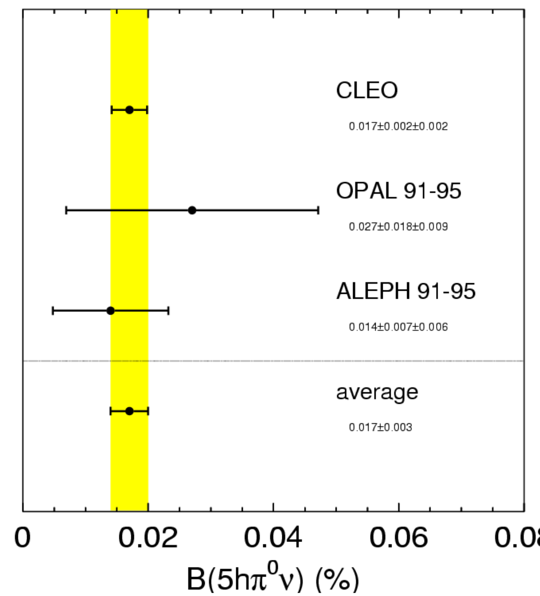
Hadronic modes (3,5-prong)











Lepton Universality in the Charged Current

B_e and B_μ

$$g_\mu / g_e = 0.9991 \pm 0.0033$$

B_e , B_μ and τ_τ WA (290.6 ± 1.1) fs

$$g_\tau / g_\mu = 1.0009 \pm 0.0023 \pm 0.0019 \pm 0.0004$$

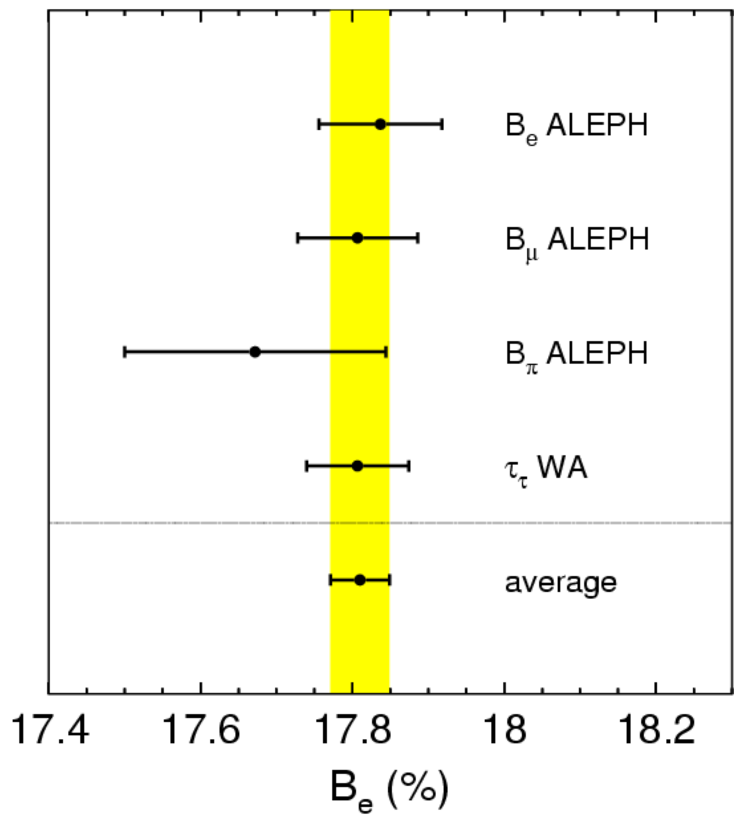
$$g_\tau / g_e = 1.0001 \pm 0.0022 \pm 0.0019 \pm 0.0004$$

(B_e , B_μ) (τ_τ) (m_τ)

B_π and τ_τ WA

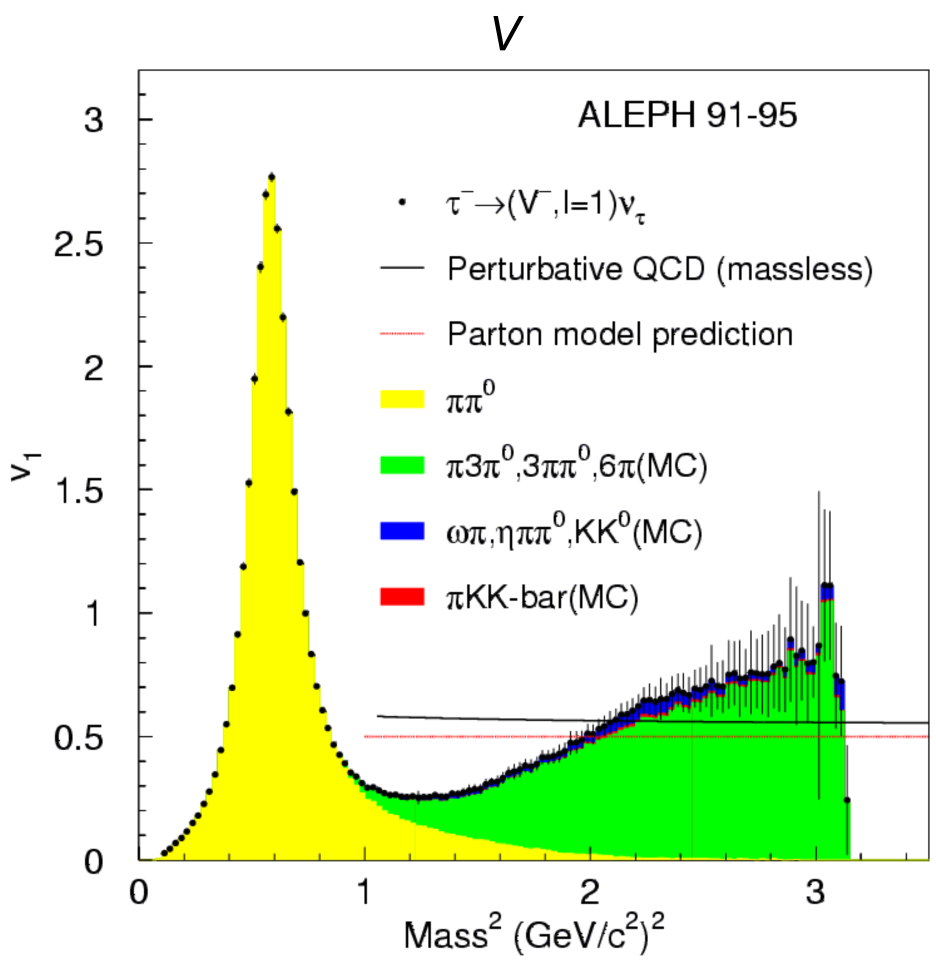
$$g_\tau / g_\mu = 0.9962 \pm 0.0048 \pm 0.0019 \pm 0.0004$$

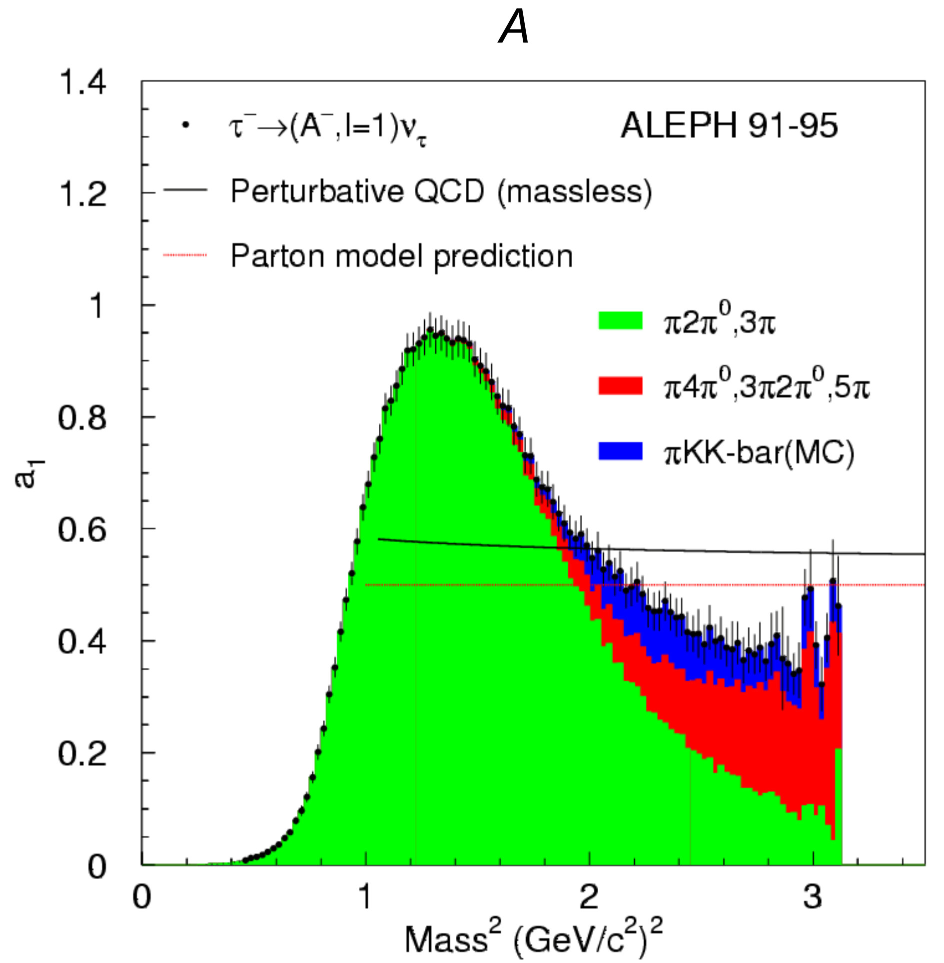
$$\pm 0.0007$$ (radiative corrections)



$B_e^{uni} = (17.810 \pm 0.039)$ %

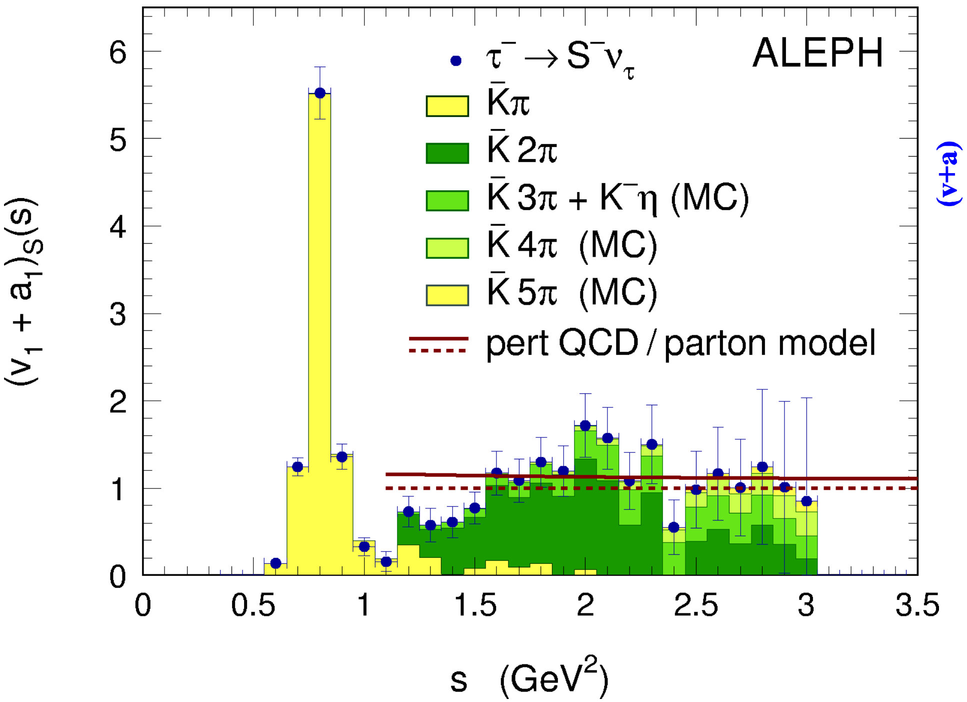
Spectral Functions: *V*, *A*

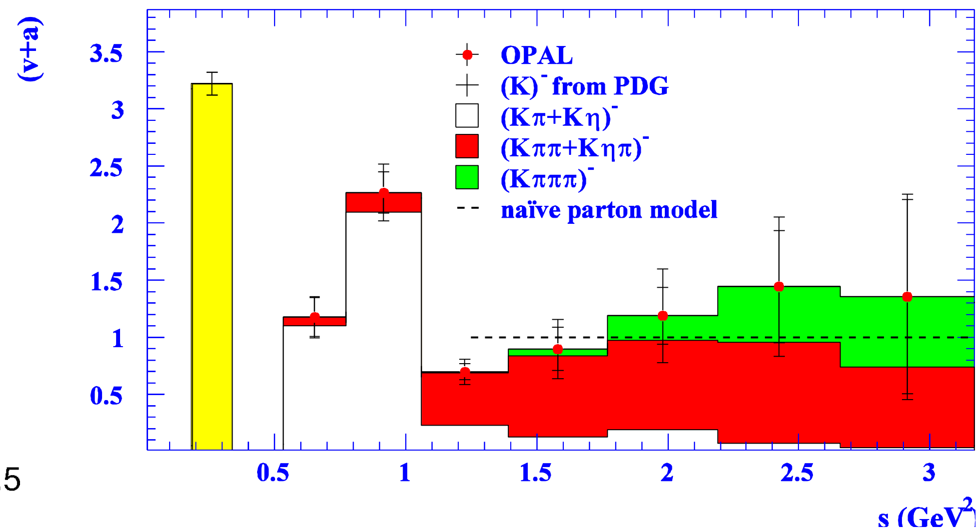




Inclusive S=1 Spectral Functions

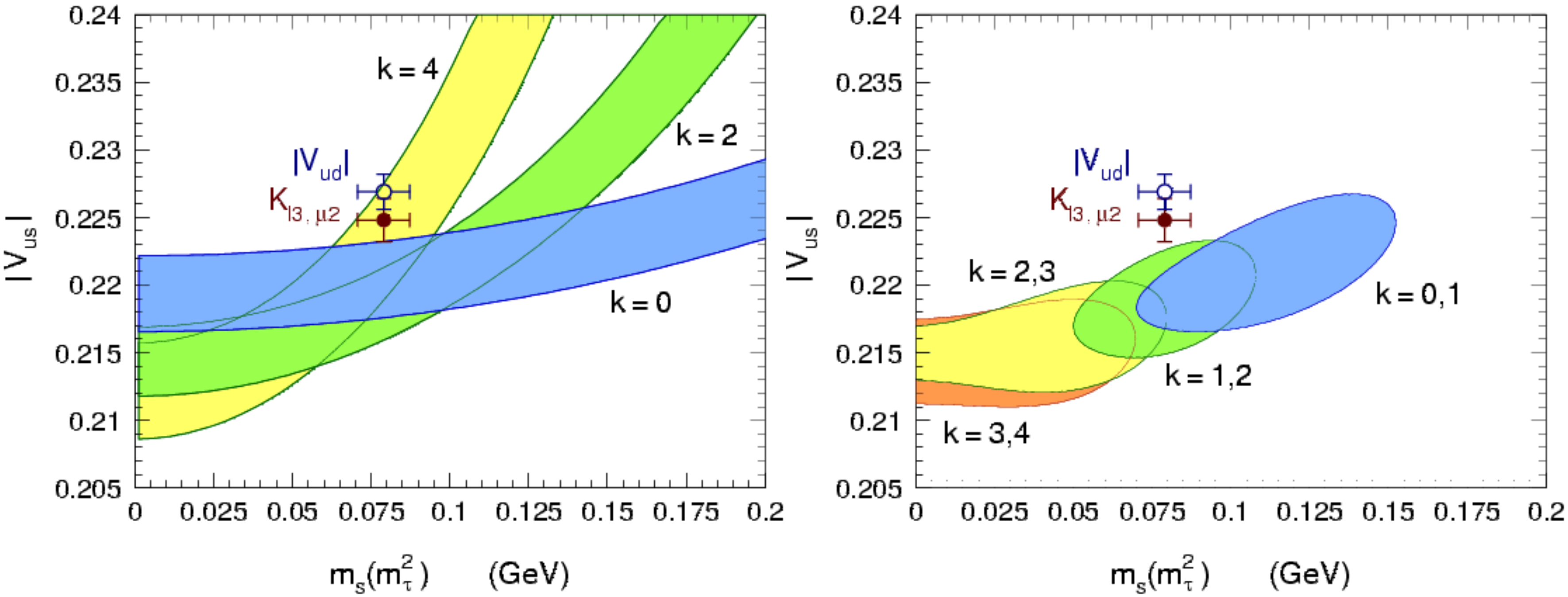
- Results from ALEPH and OPAL — and their comparison





ALEPH, EPJ C11, 599 (1999)
OPAL, EPJ C35, 437 (2004)

V_{us} and m_s determinations from strange SF



Asymptotic Series and Borel Transform

- Asymptotic perturbative series in field theories (be they abelian or not) are divergent for large orders N:

$$R_N = \sum_{n=0}^{N} r_n \alpha^{n+1}$$

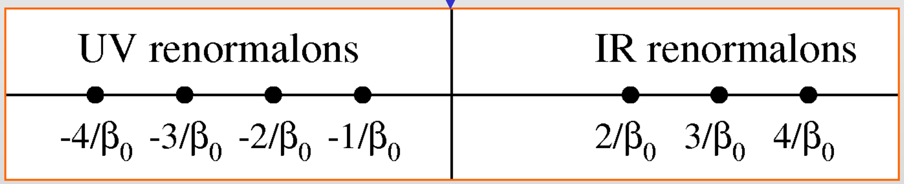
  - one expects factorial growth of perturbative coefficient $r_N \sim n!$
  - divergence may indicate nonperturbative structure of vacuum
  - asymptotic series have order of best convergence (= point of minimal sensitivity), where series should be truncated (below this point: no difference between convergent and divergent series!)

- convenient handle of this divergence: Borel transform

$$B[R](t) \equiv \sum_{n=0}^{\infty} \frac{r_n}{n!} t^n \quad \Rightarrow \quad r_n = \left. \frac{d^n B[R](t)}{dt^n} \right|_{t=0}$$

Factor makes Borel-transformed series much better behaved; <u>but</u>: the divergent behavior of the original series is encoded in singularities of Borel transform

Infrared (IR) renormalons are ~ $(\Lambda/s)^{u>1}$ absorbed in nonperturbative terms of OPE;
Ultraviolet (UV) renormalons cause divergence: apparent UV D=2 term due to series truncation



Singularities (“renormalons”) in Borel plane of Adler function